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F 75

George T. Foster
No. 14

AN ELEMENTARY TREATISE ON ASTRONOMY.

IN TWO PARTS.

THE FIRST CONTAINING,

A CLEAR AND COMPENDIOUS VIEW OF THE THEORY;

THE SECOND,

A NUMBER OF PRACTICAL PROBLEMS.

TO WHICH ARE ADDED,

SOLAR, LUNAR, AND OTHER ASTRONOMICAL TABLES.

BY JOHN GUMMERE, A. M.
Fellow of the American Philosophical Society, and Corresponding Member of the Academy of Natural Sciences, Philadelphia.

SECOND EDITION, ENLARGED AND IMPROVED.

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1837.

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A. WALDIE, PR.

PREFACE.

The object in writing the present treatise has been to give, in a moderate compass, a methodical and scientific exhibition of the elementary principles of Astronomy, and to furnish the student with rules and tables for making some of the more useful and more interesting calculations. The work is divided into two parts; the first containing the theory, and the second, practical problems.

Particular attention has been given to the arrangement of the first part. The different subjects are introduced in such order as to make it unnecessary for the student to anticipate propositions in advance of those which he is studying. The definitions are given as they are wanted in the course of the work, and after previous investigations have served to render them easily understood.

Astronomy, when taken in its whole extent, and with all its different methods, necessarily forms a large treatise. It is not therefore practicable to give those various methods, in a work of the size to which it has been thought proper to limit this. Neither are they important, except to those who devote very decided attention to this interesting science; and they must have recourse to more extended works. Most students are satisfied with obtaining a correct general knowledge of the subject, and of the means by which the principal facts have been discovered or can be established, without entering into all the investigations necessary to render those means the most efficacious in giving precision to the results. In conformity with these views, I have seldom given more than one method of determining any particular fact, and have avoided entering into minute details that did not appear necessary to a proper comprehension of the subject. In the demonstrations, the student is supposed to be acquainted with Algebra, Geometry, Plane and Spherical Trigonometry, and Conic Sections, or at least the properties of the Ellipse. As many persons study Astronomy who have no knowledge of the Differential Calculus, it has not been used, though in a few cases it might have been introduced with advantage.

The Problems in the Second Part are principally for making calculations relative to the Sun, Moon, and Fixed Stars. The Tables of the Sun and Moon, which are used in these, have been abridged from the tables of Delambre and Burckhardt, and reduced to the meridian of Greenwich. Although the quantities are only given to whole seconds and several small equations have been omitted, the places of the sun and moon obtained from these tables will be very nearly correct, for any time within the period to which the tables of Epochs extend. Rules are also given for obtaining the places and motions of the sun and moon for a given time from the Nautical Almanac. Each of the problems is illustrated by one wrought example; and to most of them are added one or two unwrought questions, with the answers annexed, to serve as exercises for the student.

While writing the present treatise, I have had recourse to several of the best modern publications on the subject; among which may be particularly mentioned those of *Vince*, *Woodhouse*, *Playfair*, *Delambre*, *Biot*, and *Laplace*. From these I have adopted the methods which best suited my purpose, making, when it appeared necessary, such modifications in them as the plan of the work required. In the Projection of Eclipses and Occultations, a method is given which is believed to be new, and which renders the operation more simple, without materially affecting the accuracy of the results. An easy method, derived from the former, is also given for tracing the central path of an eclipse of the sun.

In a work of this description, particularly when printed from manuscript, errors must be expected to occur; some proceeding from inadvertencies on the part of the author, and others occurring in the press. Such as have been discovered, which, it is believed, include all that are important, are enumerated at the end of the volume.

JOHN GUMMERE.

ADVERTISEMENT TO THE SECOND EDITION.

The work has undergone a careful revision, and has received considerable additions in both parts. In the appendix to part first, a general analytical investigation of eclipses of the sun, occultations, and transits, has been introduced; and in part second, the formulæ obtained are applied in practical problems for calculating these phenomena. The table of the sun's epochs, commencing with the year 1836, has been improved by the application of Bessel's corrections. The epochs for the preceding years are retained as in the former edition, as several of them are used in examples which it was not deemed necessary to change. The tables of epochs both for the sun and moon have been extended. And tables of the planet Mercury, abridged from Lindenau's tables and adapted to the meridian of Greenwich, have been introduced. These serve to illustrate the method of calculating the place of a planet, and they are sufficiently accurate for determining, nearly, the times of a transit of this planet over the sun's disc. Several other useful tables have also been added.

J. G.

Haverford School, 1mo. 1837.

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The following Alphabet is given in order to facilitate, to the student who is unacquainted with it, the reading of those parts in which the Greek letters are used:

Letters.		Names.
Α	α	Alpha
Β	β ϐ	Beta
Γ	γ ſ	Gamma
Δ	δ	Delta
Ε	ε	Epsilon
Ζ	ζ ζ	Zeta
Η	η	Eta
Θ	ϑ θ	Theta
Ι	ι	Iota
Κ	κ	Kappa
Λ	λ	Lambda
Μ	μ	Mu
Ν	ν	Nu
Ξ	ξ	Xi
Ο	ο	Omicron
Π	ϖ ϖ	Pi
Ρ	ρ ϱ	Rho
Σ	σ ς	Sigma
Τ	τ ꞇ	Tau
Υ	υ	Upsilon
Φ	φ	Phi
Χ	χ	Chi
Ψ	ψ	Psi
Ω	ω	Omega

AN
ELEMENTARY TREATISE
ON
ASTRONOMY.

PART I.

CHAPTER I.

GENERAL PHENOMENA OF THE HEAVENS.

1. *Astronomy* is the science which treats of the appearances, motions, distances, and magnitudes of the heavenly bodies. *Physical astronomy* is that part of the science in which the causes of their motions are considered, and the effects of those causes explained.

2. If, in a clear night, we fix our attention on the heavens, and continue to observe them at intervals for a few hours, we may, without the aid of any instruments, obtain several useful results. It will be seen that the stars retain the same positions with regard to one another; but that their positions with reference to the earth, are continually changing. Those in the eastern part of the heavens become more and more elevated, and others which were not at first visible, come into view, or *rise*. Those in the western part descend lower and lower, till they go out of view, or *set*. In the southern part, some will be observed to come into view, rise to a small elevation, then descend

and go out of view, to the west of their places of rising.*

3. If we direct our attention towards the north, different phenomena present themselves. It will be observed that there are many stars in that part of the heavens, which do not set. Those that are descending, continue to do so till they arrive at certain lowest points, and then begin to ascend. They *appear* to revolve or describe circles about a certain star, which seems to remain stationary. This star is called the *Pole Star.*

All the stars that do not set are called *Circumpolar Stars.*

4. When the pole star is more accurately observed, by the aid of suitable instruments, it ceases to appear stationary. It is found to have an apparent motion in a small circle, around a certain point as a centre, distant about 2° from it. This point is called the *North Pole of the Heavens,* or simply the *North Pole.*

It is about this point as a centre, and not the pole star, that the apparent revolutions of the stars are performed.

5. The stars appear to move, from *east* to *west,* exactly as if attached to the concave surface of a hollow sphere, which revolves on its axis in a space of time nearly equal to 24 hours. This motion, which is common to all the heavenly bodies, is called the *Diurnal Motion.*

6. If we examine the situations of the Moon, on successive nights, we shall find that she changes her position among the stars, and advances from *west* to *east.*

7. The sun also appears to partake of this motion, *from west* to *east,* relative to the stars. This may be inferred from observing the stars in the west after the sun has set. If our observations be continued for a number of successive evenings, we shall find that the sun continually approaches the stars situated to the eastward of him.

8. Besides the sun and moon, there are ten stars which

---

* Here, and in other parts of the work, unless the contrary is mentioned, the observer is supposed to be in the United States, or in the southern or middle parts of Europe.

are found to change their situations with respect to the other stars, and have a motion among them from *west* to *east*. These are called *Planets*. Five of them, named *Mercury*, *Venus*, *Mars*, *Jupiter*, and *Saturn*, are visible to the naked eye, and were known to the ancients.

The other five are named, *Uranus*,* *Vesta*, *Juno*, *Ceres*, and *Pallas*. None of them can be seen without the aid of a telescope, except Uranus; which under favourable circumstances is discernible by a good eye. They have not been long known.

The stars which do not sensibly change their situations, with respect to one another, are called *Fixed Stars*.

9. There are some stars that occasionally appear in the heavens, which have a motion among the fixed stars, and only continue visible for a few weeks or months. They are frequently accompanied by a faint brush of light, called a tail. These are named *Comets*.

10. If a person, placed on the margin of the sea, observe a vessel receding from the land, he will first lose sight of the hull, then of the lower parts of the sails, and lastly of the topsails. This will be the case, in whatever direction the vessel pursues her course, or in whatever part of the earth the observation is made. We hence conclude that the surface of the sea is convex. It is also well known, that vessels have sailed entirely round the earth, in various directions. From these circumstances it is inferred that the form of the earth is globular. It will be shown in a subsequent chapter that the earth is very nearly, though not exactly, a sphere.

11. In astronomical investigations, except when great accuracy is required, it is usual to consider the earth as a perfect sphere.

12. The angular distance between any two of the fixed stars, is found to be the same, in whatever part of the earth's surface the observation is made. It follows, there-

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* The planet Uranus, which was discovered by Dr. Herschel, was by him named Georgium Sidus, in honour of his patron, King George III. By the French it was called Herschel. It is now generally known by the name given in the text.

fore, that the distance of the stars from the earth is so great, that the earth's diameter, compared with it, is insensible.

13. It is not supposed that the fixed stars are all at the same distance from the earth. But since their distances are so immensely great that the most accurate observations do not indicate a difference, they may be considered as placed in the concave surface of a sphere, having the same centre with the earth. This sphere is called the *Celestial Sphere.*

CHAPTER II.

DEFINITIONS OF TERMS.—ASTRONOMICAL INSTRUMENTS.

1. The *Axis of the Heavens* is an imaginary straight line, joining the north pole of the heavens and centre of the earth, and extending to the southern part of the celestial sphere. It is the line about which the heavens appear to revolve. The point in which it meets the southern part of the celestial sphere, is called the *South Pole of the Heavens*, or simply the *South Pole.*

2. The *North* and *South Poles of the Earth* are the points in which the axis of the heavens intersects the surface of the earth.

3. The *Celestial Equator,** or simply the *Equator*, is the great circle in which a plane through the earth's centre, and perpendicular to the axis of the heavens, intersects the celestial sphere. The circle in which this plane intersects the earth's surface is called the *Terrestrial Equator.*

The north and south poles of the heavens are evidently the poles of the equator.

4. A *Declination Circle* is any great circle which passes

---

* The celestial equator is sometimes called the *Equinoctial.* More commonly, however, the term equator simply, is applied both to the celestial and the terrestrial equator; the context generally serving to designate which is intended. The same observation applies to the terms pole of the heavens and terrestrial pole.

through the poles of the celestial equator. It intersects the equator at right angles.

5. The *Declination* of a heavenly body or of any point in the celestial sphere, is the arc of a declination circle, intercepted between the equator and the centre of the body, or the point. The declination is said to be *north* or *south*, according as the body or the point is on the north or south side of the equator.

6. The *Zenith* and *Nadir* of any place on the earth's surface are the points above and below, in which a straight line passing through the place in the direction of gravity, that is in the direction of a plumb line, when a plummet is freely suspended and is at rest, meets the celestial sphere.

7. The *Horizon* of a place, or as it is sometimes called the *Sensible Horizon*, is the circle in which a plane, passing through the place and perpendicular to the line joining the zenith and nadir, cuts the celestial sphere. The plane itself is also frequently called the *Horizon*.

The circle in which a plane, passing through the earth's centre and parallel to the plane of the sensible horizon, cuts the celestial sphere, is called the *Rational Horizon*.

8. The *Meridian* of a place, is a declination circle which passes through the zenith of the place.

The meridian cuts the horizon at right angles, in two points, called the *North* and *South Points* of the horizon.

9. A *Meridian Line* at any place, or *Terrestrial Meridian*, is the intersection of the plane of the meridian of the place with the earth's surface.

10. The *Latitude* or *Geographical Latitude* of a place is the arc of the meridian intercepted between the equator and the zenith of the place. The latitude is considered as being north or south, according as the zenith is to the north or south of the equator.

It follows from the fifth article, that the latitude of a place is the same thing as the *declination of the zenith* of the place.

11. A *Vertical Circle* is any great circle which passes through the zenith and nadir of a place. It cuts the

horizon at right angles. A straight line in the direction of gravity at any place is called a *Vertical Line.*

12. The *Prime Vertical* is that vertical circle which is at right angles to the meridian of any place.

The prime vertical cuts the horizon in two points, called the *East* and *West Points* of the horizon.

13. The *Altitude* of a heavenly body or of a point in the celestial sphere, is the arc of a vertical circle intercepted between the horizon and the centre of the body, or the point. The *Azimuth* of a body is the arc of the horizon intercepted between a declination circle through the centre of the body, and the north or south point of the horizon.

The complement of the altitude of a body expresses the distance of the body from the zenith, and is called the *Zenith Distance* of the body.

14. The *Culmination* of a celestial body is the passage of its centre over the meridian. This is also called the *Transit* of the body over the meridian.

The definitions of other astronomical terms will be found in succeeding parts of the work.

ASTRONOMICAL INSTRUMENTS.

15. For making accurate astronomical observations, various instruments are necessary. Some of these, with the purposes to which they are applied, it will be proper briefly to mention.

16. The *Astronomical clock* differs from the common clock, in having its different parts constructed with much greater care and precision, and in having what is called a *Compensating Pendulum;* that is, a pendulum with a rod so formed by a combination of materials that its length is not sensibly affected by changes in the temperature of the air.

17. A *Chronometer* is a balance watch, constructed with various improvements and refinements of modern art, so as to insure great precision in its rate of going.

The perfection to which the astronomical clock and chronometer are at present brought, is such, that the error to which a good instrument of either kind is liable in its rate of going, falls considerably short of a single second in twenty-four hours.

18. The *Transit Instrument* is an instrument used for observing the culminations of the heavenly bodies. It consists of a telescope firmly fixed to a strong axis, placed exactly at right angles to its length. This axis terminates in cylindrical pivots of equal size, which rest in notches formed in supports for the two ends. When the instrument is fitted up for use, this axis is so adjusted by means of appendages for the purpose, that its central line is exactly parallel to the horizon and at right angles with the plane of the meridian at the place of observation. Consequently the telescope must be in the plane of the meridian.

In the tube of the telescope, near to the eye glass, a ring is placed, across the middle of which, an extremely fine wire or spider's-line is fixed in a horizontal direction. This is crossed at right angles by five equidistant ones. The ring is capable of being moved by screws, and may be so adjusted that the straight line joining the centre of the eye-glass and the intersection of the horizontal wire with the middle one of the five vertical wires, technically called the *Line of Collimation*, may exactly coincide with the central line of the telescope; that is, with the line joining the centres of the eye-glass and object-glass.

When the whole instrument is properly adjusted for observations, the line of collimation coincides with the plane of the meridian, in every position of the telescope as it is made to turn round on the axis of the instrument.

Transit instruments are made of very different dimensions; the length of the telescope varying from 20 inches to 10 feet. Those of the smaller sizes have metallic stands capable of being moved from place to place, and are called *portable* instruments. In those of larger dimensions, the immediate supports of the instrument rest on stone piers. These are called *fixed* instruments.

19. The *Transit Circle* differs from the transit instrument

in having an accurately graduated vertical circle, usually of considerable size, firmly connected with the telescope and axis; its centre being in the central line of the axis, and its graduated face at right angles with this line. The graduations of the limb are such, that by means of vernier indices or microscopes of a peculiar construction, an arc of the limb may be read off to a second or even to the fraction of a second.

This instrument may be used as a transit instrument; but its prominent use is to determine with great accuracy the *meridian altitudes* of the celestial bodies.

20. The *Altitude and Azimuth Instrument* is an instrument the principal parts of which consist of a graduated vertical circle with an attached telescope, and a graduated horizontal circle; the former being placed perpendicularly over the centre of the latter. These are so combined that, by motions of the frame supporting the vertical circle and of this circle itself on its axis, the telescope may be pointed to an object in any part of the heavens. When the instrument has been properly adjusted for observations, and the line of collimation of the telescope has been directed to an object, the altitude of the object will be exhibited on the vertical circle and its azimuth on the horizontal one.

21. The *Sextant* is an instrument by means of which the angular distance between any two of the heavenly bodies may be measured with considerable precision. It also serves, with the aid of an instrument called an *Artificial Horizon*, to observe the altitude of a body.

The *Quadrant*, or as it is sometimes called, *Hadley's Quadrant*, is nearly similar to the sextant, but it is made of inferior materials, and is not capable of measuring angles with the same accuracy. It is principally used for observing altitudes at sea.

22. The *Reflecting Circle* is an instrument used for the same purposes as the sextant; but it is more complete in its construction, and is capable of giving greater precision in its measurements.

23. A *Micrometer* is an instrument which, when attached

to a telescope, serves to measure small angles with great precision.

Dr. Pearson's *Treatise on Practical Astronomy* contains very full and explicit descriptions of most astronomical instruments, with the methods of adjusting and using them, illustrated by excellent engravings. Descriptions of the principal instruments may also be found in Rees' Cyclopedia and in the Edinburgh Encyclopedia.

The quadrant, sextant, and reflecting circle, at least the two former, are extensively used at sea, and are usually described in treatises on navigation. The student will find ample information relative to their construction and the manner of using them, in Dr. Bowditch's "Practical Navigator."

CHAPTER III.

MERIDIAN LINE.—SIDEREAL DAY.—DIURNAL MOTION.—REFRACTION.

1. Let ZR, Fig 1. represent the northern part of the meridian of a place, Z the zenith, P the pole, HRO the horizon, SS′G the circle which one of the fixed stars appears to describe in its diurnal motion, and S and S′ different situations of the star, the former in the eastern and the latter in the western part of the heavens; also let PS and PS′ be arcs of declination circles, ZSA and ZS′B arcs of vertical circles, and let the situations S and S′ of the star in its apparent circle be such that the altitudes AS and BS′ are equal.

Then if, as it appears to do, the star continues at the same distance from the pole P, the arc PS = PS′; also because ZA = ZB, being each quadrants, and AS = BS′, we have ZS = ZS′, and PZ is common to the two triangles PZS and PZS′; therefore the angle PZS = PZS′, and the arc AR = BR, these arcs being the measures of the angles PZS and PZS′. Now, RA and RB are the azimuths of the star when in the situations S and S′ (2. 13.)* If therefore the altitude and bearing of a star be observed when in the eastern part of the heavens, and its bearing be again observed when it arrives at the same altitude in the western part of the heavens, the line bisecting the angle made by these bearings will be a meridian line, (2. 9.)

2. In conformity with appearance, we have, in the preceding article, made the assumption, that the apparent diurnal motion of a star is in a circle. The probability that this assumption is true, is increased by the fact that repeated accurate observations on the same star with different equal altitudes, or similar observations on any other star, give the same situation for the meridian line.

3. When an accurate meridian line has been thus obtained, a transit instrument may be so adjusted, that the line of collimation of its telescope shall move in the plane of the meridian.

4. When by a good clock the exact time is observed from the time of a fixed star passing the meridian on any evening to the time of its passage on the next evening, and this observation is repeated on several successive evenings, it is found that the interval of time between its passages on any two succeeding evenings is the same. Similar observations on different stars give the same interval of time. This is true, not only for the time between two successive passages of a star over the meridian, but also for the time from a star being at any altitude to its return to the same altitude on the succeeding evening. It appears therefore very probable that the diurnal motion of a star is uniform.

5. The time between two successive passages of a star over the meridian is called a *Sidereal Day*. And a clock that is so regulated as to move through 24 hours in the course of a sidereal day is said to be regulated to *Sidereal Time*.

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* The first number refers to the chapter, and the second to the article. When a reference is made to an article in the same chapter, the number of the article only is given.

6. We have inferred as probable, that the diurnal motion of a star is performed in a circle about the pole of the heavens, and that its motion in that circle is uniform. If this is the case it is evident that the distance PS′ is constant, and that the angle ZPS′ must increase uniformly with the time. As the star moves through the whole circle or 360° in 24 sidereal hours, it must, if its motion is uniform, move through 15° in each hour, and consequently the angle ZPS′ must increase at the rate of 15° per hour. Now, supposing PZ, the distance of the pole from the zenith, and PS′, the distance of a star from the pole, to be known, and the altitude BS′, or its complement, the zenith distance ZS′ of the star, to be observed, the angle ZPS′ may be calculated. Observations and calculations thus made on a star at different times during the same night, prove that the angle ZPS′ varies as the time; and, therefore, that *the diurnal motion of a star is uniform.*

7. In the preceding article the distances PZ and PS′ are supposed to be known. A method of obtaining them is now to be explained. Let F and G be the situations of the star on the meridian above and below the pole. Then we have

$$ZF = PZ - PF$$
$$ZG = PZ + PG = PZ + PF$$

therefore $ZG - ZF = 2\,PF$, or $PF = \frac{1}{2}(ZG - ZF)$

also $ZG + ZF = 2\,PZ$, or $PZ = \frac{1}{2}(ZG + ZF)$

But PF is the same as PS or PS′. It therefore follows that if the zenith distance of a circumpolar star when it is on the meridian above the pole, be obtained from its observed altitude, and again when on the meridian below the pole, the half difference of these zenith distances will give PS, the distance of the star from the pole, and the half sum will give PZ, the distance of the pole from the zenith.

Now it is obvious that the same value for PZ, the distance of the pole from the zenith, should be obtained, whatever circumpolar star is used in obtaining it. It is, however, found that if the distance PZ is determined from observations on one star, and then from observations on another star, considerably more or less distant from the pole, taking all the altitudes as they are given by the instrument, the results *do not agree.* If the two stars are the pole star and another whose distance from the pole is not less than 30° or 40°, and which, when at its lowest point, is very near the horizon, the difference in the results will amount to nearly a quarter of a degree. Consequently, if our conclusion, that the stars remain at the same distances from the pole, is correct, there must some cause operate, in consequence of which, the altitudes given by the instrument are not the true altitudes. Such a cause is found to exist in the action of the earth's atmosphere on the rays of light from the stars in their passage through it to the eye of the observer. The effect of this action is called *Astronomical Refraction*, or simply *Refraction.*

OF REFRACTION.

8. It is known that when a ray of light passes obliquely from one medium to another of different density, its direction is changed.

Let FHGB, Fig. 2, be a vertical section of a vessel whose sides are opaque. An object placed on the bottom at E could not, when the vessel is empty, be seen by an eye placed at O. But if the vessel be filled with water, the object will become visible in the direction OB, and will appear as though it were really at D. A ray of light, therefore, which passes obliquely from water to air, is refracted so as to make a greater angle with the perpendicular to the common surface, than if it passed on, without suffering a change in its direction. Again, a ray of light passing from an object at O, in the direction OB, will, when the vessel is empty, meet the bottom in D. But if the vessel be filled with water, the ray of light will be refracted on entering the water, and will take the direction BE, so that, to an eye at E, it would appear to come from the point A, and therefore the object would appear to be more elevated than it really is. The same effects, though different in degree, take place when a ray of light passes from air into a vacuum, or the contrary.

9. The angle contained between the directions of the direct and refracted rays, is called the *Angle of Refraction*, or simply the *Refraction.*

10. It is found by experiment, that for the same two mediums, except when the ray of light passes very obliquely from one to the other, the sine of the angle contained between the direct ray and the perpendicular to the common surface, is equal to the product of the sine of the angle contained between the refracted ray and the same perpendicular, multiplied by some *constant* quantity.

If ZB be perpendicular to the common surface FB, of the two mediums, and OB be the direct ray, AB the direction of the refracted ray, and m, some constant quantity, we have $m \sin ZBA = \sin ZBO = \sin (ZBA + ABO.)$ The value of m is constant for the same two mediums, but is greater or less, according as the difference of density of the mediums is greater or less.

11. The atmosphere is known to extend to the height of some miles, and its surface is supposed to be nearly concentric with the surface of the earth. It has been found by experiments, in ascending high mountains, that its density gradually decreases with an increase of distance from the general surface of the earth. Hence a ray of light which enters it obliquely, passing continually from a rarer to a denser medium, has its direction continually changed, being bent more and more downwards, so that its path will be a curve, concave towards the earth. This curve coincides with a vertical plane, because as the density of the atmosphere, on each side of such plane, is the same, there is no cause for its deviating either way. *Refraction, therefore, makes the apparent altitude of a star greater than the true; but it does not change its azimuth.*

The curvilinear path of a ray of light, passing through the atmosphere, differs but little from a right line, except near the horizon, where, on account of its greater obliquity, its direction undergoes a greater change.

12. The refraction, except near the horizon, varies, nearly as the tangent of the apparent zenith distance.

Let N represent the apparent zenith distance of a star, and r, the corresponding refraction. Then $N + r =$ the true zenith distance; and, therefore, (10.)

$m \sin N = \sin(N + r) = \sin N \cos r + \cos N \sin r$ (App.* 13.) or $m = \cos r + \frac{\cos N}{\sin N} \sin r = \cos r + \cot N \sin r$.

But as r is small, $\cos r = 1$, and $\sin r = r$, nearly. Therefore $m = 1 + r \cot N$, or $r = \frac{m - 1}{\cot N} = (m - 1) \tan N$.

13. It is evident that the refraction cannot vary accurately as the tangent of the apparent zenith distance, because in that case, when the zenith distance is 90°, it would be infinite. And in all cases, when the altitude is small, and consequently when the rays of light enter the atmosphere very obliquely, it would be too great.

Dr. Bradley found that the refraction is more nearly equal to the product of the tangent of the difference between the apparent zenith distance and three times the refraction, multiplied by a constant quantity.

His formula is, $r = 57'' \tan(N - 3r)$; in which 57″ is the refraction at 45° apparent zenith distance.

14. From the observed altitudes or zenith distances of two circumpolar stars, when on the meridian, both above and below the pole, the true distance of the pole from the zenith, the refraction for each meridian altitude of the stars, and the true polar distances of the stars, may be obtained.

In Fig. 1. let P be the true place of the pole, G and F the true situations of a star on the meridian, below and above the pole, and L and K those of another star. Also let N and n represent the apparent zenith distances of the star at G and F, R and r the corresponding refractions, N′ and n' the apparent zenith distances of the other star at L and K, and R′ and r' the corresponding refractions. Then,

$$ZG = N+R,\ ZF = n+r,\ ZL = N'+R',\ \text{and}\ ZK = n'+r';$$
$$\text{therefore (7)},\ PZ = \tfrac{1}{2}(N+R+n+r) = \tfrac{1}{2}(N'+R'+n'+r',)$$
$$\text{and},\ N+R+n+r = N'+R'+n'+r',$$
$$\text{or},\ R+r-R'-r' = N'+n'-N-n.$$

Now supposing the refraction to vary as the tangent of the apparent zenith distance, (12.) we have,

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* Appendix at the end of part 1st.

$$m - 1 = \frac{R}{\tan N} = \frac{r}{\tan n} = \frac{R'}{\tan N'} = \frac{r'}{\tan n'}; \text{ whence,}$$

$$r = \frac{R \tan n}{\tan N},\ R' = \frac{R \tan N'}{\tan N} \text{ and } r' = \frac{R \tan n'}{\tan N}; \text{ therefore,}$$

$$R + \frac{R \tan n}{\tan N} - \frac{R \tan N'}{\tan N} - \frac{R \tan n'}{\tan N} = N' + n' - N - n.$$

From which we obtain $R = \dfrac{(N'+n'-N-n) \tan N}{\tan N + \tan n - \tan N' - \tan n'}$

Whence r, R' and r' become known, and consequently $PZ = \frac{1}{2}(N+n+R+r)$; also the polar distance $PF = ZP - ZF = ZP - (n+r)$, and $PK = ZP - ZK = ZP - (n'+r')$.

This method of finding the refraction is by Boscovich. When neither of the zenith distances exceeds 70° or 75°, the results obtained by this method may be regarded as very nearly accurate.

15. When the true distance of the pole from the zenith, and the apparent zenith distance and corresponding refraction, of one star on the meridian are known, the true polar distance of any other star may be determined from its observed meridian altitude.

If M be the true situation of the star on the meridian, and we put n'' for the observed apparent zenith distance, and r'' for the corresponding refraction, we have,

$$\frac{\tan n}{r} = \frac{\tan n''}{r''} \text{ or } r'' = \frac{r \tan n''}{\tan n};$$

consequently $PM = ZM - PZ = n'' + r'' - PZ$, becomes known.

16. The true zenith distance of the pole and polar distance of a star being known, the refraction may be found for any observed altitude, by knowing also the interval between the times of observation and of the star's passage over the meridian.

Let S″ be the place of the star. Then in the triangle ZPS″, the sides ZP and PS″ are given, and also from the observed interval of time, the angle ZPS″. Consequently the side ZS″ may be calculated. The difference between ZS″ thus obtained, and the observed zenith distance, is the corresponding refraction. In this way the refraction may be found for different apparent altitudes, from the horizon to the zenith.

17. The refraction at a given altitude is subject to some change depending on the variation in the state of the air

as indicated by the barometer and thermometer. The refractions which have place when the barometer stands at 29.6 inches and the thermometer at 50° are called mean refractions. Table II. contains the mean refractions for different altitudes from the horizon to the zenith. Above the altitude of 15 or 20°, these are at all times sufficiently accurate, except in cases when great precision is required.

Formulæ have been investigated for obtaining the refraction with reference to the state of the barometer and thermometer; but these investigations do not belong to an elementary treatise.

18. As refraction always increases the altitude of a heavenly body, it follows that the refraction corresponding to any observed altitude must be *subtracted* from that altitude in order to obtain the *true* altitude of the body above the sensible horizon.

OTHER EFFECTS OF REFRACTION.

19. As refraction elevates the heavenly bodies in vertical circles, and as these circles continually approach each other from the horizon till they meet in the zenith, it is evident that the *apparent* distance of any two of those bodies must be *less* than the *true* distance.

20. The refraction increases with an increase of zenith distance. The lower part of the sun or moon is therefore more refracted than the upper part, so that the vertical diameter is shortened and the body appears of an elliptical form. This effect is most observable near the horizon, where on account of the more rapid increase of the refraction, the difference between the vertical and horizontal diameters may amount to ⅛ part of the whole diameter. The horizontal diameter also suffers a slight diminution. (19)

21. At the true horizon the refraction is about 28′½. Hence it follows that when any of the heavenly bodies are really in the horizon, they appear to be 28′½ above it, and that therefore refraction *retards* their *setting*, and *accelerates* their *rising*.

22. When the sun descends below the horizon of any place, its rays continue for some time to reach the upper parts of the atmosphere, and are refracted and reflected so as to occasion considerable light, which gradually diminishes as the sun descends farther below the horizon, and prevents an immediate transition from the light of day to the darkness of night. The same effect, though in a reverse order, takes place in the morning previous to the sun's rising. The light thus produced is called the *Crepusculum* or *twilight.*

CHAPTER IV.

LATITUDE OF A PLACE.—FIGURE AND EXTENT OF THE EARTH. LONGITUDE.

1. Let HZRN, Fig. 9, represent the meridian, Pp the axis of the heavens, Z the zenith, HOR the horizon, and EOQ the equator, the latter two seen edgewise. Then ZQ is the latitude of the place (2.9). But ZQ$=$PQ—PZ $=$ZH—PZ$=$PH; therefore the *latitude of a place is equal to the altitude of the pole at that place.* A method of obtaining the zenith distance of the pole, and consequently its altitude, has been shown (3. 14).

2. Because ZR$=90°$, RQ$=90°-$ZQ; therefore the altitude of the point of the equator which is on the meridian, at any time, is equal to the complement of the latitude of the place.

FIGURE AND EXTENT OF THE EARTH.

3. By the figure of the earth is meant the general form of its surface, supposing it to be smooth, or that the surface of the land corresponded with the surface of the ocean. This excludes the consideration of the irregularities in its surface occasioned by mountains and valleys, which indeed are very minute when compared with the whole extent of the earth.

4. Experiment proves that the direction of gravity, at any place on the earth, is perpendicular to the free surface of still water: hence it is perpendicular to the general surface of the earth at that place.

5. Let EPQ p, Fig. 3, be a meridian of the earth, Pp the axis, EQ a diameter of the equator, and A and B two places on the meridian. If the earth be a sphere, the direction of gravity at each of the places will pass through the centre of the earth, and therefore the angle ECA will be the latitude of A, and ECB the latitude of B (2.9). Hence if the latitudes of A and B be determined (1), the angle ACB = ECB — ECA, becomes known. The distance AB may be obtained by actual measurement. Then as the angle ACB : 360° : : distance AB : to the circumference of the earth. Also, as the angle ACB : 1° : : distance AB : the length of a degree of latitude.

6. If the earth was a sphere, the length of a degree of latitude would be the same in every part of the meridian. But this is not the case. A number of arcs of the meridian have been measured in different parts of the earth. The mean lengths of a degree of latitude, as determined from several of these arcs, regarded as having been measured with great care, are given in the following table. They are expressed in miles and yards.

Country.	Arc measured.			Latitude of middle of the arc.			Mean length of a degree.		Observers.
	°	′	″	°	′	″	miles	yds.	
Peru	3	7	3	1	31	0	68	1256	Condamine, &c.
India	15	57	40	16	8	22	68	1335	Lambton, Everest.
France	12	22	13	44	51	2	69	72	Delambre, Mechain.
England	3	57	13	52	35	45	69	217	Roy, Kater.
Sweden	1	37	19	66	20	10	69	487	Svanberg.

It is obvious from inspection of the table that the measured length of a degree of latitude *increases* with an *increase* of the latitude. Consequently the meridian cannot be a circle. The next probable supposition is that it is an ellipse.

7. Let us suppose that the ellipse EPQp, Fig. 4. repre-

sents a meridian, and that the minor axis Pp is the axis of the earth, and the major axis EQ a diameter of the equator. Also let AD be perpendicular to the curve at some point A, and let the situation of the point B be such, that BF, perpendicular to the curve at B, may make the angle BFP = EDA. Then the angle EDA is the difference of latitude between the places E and A, and the angle BFP is the difference of latitude between the places B and P. Now it is evident, from inspection of the figure or from the consideration of the decrease of curvature from E to P, that the distance BP, corresponding to a given difference of latitude near the pole, is greater than the distance EA, corresponding to the same difference of latitude, near the equator. It is also obvious that any arc between A and B, corresponding to the same difference of latitude as that to which EA and BP correspond, will be greater than the former and less than the latter. Thus it appears that in an elliptical meridian such as we have supposed, the length of a degree of latitude will be greater as the latitude is greater.

8. From the most accurately measured arcs of the meridian in various places, it has been proved by means of analytical investigations, not adapted to the present work, that a meridian is an *ellipse*, or very nearly so; and that the earth is an *oblate spheroid*, having the *polar* radius less than the *equatorial* radius, by about $\frac{1}{300}$ part of the equatorial radius.

According to this determination, the equatorial diameter of the earth is 7925 miles, and the polar diameter is 7899 miles. Consequently the *mean* diameter is 7912 miles; and the difference of the diameters is 26 miles.

The mean length of a degree of the earth's circumference is about $69\frac{1}{10}$ miles. Hence it follows that in going north or south about 100 feet, we make a change in our latitude of one second.

9. It is frequently convenient to consider the equatorial radius of the earth a unit or 1, and to express other lengths and distances in terms of this unit. Consequently

when this is done, taking the difference of the radii as stated above, the polar radius will be $\frac{299}{300}$. The difference between the equatorial and polar radii, thus expressed, is called the *Ellipticity* or *Oblateness* of the earth. It is also sometimes called the *Compresssion* of the earth. Hence the ellipticity or compression is $\frac{1}{300}$.*

10. The *Eccentricity* of the earth is the distance between a focus of any of the elliptical meridians and the centre. It is equal to $\frac{\sqrt{599}}{300}=0.08158$.

For putting e = the eccentricity, and considering, Fig. 4, EC = 1, we have (conic sections),

$$1-e^2=CP^2=\frac{299^2}{300^2}$$

$$\text{Whence } e=\frac{\sqrt{599}}{300}.$$

11. The *Geocentric Zenith* of a place is the point in which a straight line from the earth's centre, through the place, meets the celestial sphere. And the *Geocentric Latitude*, or *Reduced Latitude* of a place, is the arc of the meridian intercepted between the equator and the geocentric zenith of the place. The difference between the geographic and geocentric latitudes of a place is called the *Reduction of Latitude.*

12. The equatorial radius being expressed by a unit, the *tangent of the geocentric latitude of a place is equal to the product of the square of the polar radius by the tangent of the geographic latitude.*

Let EPQp, *Fig.* 5, be the elliptical meridian passing through any place, EQ the equatorial diameter of the earth, Pp the polar diameter, and C the centre; also let AD be perpendicular to the meridian at A, and AG perpendicular to EQ. Put φ = angle EDA = geographic latitude of A, and φ' = angle ECA = the geocentric latitude. Then,

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* This value of the ellipticity is considered as being very nearly accurate. Dr. Bowditch, in his excellent *Translation of Laplace's Mécanique Céleste, with a commentary*, obtains, from a combination of several of the most accurately measured arcs of the meridian, a result a little less; and from experiments which have been made on the oscillations of a pendulum in different latitudes, a result a little greater. He adopts $\frac{1}{300}$ as being very near to the true value.

$$DG \tan \varphi = AG = CG \tan \varphi' \text{; hence,}$$

$$\frac{DG}{CG} = \frac{\tan \varphi'}{\tan \varphi}. \text{ But (conic sections),}$$

$$\frac{DG}{CG} = \frac{PC^2}{EC^2} = PC^2. \text{ Therefore,}$$

$$\frac{\tan \varphi'}{\tan \varphi} = PC^2, \text{ or } \tan \varphi' = PC^2 \tan \varphi.$$

But, (10), $PC^2 = 1 - e^2$.

Hence, $\tan \varphi' = (1 - e^2) \tan \varphi$ - - - - - (A)

13. The values of CG and AG, the rectangular coordinates of a place A on the earth's surface, referred to the axes of its elliptical meridian, and AC, its distance from the centre are sometimes required. Put $\varrho = AC$. Then $CG = \varrho \cos \varphi'$, and $AG = \varrho \sin \varphi'$. Expressions for $\varrho \cos \varphi'$, $\varrho \sin \varphi'$, and ϱ, in terms of the geographic latitude and earth's eccentricity, may be found as follows:

Let the semicircle EBQ be described on EQ, and GA be produced to meet it in B. Then since, by conic sections, CP : EC : : GA : GB, we have,

$$BG = AG . \frac{EC}{CP} = \varrho \sin \varphi' \frac{1}{\sqrt{(1 - e^2)}}.$$

But $BG^2 + CG^2 = BC^2 = 1$.

$$\text{Hence, } \varrho^2 \sin^2 \varphi' . \frac{1}{1 - e^2} + \varrho^2 \cos^2 \varphi' = 1,$$

$$\text{or, } \frac{1}{1 - e^2} \tan^2 \varphi' + 1 = \frac{1}{\varrho^2 \cos^2 \varphi'}.$$

But (12. A), $\tan^2 \varphi' = (1 - e^2)^2 \tan^2 \varphi$.

$$\text{Consequently, } (1 - e^2) \tan^2 \varphi + 1 = \frac{1}{\varrho^2 \cos^2 \varphi'}.$$

$$\text{or, } (1 - e^2) \frac{\sin^2 \varphi}{\cos^2 \varphi} + 1 = \frac{1}{\varrho^2 \cos^2 \varphi'}$$

$$\sin^2 \varphi - e^2 \sin^2 \varphi + \cos^2 \varphi = \frac{\cos^2 \varphi}{\varrho^2 \cos^2 \varphi'}$$

$$1 - e^2 \sin^2 \varphi = \frac{\cos^2 \varphi}{\varrho^2 \cos^2 \varphi'}.$$

$$\text{Whence, } \varrho \cos \varphi' = \frac{\cos \varphi}{\sqrt{(1 - e^2 \sin^2 \varphi)}} \quad \text{- - (A)}$$

$$\text{Again, (App. 12) } \varrho \sin \varphi' = \varrho \cos \varphi' \tan \varphi' = \varrho \cos \varphi' \tan \varphi (1 - e^2)$$

$$= \frac{(1 - e^2) \tan \varphi \cos \varphi}{\sqrt{(1 - e^2 \sin^2 \varphi)}}$$

$$\text{or, } \varrho \sin \varphi' = \frac{(1 - e^2) \sin \varphi}{\sqrt{(1 - e^2 \sin^2 \varphi)}} \quad \text{- - - (B)}$$

Adding together the squares of the equations (A) and (B) we have,

$$\varrho^2 = \frac{(1 - e^2)^2 \sin^2 \varphi + \cos^2 \varphi}{1 - e^2 \sin^2 \varphi} = \frac{\sin^2 \varphi - 2 e^2 \sin^2 \varphi + e^4 \sin^2 \varphi + \cos^2 \varphi}{1 - e^2 \sin^2 \varphi}$$

$$=\frac{1-2\,e^2\sin^2\varphi+e^4\sin^2\varphi}{1-e^2\sin^2\varphi}$$

$$\text{or, } \varrho=\sqrt{\frac{1-(2-e^2)\,e^2\sin^2\varphi}{1-e^2\sin^2\varphi}} \quad . \quad . \quad . \qquad (C)$$

LONGITUDE.

14. We have shown (1) how to obtain the latitude of a place. But it is evident that the latitude is not of itself sufficient to designate the situation of a place, because all the points in a circle on the earth's surface, parallel to the equator, have the same latitude. Something more then is necessary to designate, with precision, the situations of places. As the meridians cut the equator at right angles, they are conveniently made use of for this purpose.

15. A meridian which passes through some particular place is called the *First Meridian.* The angle contained between the first meridian and a meridian through any place, is called the *Longitude* of that place. Longitude is measured by the arc of the equator, intercepted between the first meridian and the meridian passing through the place, and is called *east* or *west* according as the latter meridian is to the east or west of the first meridian.

16. Different nations have adopted different first meridians. The English reckon longitude from the meridian which passes through their observatory, at Greenwich, near London; and the French from the meridian of their observatory at Paris. As there is no public observatory in the United States, there is not a uniformity with respect to a first meridian. Some reckon the longitude from the meridian of *Washington*, some from that of *Philadelphia*, and others from the meridians of other principal cities. But for astronomical purposes we usually reckon our longitude from the meridian of *Greenwich.*

17. Since the diurnal motion of the stars is from east to west (1.5), any particular star must come to a given first meridian, sooner than to the meridian of a place which has west longitude, and later than to the meridian of a place which has east longitude (15); and the difference of

times will be found by allowing one sidereal hour for each 15° of longitude, and in the same proportion for odd degrees, minutes, and seconds, (3.5). It follows, therefore, that if the time at which some star passes the first meridian, be observed by an accurate chronometer, regulated to keep sidereal time; and if the chronometer be then taken to a place to the east or west of the first meridian, and the time, at which the same star passes the meridian of that place, be observed by it, the difference of times, converted into degrees, by allowing 15° to the hour, will express the longitude of the place.

There are various other methods of determining the longitudes of places, some of which will be noticed in succeeding parts of the work.

Table I. contains the latitudes of some principal cities, with their longitudes from the meridian of Greenwich, expressed both in degrees and in time.

CHAPTER V.

ON PARALLAX.

1. The directions in which a body is seen at the same instant, from different places on the earth's surface, must in reality be different; but the distances of the fixed stars are so immensely great, (1.12), that for any one of them the difference is perfectly insensible. This is not the case with the sun, moon, and planets. They are sufficiently near to the earth to have the directions in which they are seen, sensibly influenced by the situation of the observer. Astronomers, therefore, in order to render their observations easily comparable, and for convenience in calculation, reduce the situation of a body as observed at any place on the earth's surface, to the situation in which it would appear from the centre.

The observed situation of the body is called its *Apparent* place, and the situation in which it would appear from the earth's centre is called its *True* place.

2. The angle contained between two right lines, conceived to be drawn from a body, one to the centre of the earth and the other to the place of the observer, is called the *Parallax*, or the *Parallax in Altitude*, of the body.

3. Let ADE, *Fig.* 6, represent the earth, considered as a sphere, C its centre, A a place on its surface, Z the zenith of the place A, and B the situation of a body; then will ZAB be the apparent zenith distance of the body, ZCB its true zenith distance, and ABC its parallax in altitude.

The parallax ABC = ZAB — ZCB = apparent zenith distance — true zenith distance = 90° — apparent altitude — (90° — true altitude) = true altitude — apparent altitude.

4. The parallax in altitude of a body, when its apparent zenith distance is 90°, is called the *Horizontal Parallax.*

5. Supposing a body to continue at the same distance from the earth, the sine of the parallax in altitude is equal to the product of the sine of the horizontal parallax, multiplied by the sine of the apparent zenith distance.

Put $R=AC=$ mean radius of the earth,
$D=CB=$ distance of the body from the earth's centre,
$N=ZCB=$ true zenith distance,
$p=ABC=$ parallax in altitude,
and $\varpi=$ the horizontal parallax;
then $N+p=ZAB=$ apparent zenith distance,
And $\sin p=\frac{AC}{BC}\sin CAB=\frac{AC}{BC}\sin ZAB=\frac{R}{D}\sin(N+p.)$
But when $N+p=90^\circ$, p becomes ϖ,
Hence $\sin\varpi=\frac{R}{D}\sin 90^\circ=\frac{R}{D}.$
Consequently $\sin p=\sin\varpi\sin(N+p.)$

6. The distance of a body from the centre of the earth is equal to the radius of the earth, divided by the sine of the horizontal parallax.

Since $\sin\varpi=\frac{R}{D}$ (5),
We have $D=\frac{R}{\sin\varpi}.$

Hence, as the radius of the earth has been determined (4.8), when the horizontal parallax of a body is known, its distance from the centre of the earth is easily found.

7. The distances of the heavenly bodies are so great that p, the parallax in altitude, and ϖ, the horizontal parallax, are always very small angles; even for the moon which is much the nearest, the value of ϖ is always less than 62′. We may therefore, without material error, use p and ϖ themselves, instead of their sines. If this be done, the last formulæ, in the two preceding articles, become,

$$p = \varpi \sin (\mathrm{N} + p,)$$
$$\text{and } \mathrm{D} = \frac{\mathrm{R}}{\varpi} = \mathrm{R}\frac{1}{\varpi}.$$

8. In the fraction $\frac{1}{\varpi}$ of the last formula, 1 represents the radius and ϖ the measure of the horizontal parallax. Hence, in order to render the numerator and denominator of the fraction homogeneous, if ϖ be expressed in seconds, we must also express the radius in seconds.

Since 6.2831853 is the length of the circumference when the radius is 1, and 1296000 is the number of seconds in the circumference; we have 6.2831853 : 1 :: 1296000″ : 206264″.8 = the length of the radius expressed in seconds.

Hence if the value of ϖ is expressed in seconds,

$$\mathrm{D} = \mathrm{R}\,\frac{206264''.8}{\varpi}.$$

9. If the meridian zenith distances of a body be observed on the same day, by two observers, remote from each other on the same meridian, and at places whose latitudes are known, its horizontal parallax may, from thence, be determined.

Let AEA′Q, *Fig.* 7, represent a meridian of the earth, considered as a sphere, C its centre, EQ a diameter of the equator, A and A′ the situations of two observers, Z and Z′ their zeniths, and B the situation of a body on the meridian.

Put L = ECZ = latitude of the place A,
L′ = ECZ′ = do. A′,

$d = \mathrm{ZAB} =$ apparent zenith distance at A,
and $d' = \mathrm{Z'A'B} =$ do A′.
Then,

$$\mathrm{ACA'} = \mathrm{ECZ} + \mathrm{ECZ'} = \mathrm{L} + \mathrm{L'},$$
$$\mathrm{BAC} = 180^\circ - \mathrm{ZAB} = 180^\circ - d,$$
$$\mathrm{BA'C} = 180^\circ - \mathrm{Z'A'B} = 180^\circ - d',$$

and $\mathrm{ABA'} = 360^\circ - (\mathrm{ACA'} + \mathrm{BAC} + \mathrm{BA'C}) = d + d' - (\mathrm{L} + \mathrm{L'})$.
But (7) $\mathrm{ABC} = \varpi \sin d$, $\mathrm{A'BC} = \varpi \sin d'$,
and $\mathrm{ABA'} = \mathrm{ABC} + \mathrm{A'BC} = \varpi \sin d + \varpi \sin d'$.

$$\text{Hence } \varpi = \frac{\mathrm{ABA'}}{\sin d + \sin d'} = \frac{d + d' - (\mathrm{L} + \mathrm{L'})}{\sin d + \sin d'}.$$

10. If the meridian zenith distances of a fixed star, which passes the meridian at the same time with the body, be observed, as well as those of the body, the horizontal parallax may be obtained, independent of the latitudes of the places.

For if S be the situation of the star when on the meridian, we then know,

$$\mathrm{BAS} = \mathrm{ZAS} - \mathrm{ZAB}, \text{ and } \mathrm{BA'S} = \mathrm{Z'A'B} - \mathrm{Z'A'S}.$$

But $\mathrm{ABA'} + \mathrm{BAS} = \mathrm{BLS} = \mathrm{ASA'} + \mathrm{BA'S}$,
or $\mathrm{ABA'} = \mathrm{BA'S} - \mathrm{BAS} + \mathrm{ASA'}$.
Or since the angle ASA′ is insensible (1), we have,

$$\mathrm{ABA'} = \mathrm{BA'S} - \mathrm{BAS},$$
$$\text{and } \varpi = \frac{\mathrm{BA'S} - \mathrm{BAS}}{\sin d + \sin d'}.$$

11. It is not necessary that the two observers should be on precisely the same meridian; for if the meridian zenith distances of the body be observed on several successive days, its change of meridian zenith distance in a given time will become known. Then if the difference of the longitudes of the places is known (4.17), the zenith distance of the body as observed on one of the meridians, may be reduced to what it would be, if the observation had been made, in the same latitude on the other meridian.

In the year 1751, Wargentin, at Stockholm, and Lacaille, at the Cape of Good Hope, made the requisite observations on the planet *Mars*, and determined its parallax at the time of observation to be 24″.64. Hence, (8),

$$D = R.\frac{206264''.8}{24''.64} = R \times 8371.$$

The distance of Mars from the earth's centre was, therefore, at the time of observation, 8371 times the mean radius of the earth.

12. For the moon, whose parallax is much greater than that of any other of the heavenly bodies, it is necessary to take into view the spheroidal figure of the earth.

Let the ellipse PEpQ, *Fig.* 8, represent a meridian of the earth, C its centre, EQ a diameter of the equator, Z and Z′ the true zeniths of the places A and A′, and z and z', their geocentric zeniths. Then zAZ = CAd = EdZ — ECz = difference between the geographic and geocentric latitudes of the place A. In like manner z'A′Z′ = difference of the geographic and geocentric latitudes of the place A′. The angles zAZ and z'A′Z′ being known for the two places A and A′, and the angles ZAB and Z′A′B, being determined from observation, the angle zAB = ZAB — zAZ, and z'A′B = Z′A′B — z'A′Z′, become known; and thence (10) ABA′ is also known.

Now if ϖ and ϖ' be the horizontal parallaxes of the moon at A and A′, and R and R′ stand for the radii CA and CA′, we have (7),

$$\frac{R}{\varpi} = D = \frac{R'}{\varpi'};\ \text{whence}\ \varpi' = \frac{R'}{R}\varpi.$$

Let d and d' stand for the reduced zenith distances zAB and z'A′B; then (7),

$$ABC = \varpi \sin d,\ A'BC = \varpi' \sin d' = \frac{R'}{R}\varpi \sin d',$$

$$\text{and } ABA' = ABC + A'BC = \varpi \sin d + \frac{R'}{R}\varpi \sin d',$$

$$\text{or } R\,\varpi \sin d + R'\,\varpi \sin d' = R \times ABA'.$$

$$\text{Hence } \varpi = \frac{R \times ABA'}{R \sin d + R' \sin d'}.$$

13. The horizontal parallax of the moon, to an observer at the equator, is called the *Equatorial Parallax.*

If ϖ'' = the equatorial parallax, and R″ = CE = the radius of the equator, then,

$$\varpi'' = \frac{R''}{R}\varpi = \frac{R'' \times ABA'}{R \sin d + R' \sin d'}.$$

14. From observations made in the year 1751, by Lacaille, at the Cape, and Lalande at Berlin, and from other

methods, which have been used for the same purpose, the moon's equatorial parallax is found to vary from 53′ 52″ to 61′ 32″. Hence,

$$D = R''\frac{206264''.8}{3212''} = R'' \times 64,\text{ nearly} = \text{its greatest distance, and}$$

$$D = R''.\frac{206264''.8}{3692''} = R'' \times 56,\text{ nearly} = \text{its least distance.}$$

Consequently the moon's mean distance is about 60 times the equatorial radius of the earth.

15. The mean equatorial parallax of the moon is ½ (53′ 52″+61′ 32″)=57′ 42″. But the parallax at the mean distance is only 57′ 22″.

Let D, D′, and D″, be the least, greatest, and mean distances of the moon from the earth, and ϖ, ϖ', ϖ'', the corresponding parallaxes. Then (4),

$$D = \frac{R''}{\sin\varpi},\ D' = \frac{R''}{\sin\varpi'},\text{ and } D'' = \frac{R''}{\sin\varpi''},$$

$$\tfrac{1}{2}(D + D') = \tfrac{1}{2}\left(\frac{R''}{\sin\varpi} + \frac{R''}{\sin\varpi'}\right).$$

But $D'' = \tfrac{1}{2}(D + D')$.

$$\text{Hence } \frac{R''}{\sin\varpi''} = \tfrac{1}{2}\left(\frac{R''}{\sin\varpi} + \frac{R''}{\sin\varpi'}\right)$$

$$\frac{2}{\sin\varpi''} = \frac{\sin\varpi + \sin\varpi'}{\sin\varpi\sin\varpi'} = \frac{2\sin\tfrac{1}{2}(\varpi + \varpi')\cos\tfrac{1}{2}(\varpi - \varpi')}{\sin\varpi\sin\varpi'}\ \text{(Ap. 20.)}$$

$$\sin\varpi'' = \frac{\sin\varpi\sin\varpi'}{\sin\tfrac{1}{2}(\varpi + \varpi')\cos\tfrac{1}{2}(\varpi - \varpi')}\ \text{- - - - - (A)}$$

As the arcs are small, we may, without material error, consider $\cos\tfrac{1}{2}(\varpi - \varpi') = 1$, and for the other terms take the arcs instead of their sines. We shall then have,

$$\varpi'' = \frac{\varpi.\varpi'}{\tfrac{1}{2}(\varpi + \varpi')} = \frac{2\,\varpi.\varpi'}{\varpi + \varpi'}\ \text{- - - - - - - (B)}$$

From either of the expressions, A and B, the value of ϖ'' is found equal 57′ 22″.

16. The sun's distance is so great, that its parallax cannot be determined with precision by the preceding method. It may, however, be shown to be about 8″ or 9″.

By a method that will be noticed hereafter, the sun's mean horizontal parallax is found to be 8″.6. Hence its distance is,

$$D = R'' \frac{206264''.8}{8''.6} = R'' \times 24000, \text{ nearly.}$$

17. It is obvious (3) that *the effect of parallax is to diminish the altitude of a body.* And it has been shown (3. 11) that the effect of refraction is to *increase* the altitude. Hence to obtain, from the *observed* altitude of a body, its *true* altitude, that is, its altitude above the rational horizon, as seen from the centre of the earth, the *refraction* must be *subtracted*, and the *parallax added.*

As the fixed stars, in consequence of their immense distance, have no sensible parallax, the true altitude of a star is obtained by subtracting the refraction from the observed altitude.

When the body has a sensible diameter, which is the case with the sun, moon, and planets, the altitude of the lower or upper limb is observed. Consequently to obtain the true altitude of the centre, the refraction must be subtracted, the parallax added, and the semidiameter added or subtracted, according as it was the lower or upper limb that was observed.

CHAPTER VI.

APPARENT PATH OF THE SUN.—FIXED STARS.

1. The sun partakes with the stars in the apparent diurnal motion; but the time between his passing the meridian on any day, and his passing it the next, is found to be greater than a sidereal day. The sun, therefore, appears to have a motion *eastward* among the fixed stars. The altitude of the sun, when on the meridian, is not the same on two successive days. On the 20th of March and 22d of September, it is about the same as the meridian altitude of the equator; from the former time to the latter, it is greater; and during the other part of the year it is less. On the 21st of June the sun's meridian altitude is greatest,

and it then exceeds the meridian altitude of the equator about 23° 28′; on the 21st of December it is least, and is then less than that of the equator by the same quantity 23° 28′. The sun's motion appears, therefore, to be in a plane, cutting the equator in two opposite points.

2. Let HZRN, *Fig.* 9, represent the meridian, HOR the horizon, P*p* the axis of the heavens, AQE the equator, ASFG the apparent path of the sun, P the north pole, and Z the zenith. Also, let *s* be the situation of some bright star, which, in the latter part of March or in April, passes the meridian a short time before the sun.* Let the time at which the star passes the meridian be observed by a clock, accurately regulated to sidereal time. If then the altitude SR of the sun's centre, when on the meridian, be determined from observation, and also the time be observed, we have the polar distance PS = 180° — (PH + RS) = 180° — (latitude of the place + altitude of the sun's centre,) and the angle BPQ = the difference of times, converted into degrees (3.6).

If similar observations be made on the same star and the sun, a few weeks after, when the sun has moved in its apparent path to S′, we shall have PS′ and the angle BPD. Consequently the angle SPS′ = BPD — BPQ, becomes known.

If the sun's apparent path ASF be a great circle, SPS′ will be a spherical triangle, in which we know the two sides PS, PS′ and the contained angle SPS′; whence the angles PSS′ and PS′S may be found. Then in the right angled triangle AQS, we have ASQ = PSS′ and QS = 90° — PS, with which the angle A may be found. We may also find the angle A from the triangle ADS′, in which are given AS′D = 180° — PS′S and DS′ = 90° — PS′. The value of the angle A, thus determined from the two triangles AQS and ADS′, is found to be the same. Hence the apparent path ASG of the sun, is a great circle.

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* The brighter stars may be distinctly seen in the day time, with an astronomical telescope.

It therefore cuts the equator in two points, A and F, at the distance of 180° from each other.

But little more than half of the circle is shown in the figure, as the whole, if accurately represented, would occupy too much room.

3. The great circle which the sun appears to describe, is called the *Ecliptic.*

4. The points in which the ecliptic cuts the equator, are called the *Equinoctial Points.* The time when the sun is at the equinoctial point, in his passage from the south to the north side of the equator, is called the *Vernal Equinox;* and the time when he is at the other equinoctial point, is called the *Autumnal Equinox.*

The terms *Vernal Equinox* and *Autumnal Equinox* are frequently applied to the equinoctial points themselves.

5. The two points in the ecliptic, which are at 90° distance from the equinoctial points, are called the *Solstitial Points.* The point T represents the situation of the solstitial point on the north side of the equator; the other is on the part of the ecliptic left out of the figure. The time, when the sun is at the northern solstitial point, is called the *Summer Solstice,* and the time, when he is at the southern solstitial point, is called the *Winter Solstice.*

6. A great circle, passing through the equinoctial points and the poles of the heavens, is called the *Equinoctial Colure.* Another great circle, passing through the solstitial points, is called the *Solstitial Colure.*

7. The angle which the ecliptic makes with the equator, is called the *Obliquity of the Ecliptic.* The obliquity of the ecliptic is found to be 23° 28′ nearly (2).

8. Two small circles parallel to the equator and touching the ecliptic at the solstitial points, are called the *Tropics.* That which is on the north side of the equator is called the *Tropic of Cancer,* and the other, the *Tropic of Capricorn.* Thus *a*T*b* is the tropic of Cancer, and *cde* the tropic of Capricorn.

9. Two small circles parallel to the equator, and at a distance from the poles equal to the obliquity of the eclip-

tic, are called *Polar Circles.* The one about the north pole is called the *Arctic Circle;* the other, about the south pole, is called the *Antarctic Circle.* Thus *fgh* is the Arctic Circle, and *kmn* the Antarctic.

10. Circles, corresponding to the tropics and polar circles, and bearing the same name, conceived to be drawn on the earth, divide its surface into five parts, called *Zones.* The part contained between the tropics, is called the *Torrid Zone,* the two parts between the tropics and polar circles, are called the *Temperate Zones,* and the other two parts within the polar circles, are called the *Frigid Zones.*

11. The ecliptic is supposed to be divided into twelve equal parts, which are called Signs. Each sign, therefore, contains 30 degrees. The division of the signs commences at the vernal equinox, and they are numbered in the direction of the sun's apparent motion in the ecliptic. The signs of the ecliptic are, sometimes, designated by names or characters, instead of numbers.

The names of the twelve signs with their corresponding numbers, and the characters by which they are usually denoted, are,

S.			S.		
0.	Aries	♈.	6.	Libra	♎.
1.	Taurus	♉.	7.	Scorpio	♏.
2.	Gemini	♊.	8.	Sagittarius	♐.
3.	Cancer	♋.	9.	Capricornus	♑.
4.	Leo	♌.	10.	Aquarius	♒.
5.	Virgo	♍.	11.	Pisces	♓.

Aries, Taurus, Gemini, Cancer, Leo and Virgo, lie on the north side of the equator and are called *Northern Signs.* The others lie on the south side, and are called *Southern Signs.*

Capricornus, Aquarius, Pisces, Aries, Taurus, and Gemini, are called *Ascending Signs,* because while the sun is in them, his meridian altitude continually increases. Cancer, Leo, Virgo, Libra, Scorpio, and Sagittarius, are called *Descending Signs*, because the sun's meridian-altitude continually decreases, while he is in them.

12. A zone of the heavens, extending in breadth to 8 or 9° on each side of the ecliptic, is called the *Zodiac.* Within the zodiac, all the planets perform their motions, except three of those recently discovered.

13. Any great circle, which passes through the poles of the ecliptic, is called a *Circle of Latitude*.

14. The arc of the ecliptic, intercepted in the order of the signs, between the vernal equinox and a circle of latitude, which passes through a star, is called the *Longitude* of the star. And the arc of the circle of latitude, intercepted between the star and the ecliptic, is called the *Latitude* of the star. Latitude is said to be *north* or *south*, according as the body is on the north or south side of the ecliptic.

15. The arc of the equator, intercepted between the vernal equinox and a declination circle, which passes through a star, is called the *Right Ascension* of the star.

Longitude and Right Ascension are both reckoned from the vernal equinox, round to it again, in the order of the signs.

16. The situations of the fixed stars are generally expressed by right ascension and declination; those of the sun, moon, and planets, sometimes by longitude and latitude, and sometimes by right ascension and declination. With the obliquity of the ecliptic known, the longitude and latitude of a body may be obtained from the right ascension and declination, by means of spherical trigonometry. On the contrary, from the longitude and latitude the right ascension and declination may be found.

17. Let EQ, *Fig.* 10, represent the equator, EC the ecliptic, P and P′ their poles, E the vernal equinox, PSR, a circle of declination, and P′SL a circle of latitude, both passing through a body at S. Then will ER be the right ascension, RS the declination, EL the longitude, and LS the latitude of the body.

Put R = ER = right ascension,
D = RS = declination,
L = EL = longitude,
λ = LS = latitude,
ω = REL = obliquity of the ecliptic,
x = RES,
and y = LES.

18. *When the right ascension and declination are given, to find the longitude and latitude, we have,*

$$\tan \text{RES} = \frac{\tan \text{RS}}{\sin \text{ER}}; \ \tan \text{ES} = \frac{\tan \text{ER}}{\cos \text{RES}};$$

$$\tan \text{EL} = \cos \text{LES} \tan \text{ES} = \frac{\cos \text{LES} \tan \text{ER}}{\cos \text{RES}};$$

$$\text{and } \tan \text{LS} = \tan \text{LES} \sin \text{EL}.$$

$$\text{Or } \tan x = \frac{\tan \text{D}}{\sin \text{R}}; \ \tan \text{L} = \frac{\cos (x - \omega) \tan \text{R}}{\cos x};$$

$$\text{and } \tan \lambda = \tan (x - \omega) \sin \text{L}.$$

If attention be given to the rules for trigonometrical signs, and tang D be considered *negative* when the declination is *south*, these formulæ will apply, whatever be the situation of the body; observing that the longitude and right ascension are always, either both between 90° and 270° or both between 270° and 90°, and that, when the tang λ comes out *negative*, the latitude is *south*.

Let S′ be the sun's place in the ecliptic; then ES′ = sun's longitude. Hence,

$$\text{tang. Sun's longitude} = \tan \text{ES}' = \frac{\tan \text{ER}}{\cos \text{RES}'} = \frac{\textit{tang Sun's right ascension}}{\textit{cos Obliquity of the ecliptic}}.$$

19. *When the longitude and latitude are given, to find the right ascension and declination, we have,*

$$\tan \text{LES} = \frac{\tan \text{LS}}{\sin \text{EL}}; \quad \tan \text{ES} = \frac{\tan \text{EL}}{\cos \text{LES}};$$

$$\tan \text{ER} = \cos \text{RES} \tan \text{ES} = \frac{\cos \text{RES} \tan \text{EL}}{\cos \text{LES}};$$

$$\text{and } \tan \text{RS} = \tan \text{RES} \sin \text{ER}.$$

$$\text{Or } \tan y = \frac{\tan \lambda}{\sin \text{L}}; \ \tan \text{R} = \frac{\cos (y + \omega) \tan \text{L}}{\cos y};$$

$$\text{and } \tan \text{D} = \tan (y + \omega) \sin \text{R}.$$

When the latitude is south, tang λ must be considered negative ; and if tang D come out negative, the declination will be south.

For the sun we have, tang *Sun's right ascension* = tang ER = cos RES′ tang ES′ = cos *Obliquity of the ecliptic* × *tang Sun's longitude;* and sin *Sun's decl.* = sin RS′ = sin RES′ sin ES′ = sin *obliquity of the ecliptic* × sin *Sun's long.*

20. The angle contained between a circle of latitude and circle of declination, both passing through the sun or a star, is called the *Angle of Position* of the sun or star.

If P′P be produced to meet the equator in N, then in the triangle P′SP, P′S = complement of latitude, PS = complement of declination, P′P = obliquity of the ecliptic, P′PS = 180° — NPR = 180° — NR = 180° — (EN — ER) = 180° — (90° — ER) = 90° + ER = 90° + right ascension, and PP′S = EM — EL = 90° — longitude. With any three of these five parts given, the angle of Position PSP′ may be found.

When the longitude, latitude, and obliquity of the ecliptic are given, we have, putting S = PSP′ the angle of position, (App. 37).

$$\cot S = \frac{\cot PP' \sin P'S - \cos P'S \cos PP'S}{\sin PP'S}$$

$$= \frac{\cot \omega \cos \lambda - \sin \lambda \cos (90^\circ - L)}{\sin (90^\circ - L)}$$

$$= \frac{\cot \omega \cos \lambda - \sin \lambda \sin L}{\cos L}$$

$$= \frac{\sin L}{\cos L} \cdot \left(\frac{\cot \omega}{\sin L} . \cos \lambda - \sin \lambda\right)$$

$$= \tan L . \left(\frac{\cot \omega}{\sin L} . \cos \lambda - \sin \lambda\right)$$

Make $\tan z = \sin L \tan \omega$.

Then $\dfrac{\cot \omega}{\sin L} = \dfrac{1}{\sin L \tan \omega} = \dfrac{1}{\tan z} = \cot z = \dfrac{\cos z}{\sin z}$

Hence $\cot S = \tan L . \left(\dfrac{\cos z}{\sin z} . \cos \lambda - \sin \lambda\right)$

$$= \frac{\tan L}{\sin z} . (\cos z \cos \lambda - \sin z \sin \lambda).$$

But (App. 14) $\cos z \cos \lambda - \sin z \sin \lambda = \cos (z + \lambda)$.

Therefore $\cot S = \dfrac{\tan L}{\sin z} . \cos (z + \lambda)$. - - - - (A)

When the longitude, declination, and obliquity of the ecliptic, are given, we have,

$\sin S = \dfrac{\sin PP' \sin PP'S}{\sin PS} = \dfrac{\cos L \sin \omega}{\cos D}$ - - - - (B)

For the sun, $\lambda = o$, and the formula

$$\cot S = \frac{\cot \omega \cos \lambda - \sin \lambda \sin L}{\cos L} \text{ becomes,}$$

$$\cot S = \frac{\cot \omega}{\cos L},$$

$$\text{Or } \tan S = \frac{\cos L}{\cot \omega} = \cos L \tan \omega \quad . \quad . \quad . \quad . \quad . \quad . \qquad (C)$$

It is easy to see that the northern part P′S of the circle of latitude is to the west of the northern part PS of the circle of declination, when the longitude is less than 90° or more than 270°, and to the east when it is between 90° and 270°.

SITUATIONS OF THE FIXED STARS.

21. In order to distinguish the different groups or clusters of stars, spread over the whole extent of the visible heavens, the ancients conceived the figures of men, animals and other objects to be drawn on the concave surface of the celestial sphere, and gave to the groups the names of the figures thus conceived to be drawn. This mode of distinction still continues in use. The group of stars contained within the contour of any such figure, is called a *Constellation.*

The following tables exhibit the names of the principal constellations, as they are usually expressed on celestial charts or globes, with their significations in English.

1. ANCIENT CONSTELLATIONS.

Northern.

1.	Ursa Minor,	The Little Bear.
2.	Ursa Major,	The Great Bear.
3.	Draco,	The Dragon.
4.	Cæpheus,	Cæpheus.
5.	Boötes,	Boötes.
6.	Corona Borealis,	The Northern Crown.
7.	Hercules,	Hercules kneeling.
8.	Lyra,	The Lyre.
9.	Cygnus,	The Swan.
10.	Cassiopeia,	The Lady in her Chair,

11. Perseus,	Perseus.
12. Auriga,	The Wagoner.
13. Serpentarius,	Serpentarius.
14. Serpens,	The Serpent.
15. Sagitta,	The Arrow.
16. Aquila,	The Eagle.
17. Delphinus,	The Dolphin.
18. Equulus,	The Horse's Head.
19. Pegasus,	The Flying Horse.
20. Andromeda,	Andromeda.
21. Triangulum,	The Triangle.

Constellations of the Zodiac.

22. Aries,	The Ram.
23. Taurus,	The Bull.
24. Gemini,	The Twins.
25. Cancer,	The Crab.
26. Leo,	The Lion.
27. Virgo,	The Virgin.
28. Libra,	The Scales.
29. Scorpio,	The Scorpion.
30. Sagittarius,	The Archer.
31. Capricornus,	The Goat.
32. Aquarius,	The Water-bearer.
33. Pisces,	The Fishes.

Southern.

34. Cetus,	The Whale.
35. Orion,	Orion.
36. Eridanus,	Eridanus.
37. Lepus,	The Hare.
38. Canis Major,	The Great Dog.
39. Canis Minor,	The Little Dog.
40. Argo,	The Ship.
41. Hydra,	The Hydra.
42. Crater,	The Cup.
43. Corvus,	The Crow.

44.	Centaurus,	The Centaur.
45.	Lupus,	The Wolf.
46.	Ara,	The Altar.
47.	Corona Australis,	The Southern Crown.
48.	Piscis Australis,	The Southern Fish.

2. NEW SOUTHERN CONSTELLATIONS.

1.	Columba Noachi,	Noah's Dove.
2.	Robur Carolinum,	The Royal Oak.
3.	Grus,	The Crane.
4.	Phœnix,	The Phœnix.
5.	Indus,	The Indian.
6.	Pavo,	The Peacock.
7.	Apus, *Avis Indica*,	The Bird of Paradise.
8.	Apis, Musca,	The Bee or Fly.
9.	Chamelion.	The Chamelion.
10.	Triangulum Australe,	The Southern Triangle.
11.	Piscis volans, *Passer*,	The Flying Fish.
12.	Dorado, *Xiphias*,	The Sword Fish.
13.	Toucan,	The American Goose.
14.	Hydrus,	The Water Snake.

3. HEVELIUS'S CONSTELLATIONS.

Made out of the Unformed Stars.

1.	Lynx,	The Lynx.
2.	Leo Minor,	The Little Lion.
3.	Coma Berenices,	Berenice's Hair,
4.	Asteron and Chara,	The Greyhounds,
5.	Cerberus,	Cerberus.
6.	Vulpecula and Anser,	The Fox and Goose.
7.	Antinoüs,	Antinoüs.
8.	Scutum Sobieski,	Sobieski's Shield.
9.	Lacerta,	The Lizard.
10.	Camelopardalis,	The Camelopard.
11.	Monoceros,	The Unicorn.
12.	Sextans,	The Sextant.

22. The stars of a constellation are distinguished by the letters of the Greek alphabet, which are applied to them according to their apparent relative size, or brightness. The principal star in the constellation is named α, the second in order β, the third γ, and so on. When the number of stars in a constellation exceeds the letters in the Greek alphabet, the letters of the Roman alphabet are applied to the remainder in the same manner; and when these are not sufficient, the numbers 1, 2, 3, &c. are used to designate those that are left.

Some of the fixed stars have particular names, as *Sirius*, *Aldebaran*, *Arcturus*, *Regulus*, *&c.*

23. The stars are also divided into classes, depending on their apparent magnitudes or brightness. Those of the first class, are called stars of the *first magnitude*, those of the second, stars of the *second magnitude*, and so on to stars of the *sixth or seventh magnitude*, which includes all those that are just visible to the naked eye. Those stars which are not visible without the aid of a telescope, are called *telescopic stars*. The distinction of magnitudes is continued in the telescopic stars, from the *seventh* or *eighth* to the *sixteenth;* the latter including those stars that are just visible with the best telescopes.

24. In the triangle ASQ, Fig. 9. having QS and the angle ASQ (2), AQ may be found; then AQ — QB = AB = the right ascension of the star s. When the right ascension of one star is obtained, the right ascension of any other may be found by observing the difference between the time of its passing the meridian and the time of the known star doing the same.

Let s' be the situation of the star s when on the meridian, then its declination $Bs = Qs' = Rs' - RQ = Rs' - (90^\circ - PH) = Rs' + PH - 90^\circ =$ the correct meridian altitude of the star + latitude of the place — 90°.

When the right ascensions and declinations of the stars have been obtained, their longitudes and latitudes may be calculated (18).

25. A table containing a list of fixed stars, designated

by their proper characters, and giving their right ascensions and declinations or their longitudes and latitudes, is called a *Catalogue* of those stars.

26. Hipparchus began the first catalogue of the fixed stars 120 years before the Christian era. This catalogue, with some additions, was afterwards published by Ptolemy, and contained the situations of 1022 stars. The Britannic catalogue, published by Flamstead, in 1689, contained the situations of 3000 stars. Since that period various other catalogues have been published, some of which are very extensive. Bode's Atlas and Catalogue contain the situations of 17,000 stars. The catalogues of Lacaille, Bradley, Mayer, and Maskelyne, are not extensive, but they are valued for their accuracy.

All the fixed stars visible to the naked eye, with some others, are represented on celestial globes of 12 or 18 inches diameter.

27. The number of stars visible with the best telescopes, amounts to several millions: but the number visible to the naked eye, is much less than is generally supposed by those who only judge from the impressions made, when noticing them on a fine evening. The number thus visible at any one time above the horizon does not much exceed 1000.

28. Many of the stars, which to the naked eye, or through telescopes of small power, appear single, are found with high magnifiers to consist of two, three, or more stars, extremely near to each other. These are called *Double Stars.*

29. The fixed stars are not entirely exempt from change. Several stars, which are mentioned by the ancient astronomers, have now ceased to be visible, and some are now visible to the naked eye, which are not in the ancient catalogues.

30. There are some stars which undergo a regular increase and diminution of lustre. These are called *Variable* or *Periodical Stars.*

31. Many spaces are discovered in the heavens, which

are faintly luminous, and shine with a pale white light. On applying to them telescopes of great power, they are found to consist of a multitude of small stars, distinctly separate, but very near to each other. These are called *Nebulæ*. The *Milky Way* is a space of this kind, and is visible to the naked eye.

PRECESSION OF THE EQUINOXES.

32. By comparing catalogues of the same fixed stars, formed at different periods, it is found that their latitudes continue very nearly the same, but that all their longitudes *increase* at the rate of 50″.2 in a year. As the latitude of a star is its distance from the ecliptic (14), it follows from the first mentioned circumstance, that the plane of the ecliptic remains fixed, or very nearly so, with respect to the situations of the fixed stars.

33. The longitude of a star being the arc of the ecliptic, intercepted in the order of the signs, between the vernal equinox, and a circle of latitude passing through the star, (14), it follows from the circumstance of all the stars having the same increase of longitude (32), that the vernal equinox must have a retrograde motion along the ecliptic, that is, a motion in a direction contrary to the order of the signs, amounting to 50″.2 in a year. As the autumnal equinox is always at the distance of 180° from the vernal equinox (2 and 4), it must have the same motion. This retrograde motion of the equinoctial points, is called the *Precession of the Equinoxes.*

34. As the ecliptic remains fixed (32), its pole must also continue in the same place; but the equator and its pole must change their situations, otherwise there could not be a motion in the equinoctial points. Let EC, *Fig.* 11, be the ecliptic, P its pole, $p'p$D a circle about the pole P, at a distance equal to the obliquity of the ecliptic, EQC the equator, and PpLQ the solstitial colure. Then because E is the pole of PLQ, the pole of the equator EQC is in PLQ; it is also in the small circle $p'p$D; it is therefore at

p. Now, if the vernal equinox E, move by a retrograde motion to E′, the solstitial point L will move a like distance to L′; therefore, E′Q′C′ will then be the equator, Pp'L′Q′ the solstitial colure, and p' the pole of the equator. Hence the pole of the equator has a retrograde motion, in a small circle about the pole of the ecliptic, and at a distance from the latter pole, equal to the obliquity of the ecliptic.

The precession of the equinoxes being only 50″.2 in a year, it must require 25817 years for them to move through the whole of the ecliptic; and the pole of the equator will evidently require the same time to make its retrograde revolution about the pole of the ecliptic.

35. The change in the situation of the equator, which produces the precession of the equinoxes, must also produce changes in the right ascensions and declinations of the stars. These changes are different, according to the situations of the stars with respect to the equator and equinoctial points. The change which takes place in the right ascension or declination of a star in the course of a year is called the *Annual Variation* in right ascension or declination.

Let s be the situation of a star, $psab$ a declination circle, when EQC is the situation of the equator, EE′ the annual precession in longitude = 50″.2 (32), and Em and $p'sa'$ declination circles, when the situation of the equator is E′Q′C′. Then the difference between $a's$ and as, is the annual variation in declination of the star s, and the difference between E′a' and Ea, is its annual variation in right ascension. The annual variations in right ascension and declination, may be expressed in formulæ, involving only the right ascension and declination of the star, and the obliquity of the ecliptic.

ANNUAL VARIATION IN DECLINATION.

36. Let sn be equal to sp' and np' be the arc of a great circle; then as the arcs pp' and $p'n$ are evidently very small, we may, without sensible error, consider pp' as the arc of a great circle, and the angle pnp' as a

right angle, excepting, with respect to the latter, the case in which the star is very near the pole. We may also consider any very small arc as equal to its sine or tangent.

Put $\omega = pP = EE'm$ = obliquity of the ecliptic,
$R = Ea$ = right ascension,
and v = variation in declination. Then

$$pp' = \frac{pPp' \sin pP}{\sin pp'P} = \frac{50''.2 \sin \omega}{\sin 90^\circ} = 50''.2 \sin \omega$$

$pn = pp' \cos p'ps = 50''.2 \sin \omega \cos p'ps$.

But $p'ps = 90^\circ - apQ = EQ - aQ = Ea = R$,

and $pn = sp - sn = sp - sp' = 90^\circ - as - (90 - a's) = a's - as = v$.

Therefore $v = 50''.2 \sin \omega \cos R$.

When the declination is north, as in the figure, the sign of v is the same as the sign of $\cos R$; but when the declination is south, the sign of v must be contrary to that of $\cos R$.

ANNUAL VARIATION IN RIGHT ASCENSION.

37. Let $D = as$ = declination of the star, and V = its annual variation in right ascension. Then

$V = E'a' - Ea = E'a' - mb = E'm + a'b.$;

$E'm = E'E \cos EE'm = 50''.2 \cos \omega$;

$p'n = pp' \sin p'ps = 50''.2 \sin \omega \sin R$;

$$psp' = \frac{p'n}{\sin p's} = \frac{50''.2 \sin \omega \sin R}{\cos a's}; \text{ and}$$

$$a'b = a'sb \sin a's = psp' \sin a's = \frac{50''.2 \sin \omega \sin R \sin a's}{\cos a's} = 50''.2 \sin \omega \sin R \tan a's.$$

As the quantity $50''.2 \sin \omega \sin R$, which is multiplied by $\tan a's$, is very small, and as the difference between $a's$ and as can only be a few seconds, we may, without sensible error, consider $\tan a's = \tan as = \tan D$. Therefore

$a'b = 50''.2 \sin \omega \sin R \tan D$, and

$V = E'm + a'b = 50''.2 \cos \omega + 50''.2 \sin \omega \sin R \tan D$.

When the declination is south, $\tan D$ must be taken negative. The second term of the value of V is negative when the right ascension is less than 180° and declination south, or when the right ascension is more than 180° and declination north. In either of these cases, when $\sin R \tan D$ is so great as to make the second term exceed the first, the value of V is negative.

$E'm = 50''.2 \cos \omega = 46''$, is the annual, retrograde motion of the equinoctial points along the equator.

38. In catalogues of the fixed stars, which express their situations by right ascensions and declinations, the annual variations in these are also stated, with their proper signs. In some catalogues the north polar distances of the stars are given, instead of the declinations. The variations will be the same, except that they will have a *contrary* sign, when the north polar distance is less than 90°.

39. With the right ascension and declination of a star for a given time and their annual variations, its right ascension and declination may be found, with considerable accuracy, for a time a few years later or earlier. Put $t =$ the number of years, then $t. V =$ its change in right ascension and $t. v =$ its change in declination, nearly. If the time for which the right ascension and declination are required, is after the given time, $t.$ V and $t. v$ must be applied, with their signs as determined by the preceding formulæ; but, if it is before the given time, t must be considered negative, which will change the signs of $t.$ V and $t. v$.

When, from the right ascension and declination of a star for a given time, its right ascension and declination are required with accuracy, for a time several years earlier or later, they can be found by rigorous formulæ, which have been investigated for the purpose; or with nearly the same facility, by calculating the longitude and latitude of the star for the given time (18), adding to the longitude the precession in longitude, which will be the product of $50''.2$ by the interval of time expressed in years and parts, and then with the longitude thus obtained and the latitude, calculating the right ascension and declination.

40. In consequence of the precession of the equinoxes, the twelve signs of the ecliptic, which about 2000 years ago, respectively corresponded with the twelve constellations of the zodiac, bearing the same names, have receded so far, that the sign Taurus now corresponds nearly with the constellation Aries.

41. The visible effect of the precession on the aspect of the heavens becomes, in process of time, particularly

conspicuous in the positions of the northern stars, with reference to the north pole of the heavens. Since this pole has a continual retrograde motion in a circle distant about 23½° from the pole of the ecliptic, which remains nearly fixed, its distance from any of the surrounding stars must in time materially change. The star which is called the pole star, the distance of which from the pole is at present only 1° 34′, was at the time of the formation of the earliest catalogues, at the distance of 12°. It will continue to approach the pole till its distance will be within half a degree, and will then begin to recede. In about 12,000 years from the present time, the very bright star α Lyræ, which is now at a distance of more than 50° from the pole, will approach it within about 5°, and will of course be the pole star. At that period our present pole star will be more than 40° from the pole.

42. In the preceding investigations, we have considered the plane of the ecliptic as fixed and the obliquity of the ecliptic as continuing always the same. This, though very nearly, is not strictly true. A comparison of accurate observations, made at long intervals of time, proves that each is subject to a slight change. These changes will be noticed in a succeeding chapter.

CHAPTER VII.

SUN'S APPARENT ORBIT.—KEPLER'S LAWS.—KEPLER'S PROBLEM.

1. It has been shown (6.37) that the vernal equinox has a retrograde motion along the equator of 46″ a year. This is its mean motion. It has been found from accurate observations that this motion is not uniform.

The place at which the vernal equinox would, at any time, be, if its motion was uniform, is called its *Mean* place, or the *Mean Equinox*.

2. The motion of the mean equinox along the equator

being 46″ a year, its motion in one day must be $\frac{1}{8}$ of a second nearly, which corresponds to $\frac{1}{120}$ of a second in time (3.6). If therefore on any particular day the mean equinox be on the meridian at precisely the same instant with some fixed star, it would, in consequence of its retrograde motion, come to the meridian on the succeeding day $\frac{1}{120}$ of a second earlier than the star.

3. Some astronomers, with a view to convenience in observing the right ascensions of the heavenly bodies, regulate their clocks so as to mark 0 h. 0 m. 0 sec. when the mean equinox is on the meridian, and they call the interval between two of its consecutive passages over the meridian, a *Sidereal Day.*

The term sidereal day is now generally used as here defined, and is to be thus understood in the following parts of the work. But on account of the very small difference between its length and the length of the sidereal day as defined in a preceding chapter (3.5), we may consider them as equal in all cases that regard observations made during a single day or a small number of days.

4. The time between two consecutive passages of the sun's centre, over the meridian, is called a *True Solar Day.* In consequence of the sun's motion eastward among the fixed stars (6.1), the length of a solar day is greater than that of a sidereal day.

It is ascertained by observations that the length of a solar day is different at different times in the year, but that at the same time in different years it is very nearly the same. By comparing the number of solar days that elapse from the time that the sun passes the meridian on a given day in any year, to the time of its passage on the same day in some succeeding year, with the number of sidereal days and parts of a day, that elapse during the same time, it is found that the mean length of a solar day, called a *Mean Solar Day*, is equal to 24 h. 3 m. 56.555 sec. of sidereal time.

5. The ratio of 24 h. : 24 h. 3 m. 56.555 sec. is the same as 1 : 1.0027379; and therefore one solar day is equivalent

to 1.0027379 days of sidereal time. Hence to reduce a given portion of mean solar time to the corresponding sidereal time, we must multiply by 1.0027379; and on the contrary, to reduce sidereal to mean solar time, we must divide by the same number.

The excess of a mean solar day above a sidereal day is 3 m. 56.555 sec. in sidereal time; and in mean solar time it is 3 m. 55.91 sec.

6. By observing the altitude of the sun when on the meridian of a place whose latitude is accurately known, its north polar distance may be determined (6.2). If this be done on several successive days, about the 20th of March, it will be found, either that on some one of these days, the north polar distance is exactly 90°, and consequently that the sun is then at the equinox, or which is much more probable, that on the first of some two consecutive days the north polar distance is greater, and on the second less, than 90°. From these observations, the time that the sun is at the vernal equinox may be determined.

Let A, *Fig.* 12, be the sun's place on the first of these two days, B its place on the second, CD a portion of the equator, P its north pole, and AB a portion of the ecliptic. Then will E be the place of the vernal equinox.

The arcs PA and PB are known; and from the interval in sidereal time between the two observations, the angle APB or its measure, the arc CD, is likewise known; it being evidently equal to the excess of the interval above 24 hours, converted into degrees or parts of a degree. But,

$$\tan AC = \tan E \sin EC,$$

$$\tan BD = \tan E \sin ED;$$

$$\text{therefore } \tan AC : \tan BD :: \sin EC : \sin ED,$$

or, $\tan AC + \tan BD : \tan AC - \tan BD :: \sin EC + \sin ED : \sin EC - \sin ED$.

From whence we have (App. 32),

$\sin (AC + BD) : \sin (AC - BD) :: \tan \frac{1}{2} (EC + ED) : \tan \frac{1}{2} (EC - ED,)$

$$\text{or } \tan \tfrac{1}{2} (EC - ED) = \frac{\tan \frac{1}{2} (EC + ED) \sin (AC - BD)}{\sin (AC + BD)}$$

$$= \frac{\tan \frac{1}{2} CD \sin (PA - 90^\circ - 90^\circ + PB)}{\sin (PA - 90^\circ + 90^\circ - PB)}$$

$$= \frac{\tan \frac{1}{2} CD \sin (PA + PB - 180^\circ)}{\sin (PA - PB)}.$$

Now knowing CD and ½ (EC—ED), we know EC and ED, which are the sun's distances from the equinox at the times of observation. The sun's motion in right ascension during a day may be considered uniform, particularly near the equinox, as may be determined by observing it for several days about that time. If, therefore, CE converted into time (3.6), be added to the sidereal time of the first observation, we shall have the sidereal time at which the sun is at the equinox.

7. By similar observations made the ensuing year, the time of the sun's return to the vernal equinox will be known.

8. The interval of time between two consecutive returns of the sun to the vernal equinox is called a *Tropical Year.*

The ancient astronomers determined the length of the year from the sun's return to the same tropic, and thence applied to it the term tropical year*, which is still retained.

9. The length of the tropical year is subject to a slight variation. By observations made at intervals of 50, 60 or 100 years, its mean length, expressed in mean solar time, is found to be 365 d. 5 h. 48 m. 48 sec.

Hence 365 d. 5 h. 48 m. 48 sec. : 1 day :: 360° : 59′ 8.″ 33 = sun's mean motion in longitude during a mean solar day.

10. On account of the annual precession of the equinox in longitude, which is 50.″2 (6.33), the sun only passes through an arc of the ecliptic equal to 359° 59′ 9″.8, during a tropical year.

11. The time in which the sun passes through the whole 360° of the ecliptic, or which is the same thing, the interval of time between two consecutive returns of the sun to the same fixed star, is called a *Sidereal Year.*

Hence 359° 59′ 9″.8 : 360° :: 365 d. 5 h. 48 m. 48 sec. : 365 d. 6 h. 9 m. 10 sec. = the length of a sidereal year, expressed in mean solar time. The sidereal year therefore exceeds the solar, by 20 m. 22 sec.

12. When the sun's apparent diameter is accurately observed, at different seasons in the year, it is found to

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* The term *Equinoctial Year*, which is more appropriate, is now sometimes used.

vary. It is greatest about the first of January and continually decreases till about the first of July, when it is least. It then increases till the first of January. When greatest it is 32′ 34″.6, and when least it is 31′ 30″.1. Consequently the mean diameter is 32′ 2″.3. As there is no reason to suppose that this change in the *apparent* diameter is caused by a change in its *real* diameter, it is inferred that the sun's distance from the earth is variable.

13. From a comparison of the sun's apparent diameter, as observed at any two different times, we may obtain the ratio of its distances from the earth at those times. Let AB and A′ B′, *Fig.* 13, be the sun in two different situations, and E the place of the earth.

Put δ = apparent diameter AEB
δ' = apparent diameter A′EB′
$D = ES$ and $D' = ES'$.
Then $D \sin \frac{1}{2} \delta = AS = A'S' = D' \sin \frac{1}{2} \delta'$,

$$\text{or, } \frac{D}{D'} = \frac{\sin \frac{1}{2} \delta'}{\sin \frac{1}{2} \delta} = \frac{\frac{1}{2} \delta'}{\frac{1}{2} \delta} = \frac{\delta'}{\delta}.$$

14. The apparent diameter of the sun, when at his mean distance from the earth, is 82′ 1″.8.

In the same manner as for the moon's parallax (5.15), if δ, δ', and δ'', be the sun's apparent diameter, at its greatest, least, and mean distances, we shall have,

$$\delta'' = \frac{2\,\delta.\delta'}{\delta + \delta'} = 32'\ 1''.8.$$

15. From the sun's apparent diameter and horizontal parallax, its real diameter may be determined. It is about 111 times the earth's diameter.

If $d = 2$ AS = sun's real diameter, we have,

$$d = 2\,D \sin \tfrac{1}{2}\,\delta = (5.6)\ 2\,R \frac{\sin \frac{1}{2}\,\delta}{\sin \varpi} = R \frac{\sin\ \delta}{\sin \varpi} = R . \frac{\delta}{\varpi} = R\,\frac{1921''.8}{8''6.}$$

$= R \times 223 = 111\frac{1}{2}$ times the earth's diameter.

16. The sun's right ascension may be obtained by observing the sidereal time of its passage over the meridian.

From the right ascension, the longitude may be calculated (6.19). The sun's longitude thus obtained, at different times in the year, does not increase uniformly with the time. Its greatest motion in longitude during a mean solar day is $61'\ 10''$, and takes place at the time its apparent diameter is greatest. Its least motion is $57'\ 11''$, and takes place when the apparent diameter is least.

17. The curve which the sun's centre seems to describe in the plane of the ecliptic, during a year, is called the *Apparent Orbit* of the sun.

18. A right line, conceived to be drawn from the centre of the sun to the centre of the earth, or to the centre of a planet, is called the *Radius Vector* of the earth or planet. A right line joining the centres of the earth and moon, is called the radius vector of the moon.

19. It appears from the change in distance between the sun and earth (12), that the sun's apparent orbit is not a circle; or at least that the earth does not occupy the centre.

Let ADBF, *Fig*. 14, be the apparent orbit of the sun, E the earth's place, A the sun's place when the apparent diameter is least, B its place when the apparent diameter is greatest, and D its place at some other time. The difference between the sun's longitudes at A and B, that is when the apparent diameters are least and greatest, is found to be 180°. It follows therefore that EA and EB must form one straight line AB. If AB be bisected in C, then $AC = \frac{1}{2} AB = \frac{1}{2} (AE + EB)$.

The angle AED = sun's long. at D — sun's long. at A.

20. The apparent orbit of the sun is an ellipse, having the earth in one focus. This fact was discovered by Kepler, and it is called *Kepler's first Law*. It is deduced from investigations, founded on the observed apparent diameter of the sun at different longitudes.

Put $\delta =$ sun's apparent diameter at A,
$\gamma =$ do. B,
$\gamma' =$ do. D,

$m = \frac{1}{2}(\gamma + \delta) =$ sun's mean apparent diameter,
and $n = \frac{1}{2}(\gamma - \delta)$.

From the sun's longitude and apparent diameter, as obtained at different times in the year, it is found by observation that whatever be the situation of D, we have $\gamma'' = m - n \cos AED$.

$$\text{But (13)}\ AE : EB :: \gamma : \delta,$$
$$AE : AE + EB :: \gamma : \gamma + \delta,$$
$$AE : \tfrac{1}{2}(AE + EB) :: \gamma : \tfrac{1}{2}(\gamma + \delta),$$
$$AE : AC :: \gamma : m = \frac{AC}{AE}.\gamma.$$

$$\text{Again,}\ AE : EB :: \gamma : \delta,$$
$$AE + EB : AE - EB :: \gamma + \delta : \gamma - \delta,$$
$$2\,AC : 2\,EC :: \tfrac{1}{2}(\gamma + \delta) : \tfrac{1}{2}(\gamma - \delta),$$
$$\text{or}\ AC : EC :: m : n = \frac{EC}{AC}m = \frac{EC}{AC}.\frac{AC}{AE}\gamma = \frac{EC}{AE}\gamma.$$

Hence substituting the values of m and n, in $m - n \cos AED$, we have,

$$\gamma'' = \frac{AC}{AE}\gamma - \frac{EC}{AE}\gamma \cos AED.$$

$$\text{But (13)}\ \gamma'' = \frac{EB}{ED}\gamma;\ \text{therefore,}$$

$$\frac{AC}{AE}\gamma - \frac{EC}{AE}\gamma \cos AED = \frac{EB}{ED}\gamma,$$
$$\frac{AC}{AE} - \frac{EC}{AE}\cos AED = \frac{EB}{ED},$$

$ED \cdot (AC - EC \cos AED) = AE.\,EB = (AC + EC).\,(AC - EC) = AC^2 - EC^2$,

$$\text{or}\ ED = \frac{AC^2 - EC^2}{AC - EC \cos AED}.$$

But (Conic Sections*) the last equation expresses a property of an ellipse, of which AB is the transverse axis, C the centre, and E one of the foci.

21. The point which is the sun's place, when most distant from the earth, is called the *Apogee;* and the point which is its place when nearest the earth, is called the *Perigee.* Those points are also called *Apsides;* the most distant being called the *Higher Apsis*, and the nearest, the *Lower Apsis.* The transverse axis, which joins the apsides, is called the *Line of the Apsides.*

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* The proposition here referred to, though an important one, is omitted in several of our treatises on Conic Sections. It is therefore demonstrated in the Appendix, article 51.

22. The distance between the centre of the earth and the centre of the sun's apparent orbit, is called the *Eccentricity* of the orbit.

23. Astronomers usually call the mean distance of the earth from the sun, a unit or 1, and express other distances in conformity with this assumption.

24. Considering the earth's mean distance from the sun equal 1, the eccentricity of the sun's apparent orbit is .0168 nearly.

Put AC = 1, and e = EC = the eccentricity; then (20),

$$n = \frac{EC}{AC} m = \frac{e}{1} m = em,$$

$$\text{or } e = \frac{n}{m} = \frac{32''.25}{1922''.35} = .0168 \text{ nearly.}$$

25. Let ADBP, *Fig.* 15, be the sun's apparent orbit, E the place of the earth, D the sun's place at any time, and A and B the apogee and perigee. Then by the apparent motion of the sun, the radius vector ED, moves about E in such a manner that the area of the sector AED, increases uniformly with the time. This fact, discovered by Kepler, is usually expressed by saying, the *Radius Vector describes equal Areas in equal Times.* It is called *Kepler's Second Law.*

If the circle AGB be described on the transverse axis AB, and HDG be drawn perpendicular to AB, then (Conic Sections*),

$$\frac{AC^2 - EC^2}{AC - EC \cos AED} = ED = AC - EC \cos BCG = AC + EC \cos ACG;$$

$$\text{hence, } EC \cos ACG = \frac{AC^2 - EC^2}{AC - EC \cos AED} - AC = \frac{AC.\,EC \cos AED - EC^2}{AC - EC \cos AED}.$$

$$\text{or } \cos ACG = \frac{AC \cos AED - EC}{AC - EC \cos AED} = \frac{\cos AED - e}{1 - e \cos AED} \qquad \text{(A).}$$

Now the area of ECG = $\frac{1}{2}$ EC. CG sin ACG = $\frac{1}{2}$ e sin ACG,

area of ACG = AC. $\frac{1}{2}$ arc AG = $\frac{1}{2}$ arc AG,

area of AEG = $\frac{1}{2}$ (e sin ACG + arc AG) (B).

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* See Appendix, article 52.

Hence the area AEDA $= \frac{RC}{AC} \cdot AEG = \frac{\sqrt{(1-e^2)}}{2} \cdot (e \sin ACG + \text{arc } AG)$ (C).

The eccentricity e being known (24), and also the angle AED, from the sun's longitudes at A and D (19), the angle ACG or arc AG becomes known (A); and thence (C) the area of the elliptical sector AEDA. The area of the sector AEDA, thus obtained at different times in the year, is found to increase uniformly with the time.

26. If we suppose the sun to be situated at E, and the earth to revolve round it in the orbit ADBP, and if we admit the distances of the fixed stars to be so great that straight lines, conceived to be drawn from the points E and A to meet at any one of them, will not contain an appreciable angle, or which amounts to the same thing, that these lines may be considered as sensibly parallel; then the sun will appear to move exactly in the same manner as on the supposition that it is really in motion about the earth at rest at E.

Let us suppose some one of the fixed stars to coincide with the vernal equinox, and let EQ and TAQ′, be two straight lines which being produced would meet at this star. Then on the supposition that the earth is at rest at E, QEA is the sun's longitude at A, and QED is its longitude at D. Therefore AED = QED — QEA, is the difference of their longitudes at A and D.

On the supposition that the sun is at rest at E, and considering EQ and TQ′ as parallel, its longitude when the earth is at A, is 180° + EAT = 180° + QEA. In like manner its longitude, when the earth is at D, is 180° + QED. But 180° + QED —(180°+QEA)=QED — QEA = AED. The difference of its longitudes, and consequently its apparent motion, is therefore the same on the two suppositions.

But when we consider that the sun's diameter is more than a hundred times the diameter of the earth (15), and consequently its magnitude more than a million times the magnitude of the earth, it seems far more reasonable to suppose that the sun's apparent motion is produced by a real motion of the earth, than to retain the contrary supposition.

27. The apparent diurnal motion of the heavenly bodies from east to west may likewise be accounted for, by admitting the earth to have a diurnal motion on its axis in a contrary direction, that is from *west* to *east*. And it is certainly more reasonable to suppose this diurnal motion of the earth, than to suppose that all the heavenly bodies, situated at various and immensely great distances, should have motions so adjusted as that all should revolve round the earth in the same length of time.

Various astronomical phenomena serve to prove that the earth really has these two motions; that is, an annual motion round the sun, and a diurnal motion on its axis.

During the annual motion, the *earth's axis continues parallel to itself;* or in other words, if we suppose a right line to remain fixed in the position which the earth's axis has in one part of the orbit, the axis continues during the whole annual revolution nearly parallel to that line.

As the earth's axis is perpendicular to the plane of the equator, the angle contained between it and a right line passing through the centre of the earth perpendicular to the ecliptic, must be equal to the angle contained by these planes, that is to the obliquity of the ecliptic. The line perpendicular to the plane of the ecliptic is called the *Axis of the Ecliptic.*

28. Although the particular consideration of the planets is referred to a succeeding part of the work, it may be here observed that all of them, including the earth as one, revolve round the sun, from *west* to *east*, at different distances and in different times; and that the moon revolves round the earth and with it round the sun. This system of the sun and planets is called the *Copernican System*, from its inventor *Copernicus.*

The order of the planets with respect to their distances from the sun, is *Mercury*, *Venus*, *the Earth*, *Mars*, *Vesta*, *Juno*, *Ceres*, *Pallas*, *Jupiter*, *Saturn and Uranus.*

29. The same reasoning that has been used to prove

that the sun's apparent orbit is an ellipse, and that the radius vector describes equal areas in equal times, applies, when we suppose the sun at rest, to prove that the earth's orbit is an ellipse, having the sun in one focus, and that its radius vector describes equal areas in equal times.

Kepler, extending his researches, found that the orbits of the planets are ellipses, and that the radius vector of each describes equal areas in equal times.

30. Another important law, discovered by Kepler, is, that the square of the time in which any planet revolves round the sun, is to the square of the time in which another planet does the same, as the cube of the mean distance of the former, from the sun, is to the cube of the mean distance of the latter. This relation is usually expressed by saying the *Squares of the times of revolution of the Planets are as the Cubes of their mean distances from the Sun.* It is called *Kepler's Third Law.*

31. Although astronomers have completely established the fact of the earth's annual and diurnal motions, yet, with a view to convenience of expression, they still frequently speak of the sun's orbit and of the motion of the sun.

32. The point in the earth's or a planet's orbit, which is the most distant from the sun, is called the *Aphelion;* and the nearest point is called the *Perihelion.* The terms apogee and perigee are only used to express the greatest and least distances from the earth.

33. Let D and F, *Fig.* 14, be two situations of the sun in its apparent orbit, at which its apparent diameter is the same. Then ED = EF, and consequently the angle AED = AEF. Hence from the longitudes of the sun at F and D, the longitude of A, the apogee, becomes known. The following is however a more accurate method of determining the place of the apogee.

The transverse axis AB divides the ellipse into two equal parts. The sun will therefore be as long in moving from A to B as from B to A (25). Hence it will be half a year in moving from A to B, and in this time it will change its longitude 180°. This will not be the case for any other points in the orbit, because no other right line, passing through E, divides the ellipse into equal parts. If therefore two longitudes of the sun be observed at the interval of half a year, and such that their difference is 180°, the position of the line of the apsides becomes known.

We have supposed the position of the line of the apsides to remain fixed. But observations made at long intervals of time, prove that it has a slow motion in the order of the signs. Astronomers therefore modify the preceding methods of determining the position of the apsides, so as to take notice of this motion, which however only amounts to a few seconds in a year.

34. According to Bessel, the longitude of the apogee at the beginning of the year 1800, was 3*s*. 9° 30′ 8″, and its yearly increase of longitude is 61″.52.

35. If we reduce 3*s*. 9° 30′ 8″ to seconds, and divide the result by 61″.52, the quotient is 5823. Hence it appears that about 5823 years anterior to the year 1800, that is, a little more than four thousand years anterior to the Christian era, the longitude of the apogee was nothing, and consequently, *the line of the apsides coincided with the line of the equinoxes.* It is worthy of remark, that this is about the period on which chronologists fix, as the time of the creation of the world.

36. If from 61″.5, the annual motion of the apogee from the equinox, we subtract 50″.2, the annual precession of the equinoxes, the remainder, which is 11″.3, is the mean annual motion of the apogee.

37. The time between two consecutive returns of the sun to the apogee, is called an *Anomalistic Year.*

As 359° 59′ 9″.8 : 360° 0′ 11″3. : : 365d. 5h. 48m. 48sec. : 365d. 6h. 13m. 46sec.=the length of the anomalistic year, expressed in mean solar time. The anomalistic year, therefore, exceeds the tropical, by 24m. 58sec.

As 365d. 6h. 13m. 46sec. : 1 day :: 360° : 59′ 8″.16= sun's mean motion from the apogee during a mean solar day.

38. At the time that the true place of the sun is at D, *Fig.* 15, let F be the place at which it would have been, if it had moved from the apogee with its mean angular motion of 59′ 8″.16 a day.

39. The angle, contained between the line of the apsides and the radius vector, is called the *True Anomaly.* Thus AED is the true anomaly.*

40. The angle, contained between the line of the apsides and the straight line from the earth to the mean place of the sun, is called the *Mean Anomaly.* Thus AEF is the mean anomaly.

The angle ACG is called the *Eccentric Anomaly.*

41. The angle, which is the difference between the mean and true anomalies, is called the *Equation of the Centre.*

42. The equation of the centre DEF, expresses the difference between the mean longitude QEF and the true longitude QED. The mean longitude and mean anomaly increase uniformly with the time. They may, therefore, be easily determined for any particular point of time. Then, if the equation of the centre, corresponding to the mean anomaly, be known, the true longitude becomes also known.

43. The problem for determining the true anomaly from the mean, and which therefore determines the equation of the centre, is called *Kepler's Problem.* It is a problem of great importance, and has been solved in various ways. The following method combines simplicity with a requisite degree of accuracy.

Let CL be drawn parallel to EF, LO parallel to GC, and EM perpendicular to GC produced. Put,

$T =$ time of describing the whole ellipse.
$t =$ time of describing AD,
$e =$ EC, AC being $= 1$,
$u =$ ang. AED $=$ true anomaly,
$x =$ arc AG $=$ ang. ACG $=$ eccentric anomaly,

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* This is the anomaly reckoned from the apogee. Astronomers frequently reckon the anomaly from the perigee. The anomaly of the sun at D, reckoned from the perigee, would be 180° + ang. AED.

z = arc AL = ang. ACL = ang. AEF = mean anomaly,
M = area of the circle AGB,
N = area of the ellipse ADBP,
p = 6.28318 &c. = circumference of the circle.

Then $\mathrm{M} = \mathrm{AC} \times \text{arc AGB} = 1 \times \tfrac{1}{2} p = \tfrac{1}{2} p$.

Now (Conic Sections)

$$\mathrm{M} : \mathrm{N} :: \mathrm{AC} : \mathrm{CR} :: \mathrm{AEGA} : \mathrm{AEDA},$$

$$\mathrm{M} : \mathrm{AEGA} :: \mathrm{N} : \mathrm{AEDA} :: \mathrm{T} : t :: 360^\circ : \text{ang. AEF} :: p : z;$$

$$\text{hence } \mathrm{AEGA} = \frac{\mathrm{M}}{p} . z = \frac{\frac{1}{2} p}{p} . z = \tfrac{1}{2} z.$$

$$\text{But (25.B) } \mathrm{AEGA} = \tfrac{1}{2} (e \sin x + x).$$

$$\text{Therefore } z = x + e \sin x. \qquad \text{(D).}$$

$$\text{Again (25.A) } \cos x = \frac{\cos u - e}{1 - e \cos u},$$

$$\cos x - e \cos u \cos x = \cos u - e,$$

$$\cos u + e \cos u \cos x = e + \cos x,$$

$$\cos u = \frac{e + \cos x}{1 + e \cos x}. \qquad \text{(E).}$$

The last formula may be converted into another that will be more convenient for logarithmic computation. We have (App. 12),

$$\tan^2 \tfrac{1}{2} u = \frac{1 - \cos u}{1 + \cos u} = \frac{1 - \dfrac{e + \cos x}{1 + e \cos x}}{1 + \dfrac{e + \cos x}{1 + e \cos x}}$$

$$= \frac{1 + e \cos x - e - \cos x}{1 + e \cos x + e + \cos x}$$

$$= \frac{(1 - e) - (1 - e) \cos x}{(1 + e) + (1 + e) \cos x} = \frac{1 - e}{1 + e} . \frac{1 - \cos x}{1 + \cos x} = \frac{1 - e}{1 + e} \tan^2 \tfrac{1}{2} x.$$

$$\text{Hence } \tan \tfrac{1}{2} u = \tan \tfrac{1}{2} x \sqrt{\frac{1 - e}{1 + e}}. \qquad \text{(F).}$$

We have now an expression for the true anomaly in terms of the eccentric, and (D) an equation showing a relation between the mean and eccentric anomalies. But in consequence of the latter containing both x and $\sin x$, we cannot, in a direct manner, obtain the eccentric from the mean anomaly. There are however various methods of approximation which give its value to any required degree of accuracy.

44. We have by trigonometry,

$$\mathrm{EM} = \mathrm{EC} \sin \mathrm{ECM} = \mathrm{EC} \sin \mathrm{ACG} = e \sin x;$$

but (43.D) $e \sin x = z - x$ = arc AL — arc AG = arc LG. Therefore EM = arc LG.

Because OL is parallel to MG, and EM is perpendicular to it, OM = sin LG. The difference between EM and OM is therefore equal to the difference between the arch LG and its sine; and in all cases for the sun and planets, this difference is small, as may be thus shown. Since LG $= e \sin x$, it is evident that LG will be greatest when $x = 90°$, and then we have LG $= e \sin 90° = e$. But for the sun's apparent orbit $e = .0168$ nearly (24); therefore LG $= .0168 = 0° \; 57'\tfrac{3}{4}$. The value of e for the most eccentric orbit of the planets is about .254, the mean distance from the sun being expressed by 1. Consequently LG when greatest is about 14°½.

The difference between an arc of 14°½ and its sine, is but little, and for an arc of 57'¾ it is much less. It follows therefore that in all cases, OM is nearly equal to the arc LG or its equal, the right line EM. Consequently, because OL is parallel to CG, EL is nearly parallel to it, and the angle AEL is nearly equal to the eccentric anomaly ACG.

$$\begin{aligned} \text{Now } z &= \text{ACL} = \text{CEL} + \text{CLE}. \\ \text{Put } \theta &= \text{CEL} - \text{CLE}, \\ w &= \text{CEL}, \\ y &= \text{ACG} - \text{CEL} = x - w. \\ \text{Then } x &= w + y. \end{aligned}$$

By trigonometry,

$$\text{CL} + \text{EC} : \text{CL} - \text{EC} :: \tan \tfrac{1}{2} (\text{CEL} + \text{CLE}) : \tan \tfrac{1}{2} (\text{CEL} - \text{CLE}),$$

$$\text{or } 1 + e : 1 - e :: \tan \tfrac{1}{2} z : \tan \tfrac{1}{2} \theta.$$

$$\text{Therefore } \tan \tfrac{1}{2} \theta = \frac{1 - e}{1 + e} \tan \tfrac{1}{2} z.$$

$$w = \text{CEL} = \tfrac{1}{2} z + \tfrac{1}{2} \theta.$$

Now (43. D) and (App. 13),

$z = x + e \sin x = w + y + e \sin (w + y) = w + y + e \sin w \cos y + e \cos w \sin y$.

But because y is very small we may take $\cos y = 1$ and $\sin y = y$.

Then $z = w + y + e \sin w + e\, y \cos w$.

$$y + e\, y \cos w = z - (w + e \sin w),$$

$$y = \frac{z - (w + e \sin w)}{1 + e \cos w};$$

and $x = w + y$, becomes then known.

If greater accuracy is required, put $w + y = x'$, and $y' = x - x'$. Then reasoning as above we have,

$$y' = \frac{z - (x' + e \sin x')}{1 + e \cos x'}.$$

and $x = x' + y'$.

The last formula is not necessary except very great accuracy is required. The value of $x = w + y$, is not liable to a greater error than $\frac{1}{10}$ of a second for the most eccentric orbit of the planets. For the sun, the error cannot amount to more than about $\frac{1}{10}$ of a second if we take $x = w$.

When the value of x is found, we obtain the true anomaly from the expression

$$\tan \tfrac{1}{2} u = \tan \tfrac{1}{2} x \sqrt{\frac{1 - e}{1 + e}}.$$

45. Some astronomers have proposed that the anomaly should always be reckoned from the perigee, because it is necessary to do so for the orbits of comets.

If 180° be added to the mean anomaly, reckoned from the perigee, rejecting 360° when the sum exceeds it, the result will be the mean anomaly reckoned from the apogee. Then the true anomaly from the apogee may be found by the preceding formulæ. The difference between these will be the equation of the centre.

EXAMPLE I.

Given the sun's mean anomaly from the perigee 8*s*. 25° or from the apogee 2*s*. 25°, and the eccentricity of the orbit .016774; required the equation of the centre.

Here $z = 2s.\ 25° = 85°$,
and $e = .016774$.

Log. $(1-e)$	.983226	9.9926533
log. $(1+e)$	1.016774	0.0072244
log. $\frac{1-e}{1+e}$		9.9854289
tan $\frac{1}{2}z$	42° 30′	9.9620525
tan $\frac{1}{2}\theta$	41 32 38″.6	9.9474814
$w =$	84 2 38.6	
$\frac{1}{2}x = \frac{1}{2}w =$	42 1 19.3	
Log. $\frac{1-e}{1+e}$		9.9854289
log. $\sqrt{\frac{1-e}{1+e}}$		9.9927144
tan $\frac{1}{2}x$	42° 1′ 19″.3	9.9547732
tan $\frac{1}{2}u$	41° 32′ 40″	9.9474876
$u =$	83 5 20	
$z =$	85 0 0	
$u - z = -$	1 54 40 = equation of sun's centre.	

EXAMPLE II.

Given Mercury's mean anomaly from the aphelion 1*s*. 28°, and the eccentricity of its orbit .205513; required the equation of the centre.

$z = 58°$ and $e = .205513$.

Log. $(1 - e)$	.794487	9.9000868
log. $(1 + e)$	1.205513	0.0811719
log. $\frac{1-e}{1+e}$		9.8189149
$\tan \frac{1}{2} z$	29° 0′ 0″	9.7437520
$\tan \frac{1}{2} \theta$	20 4 4.7	9.5626669
$w =$	49 4 4.7	
Log. $e =$ .205513		9.3128393
log. 206264″.8		5.3144251
$\sin w$ 49° 4′ 4″.7		9.8782272
$e \sin w =$ 32025″.2 $=$	8° 53′ 45″.2	4.5054916
$w =$	49 4 4.7	
$w + e \sin w =$	57 57 49.9	
$z =$	58 0 0	
$z - (w + e \sin w)$	0 2 10.1 = 130″.1	
log. $e =$ .205513		9.3128393
$\cos w$ 49° 4′ 4″.7		9.8163495
$e \cos w$ .1346		9.1291888

$1 + e \cos w$ 1.1346

$$y = \frac{130''.1}{1.1346} = 114''.7 = 1' \; 54''.7$$

$$x = w + y = 49° \; 4' \; 4''.7 + 1' \; 54''.7 = 49° \; 5' \; 59''.4$$

log. $\frac{1-e}{1+e}$		9.8189149
log. $\sqrt{\frac{1-e}{1+e}}$		9.9094574
$\tan \frac{1}{2} x$	24° 32′ 59″.7	9.6591633
$\tan \frac{1}{2} u$	20 20 44.9	
$u =$	40 41 29.8	
$z =$	58 0 0	

$u - z = -$ 17 18 30.2 = equation of the centre.

46. When the eccentricity of the orbit is known, the true radius vector, corresponding to any given mean anomaly, may thence be determined.

Put $r = \text{ED} =$ the radius vector ; then because (25),
$\text{ED} = \text{AC} + \text{EC} \cos \text{ACG}$, we have,
$r = 1 + e \cos x$.

Hence, having computed the eccentric anomaly, corresponding to the given mean anomaly, we easily obtain the value of the radius vector.

47. If we divide 360° by the number of hours in the time of revolution, of the sun or a planet, in its orbit, the result will be the mean hourly motion. If H be the mean hourly motion, H′ the true hourly motion, e the eccentricity of the orbit, and r the radius vector, then we have for the true hourly motion, the following expression,

$$\text{H}' = \text{H}. \frac{\sqrt{(1 - e^2)}}{r^2}.$$

Let a and b be the places of the body, half an hour before and after it is at P. Then will the angle $a\text{E}b$, be its true hourly motion in this part of its orbit. With the centre E and radius EP describe the arc cd. Then since the angle $a\text{E}b$ is very small, we may consider the elliptical sector $a\text{E}ba$, as equal to the circular sector $c\text{E}dc$.

Now the circumference of the circle of which EP is radius, is equal EP. $p = rp$; and consequently the area of the same circle is $= r. \frac{1}{2}rp = \frac{1}{2}r^2p$. Then,

$$360^\circ : \text{H}' :: \tfrac{1}{2} r^2 p : c\text{E}dc,$$

$$c\text{E}dc = \frac{\text{H}'}{360^\circ}. \tfrac{1}{2} r^2 p.$$

Again (43) $360^\circ : \text{H} :: \text{T} : 1 :: \text{N} : ab\text{E}a :: \text{N} : c\text{E}dc$,

$$c\text{E}dc = \frac{\text{H}}{360^\circ}. \text{N} = \frac{\text{H}}{360^\circ}. \frac{\text{CR}}{\text{AC}}. \text{M} = \frac{\text{H}}{360^\circ} \sqrt{(1 - e^2)}. \tfrac{1}{2} p.$$

Hence, $\frac{\text{H}'}{360^\circ}. \frac{1}{2} r^2 p = \frac{\text{H}}{360^\circ} \sqrt{(1 - e^2)}\ \frac{1}{2} p$.

or $\text{H}' = \text{H}. \frac{\sqrt{(1 - e^2)}}{r^2}$.

48. Because H and $\sqrt{(1 - e^2)}$ are constant quantities, it follows that *the true motion of a body, in an elliptical orbit, varies inversely as the square of the radius vector.* Hence it continually increases from the apogee, where it is least, to the perigee, where it is greatest; and thence continually decreases to the apogee.

49. As the mean and true places of the body coincide at the apogee and perigee, and as near the apogee the

true motion is less than the mean motion, the mean place will be in advance of the true place, from the apogee to the perigee. From the perigee to the apogee, the true place will be in advance of the mean place. It is therefore evident, that the equation of the centre, which is the difference between the mean and true places, and which at the apogee is nothing, must continually increase, till the true motion becomes equal to the mean, when the equation will be greatest; and thence it will decrease to the perigee, where it again becomes nothing. In like manner, the equation of the centre increases from the perigee, till the true motion, which is then diminishing, becomes equal to the mean. It is then greatest; and afterwards decreases till it becomes nothing, at the apogee.

The parts of the orbit on each side of the line of the apsides, being symmetrical, the greatest equation on one side will be equal to the greatest equation on the other side.

The sun's true longitude may be obtained each day from its observed right ascension (6.18). The difference between its longitudes on any two consecutive days will be its true diurnal motion at that time. Hence we may, by repeated observations, find the time when the true diurnal motion is equal to the mean. Knowing then the mean and true longitudes, when this takes place in opposite parts of the orbit, we may obtain from thence the greatest equation of the centre.

Let $A =$ the mean longitude, and $B =$ the true longitude, at the time the motions are equal between the apogee and perigee; $A' =$ the mean longitude, and $B' =$ the true longitude, when the motions are equal between the perigee and apogee; and $V =$ the greatest equation of the centre. Then,

$$B = A - V,$$

$$B' = A' + V.$$

Hence $B' - B = A' - A + 2V$,

$$2V = (B' - B) - (A' - A),$$

$$V = \tfrac{1}{2}(B' - B) - \tfrac{1}{2}(A' - A).$$

At the time of the greatest equation, the sun's true motion continues very nearly the same for two or three days. Consequently the equation of the centre will remain very nearly the same during this time. The value of the greatest equation may therefore be determined with great accuracy by this method, without the necessity of knowing very precisely the time at which the true motion is equal to the mean.

50. In a preceding article, a method has been given of obtaining the eccentricity from the greatest and least apparent diameters. It may however be obtained much more accurately from the greatest equation. Put

$$K = \frac{\text{number of seconds in V}}{206264''.8}.$$

Then, it is found by means of an analytical investigation, which we shall omit, that

$$e = \tfrac{1}{2} K - \frac{11}{768} K^3 - \frac{587}{983040} K^5 - \&c.$$

51. By a comparison of observations made at distant periods, it has been discovered that the equation of the sun's centre, and consequently the eccentricity of the orbit, are at the present period continually diminishing. The rate of diminution in the greatest equation is about 18″ in a century. It follows, therefore, that the equation of the centre, as computed for a given time, will not be accurately true for a different time. It will, however, err but little for a few years, before and after the time, for which it is computed. A complete table of the equation of the sun's centre, has a column containing the variation of the equation in a century, called the *Secular* variation, by means of which the correct equation may be obtained for different periods.

52. The force which causes heavy bodies, when left at liberty near the surface of the earth, to fall to it, is called the *Attraction of Gravitation.* Newton was led by reasoning

which appertains to physical astronomy, to adopt as a principle, that this attraction, decreasing inversely as the square of the distance from the earth's centre, extends to the moon, and retains it in an elliptical orbit about the earth; that the sun, moon, and planets, are endued with like attractive forces, which vary according to the same law; and that it is the sun's attraction which retains the earth and planets in their orbits. This general principle of attraction is called *Newton's Theory of Universal Gravitation.* A combination of various discoveries which have been made in astronomy since the time of Newton, has served completely to establish the truth of his theory.

53. If the earth was acted on by no other force than the attraction of the sun, its orbit would be accurately an ellipse, and the areas described by its radius vector in equal times, would be precisely equal. Its true longitude would therefore be accurately expressed by its mean longitude, corrected by the equation of the centre. But the attractions of the moon and planets extend to the earth, and some of them produce sensible, though slight effects on its motion. By the aid of very refined analytical investigations, the means have been obtained of calculating these effects, which are called *Perturbations.* Their whole amount may produce a change in the sun's longitude of about 45″; but in general it is considerably less. Our best solar tables, which are those calculated by Delambre, or the more recent ones calculated by Carlini, contain the equations due to the attractions of the moon and planets.

The equation of the centre and the amount of the perturbations, applied to the mean longitude of the sun, give its true longitude from the mean equinox (1).

54. The difference between the mean place of the equinox in the ecliptic and its true place, is called the *Equation of the Equinoxes in Longitude*, or sometimes, the *Lunar Nutation.* The greatest value of this equation is about 18″. A table, from which its value may be obtained for any particular time, forms a part of a complete set of solar tables.

To obtain the sun's true longitude from the true equinox, we must correct the mean longitude by the equation of the centre, the amount of the perturbations, and the equation of the equinoxes in longitude.

55. From the sun's true longitude, we obtain its true right ascension, by the formula in the last chapter (6.19). Another method is by means of a table calculated for the purpose.

The difference between the longitude and right ascension, is called the *Reduction of the Ecliptic to the Equator.* Tables have been calculated which, for a given obliquity of the ecliptic, give the reduction corresponding to each degree or half degree of longitude, and also the variation in the reduction for a change of 100″ in the obliquity. With such a table the reduction corresponding to a given longitude is easily obtained, and being applied to the longitude, it gives the right ascension.

CHAPTER VIII.

EQUATION OF TIME.—EQUINOCTIAL TIME.—RIGHT ASCENSION OF MID-HEAVEN.

1. Solar days, being determined by the apparent diurnal motion of the sun (7.4), are used for all the common purposes of life. Astronomers also generally use solar time, except in determining the right ascensions of bodies.

2. In common or civil reckoning the day begins at midnight, and is divided into two portions of 12 hours each. The first 12 are from midnight to noon, and are usually designated by the letters A. M.* annexed to the number of the hour. The latter 12, from noon to midnight, are designated by the letters P. M.*

The astronomical day begins at noon of the common day, and the hours are reckoned on to 24. Hence, any given time from noon to midnight is expressed by the same day and hour in astronomical and in common reckoning. But to express astronomically a given time from midnight to noon, we must diminish the number of the common day by a unit, and increase the number of the hour by 12.

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* A. M. are the initials of *Ante Meridian*, forenoon, and P. M. of *Post Meridian*, afternoon.

3. The angle contained between the meridian and a declination circle passing through the sun or any one of the heavenly bodies is called the *Distance of the body from the Meridian*, or the *Hour Angle of the body*. The intercepted arc of the equator is the measure of this angle, and therefore designates the distance of the body from the meridian.

4. The point of the equator which is on the meridian of any place at the same time with the sun, will, by the diurnal motion, be 15° to the west at the end of a sidereal hour. But on account of the sun's increase in right ascension, during the time, its distance from the meridian must be less than 15°. The sun does not therefore move from the meridian at the rate of 15° in a sidereal hour.

But as the interval from the time the sun is on the meridian to its return to it again is divided into 24 solar hours, and as the distance is 360°, if we suppose the right ascension to increase uniformly during the day, the sun's diurnal motion from the meridian must be at the rate of 15° in a solar hour.

5. Since 15° of the sun's distance from the meridian correspond to 1 solar hour, 1° must correspond to 4 minutes in time, 1′ to 4 seconds, and 1″ to 4 thirds. Hence to convert the sun's distance from the meridian into time, if we multiply the distance in degrees, minutes and seconds, by 4, the product of the seconds will be thirds of time, the product of the minutes will be seconds, and the product of the degrees will be minutes. As an example let 17° 21′ 36″ be converted into time.

$$\begin{array}{r} 17^\circ\ 21'\ 36'' \\ 4 \\ \hline \text{1h. 9m. 26 sec. 24 thirds.} \end{array}$$

The sun's distance from the meridian is sometimes called the *time from noon in degrees*.

6. If the apparent annual motion of the sun was in the equator and was uniform at the rate of 59′ 8″.33 from the mean equinox, in the interval between two of its consecutive passages over the meridian, it is evident that the intervals would be mean solar days (7.9 and 4). But the motion of the sun is not in the equator, and in the ecliptic it is not uniform. There are, therefore, two causes of inequality in mean solar days.

7. Supposing, as in the last article, the sun to move uniformly in the equator with its mean daily motion in longitude from the mean equinox, the time when it would, in that case, be on the meridian, is called *Mean Noon*. And time reckoned from mean noon is called *Mean Time*.

The time when the sun is really on the meridian is called *Apparent Noon*. And time reckoned from apparent noon is called *Apparent Time*.

The difference between the apparent and mean time is called the *Equation of Time*.

8. If the sun's mean longitude be corrected by the equation of the equinoxes in right ascension, *the difference between the true right ascension and the corrected mean longitude will be the equation of time in degrees.*

When the true right ascension is *greater* than the corrected mean longitude, the equation of time must be *added* to *apparent time* to obtain *mean time*, and when it is *less*, the equation must be *subtracted*.

But on the contrary, when *mean time* is to be reduced to *apparent time*, the equation must be subtracted, if the true right ascension is *greater* than the corrected mean longitude, and must be *added* if it is *less*.

Let VQ *Fig.* 16, be the equator, VC the ecliptic, V the vernal equinox, A its mean place in the ecliptic, S the true place of the sun, and AB and SD, arcs of declination circles. Then will B be the mean place of the vernal equinox in the equator, VA be the equation of the equinoxes in longitude, VB, in right ascension, VS the sun's true longitude, and VD its true right ascension. Also let BL be equal to the sun's mean longitude from the mean equinox, at the time the true longitude is VS or true right ascension VD, and M be the point of the equator which is on the meridian at that time. Then (5 and 7),

$$\begin{aligned} \text{MD} &= \text{apparent time in degrees,} \\ \text{ML} &= \text{mean time,} \\ \text{and DL} &= \text{VD} - \text{VL} = \text{equation of time.} \end{aligned}$$

Now (App. 49) tan VB = tan VA cos V; or since VA and consequently VB, only amounts to a few seconds (7.54),

$$\text{VB} = \text{VA} \cos \text{V} = \text{VA} \cos 23° \, 28' = .917 \, \text{VA}.$$

$$\text{Hence DL} = \text{VD} - \text{VL} = \text{VD} - (\text{LB} + \text{BV}) = \text{VD} - (\text{LB} + .917 \, \text{VA}).$$

9. If the equation of the equinoxes in right ascension be omitted, the error in the equation of time will seldom exceed one second.

Since VA when greatest is only 18'' (7.54), the value of .917 VA cannot exceed 16''.5, or 1 second 6 thirds, in time.

10. The equation of time may be further resolved into its component parts. If E be the equation of the centre, P the amount of the perturbations, q the equation of the equinoxes in longitude, and R the reduction to the equator, then,

$$\text{Equation of time in degrees} = \text{E} + \text{R} + \text{P} + .083 q.$$

Put

$$\begin{aligned} \text{M} &= \text{sun's mean longitude,} \\ \text{and } \omega &= \text{obliquity of the ecliptic;} \end{aligned}$$

then (8) $q \cos \omega$ = equation of equinoxes in right ascension.

Hence,

$$\begin{aligned} \text{VL} &= \text{BL} + \text{BV} = \text{M} + q \cos \omega, \\ \text{VS} &= \text{M} + \text{E} + \text{P} + q \ (7.54), \\ \text{and VD} &= \text{M} + \text{E} + \text{P} + q + \text{R} \ (7.55). \\ \text{Therefore DL} &= \text{VD} - \text{VL} = \text{E} + \text{P} + \text{R} + q - q \cos \omega \\ &= \text{E} + \text{P} + \text{R} + q (1 - \cos \omega) \\ &= \text{E} + \text{P} + \text{R} + 2 q \sin^2 \tfrac{1}{2} \omega \ (\text{App. } 8) \\ &= \text{E} + \text{R} + \text{P} + .083 \, q. \end{aligned}$$

11. The term E depends on the sun's mean anomaly, and the eccentricity of the orbit (7.43); and the sun's mean anomaly depends on its mean longitude and the longitude of the apogee. Therefore the term E depends on the mean longitude of the sun, longitude of the apogee, and

the eccentricity of the orbit. The latter two change but little in a year. Hence for a given year, a change in E depends principally on the mean longitude.

With a given obliquity of the ecliptic, the term R depends on the sun's true longitude (7.55), and therefore principally on the mean longitude and term E.

Consequently a change in the value of the sum of E and R, depends mainly on a change in the sun's mean longitude. A table may therefore be formed, containing the value of this sum for each degree of the mean longitude.

Such tables have been calculated, having the sum of E and R expressed in time. As they assume the apogee of the sun's apparent orbit to be fixed, they are not rigorously true, except for the time for which they are calculated. But the greatest error to which they are liable does not exceed seven or eight seconds for fifty years. Sometimes a column is annexed containing the secular variations in the values; that is the change which each value undergoes in 100 years.

12. The term P must consist of several parts, each depending on the body which produces the perturbation. The greatest value of P is about $45''$ (7.53), which is three seconds, in time. The greatest value of the term, $.083q$, is about $\frac{1}{10}$ of a second in time. In calculations where great accuracy is not required, these terms may be omitted, and the sum of E and R be regarded as expressing the whole equation of time.

13. When E and R are equal and have contrary signs, their sum is nothing. Nearly at the same time, as the other terms only amount to a few seconds, the equation of time must also be nothing. This circumstance takes place four times in the year. These times are about the 15th of April, 15th of June, 1st of September, and 24th of December.

14. As the motions of clocks or watches that are well made are uniform, or nearly so, they cannot correspond with the unequal motion of the sun. They should therefore, for the common purposes of life, be regulated to

mean solar time. This is easily done by applying the equation of time to the observed time of the sun's passage over the meridian.*

EQUINOCTIAL TIME.

15. Since sidereal time, mean solar time, and apparent solar time are all *local*, the commencement of the day in each being determined by the passage of a point of the celestial equator or the sun's centre, over the meridian of a place, when either of these is employed to denote the time at which any phenomenon occurs, it is necessary to name not only the kind of time employed, but also the place for which it is reckoned. In order to avoid the necessity of naming the place and to prevent any ambiguity that may sometimes arise, it has lately been proposed to introduce a reckoning of time that is not local, being the same for all parts of the earth. This time is called *Mean Equinoctial Time.* It is reckoned from the instant at which the mean place of the sun's centre coincides with the mean place of the vernal equinox in any year, and is expressed in mean solar days and parts of a day.

RIGHT ASCENSION OF THE MID-HEAVEN.

16. The arc, contained in the order of the signs, between the vernal equinox and the point of the equator which is on the meridian at any time, is called the *Right Ascension of the Mid-heaven* at that time. Thus VM is the right ascension of the mid-heaven, when the point M is on the meridian.

17. The right ascension of the mid-heaven, VM, is equal to the sum of VD and DM; that is, to the *sum of the true right ascension of the sun and the apparent time expressed in degrees.*

It is also equal to the sum of VB, BL, and LM; that is, to the *sum of the equation of the equinoxes in right ascension,*

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* A simple method of drawing a meridian line that will be sufficiently accurate, to regulate a clock for the common purposes of society, will be given in the next chapter.

mean longitude of the sun, and the mean time expressed in degrees.

In either case, the time is to be reckoned from noon to noon; and if the sum exceeds 360°, its excess above 360° is the right ascension of the mid-heaven.

CHAPTER IX.

CIRCUMSTANCES OF THE DIURNAL MOTION.—SUN'S SPOTS, AND ROTATION ON ITS AXIS.—ZODIACAL LIGHT.

1. Let HZNR, *Fig.* 17, represent the meridian of a place, Z the zenith, P the north pole, P' the south pole, HR the horizon, EQ the equator, S the situation of a body in the eastern part of the horizon, and PSD a declination circle passing through the body.

2. If PS, the polar distance, does not change, the body rising* at S will describe the arc SB, parallel to the equator, and come to the meridian at B. In descending from the meridian to the horizon, it will describe a similar and equal arc in the western hemisphere. It will therefore be the same length of time in descending from the meridian to the horizon, as in ascending from the horizon to the meridian.

3. The angle SPZ, or its measure DE, converted into time, expresses the interval of time between the rising of the body and its passage over the meridian. This interval is called the *Semi-diurnal Arc.* As the body is 12 hours in passing from A to B, the difference between the semi-diurnal arc and 12 hours, expresses the time in which the body ascends to the horizon from the meridian below, and is called the *Semi-nocturnal Arc.*

4. When PS=90°, S coincides with O, and the body is in

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* It has been noticed that the rising and setting of the heavenly bodies are affected by refraction (3.20). It is also evident that, for the sun, moon and planets, parallax will produce some effect. But in this and the following articles, unless when the contrary is mentioned, these effects are not considered.

the equator. As OPE is 90°, the semi-diurnal arc will then be six hours. When PS is less than 90°, it is evident the angle SPZ will be more than 90°, and consequently the semi-diurnal arc more than six hours. When PS is more than 90°, as PS′, the angle S′PZ is less than 90°, and the semi-diurnal arc less than six hours.

5. When PS, the distance of the body from the elevated pole, is less than PH, the latitude of the place, the body continues above the horizon and does not set. When the distance of a body from P′, the depressed pole, is less than P′R, which is equal to PH, the latitude of the place, the body continues below the horizon and does not rise.

6. The sun's polar distance, when least, is about 66½°. Therefore, at a place whose latitude is greater than 66½°, the sun, when nearest the elevated pole, will revolve above the horizon without setting, and will continue to do so, as long as its distance from the elevated pole is less than the latitude. At the opposite season of the year, when the sun is nearest the depressed pole, it will continue a like period of time below the horizon without rising.

7. At either of the poles of the earth, the latitude is 90°; and, therefore, the sun will continue above the horizon all the time it is on the same side of the equator with the elevated pole, which is about half the year. During the other half of the year it will be below the horizon.

8. At the equator, the latitude is nothing, the axis PP′ coincides with HR, and the angle SPZ becomes SHZ = 90°. The semi-diurnal arc is therefore six hours, whatever be the distance of the body from the pole.

9. It follows from the preceding articles, that at the equator, the days* are always 12 hours long; from the equator to 66½° latitude, the longest varies from 12 to 24 hours; and from thence to the pole it varies from 24 hours to six months.

10. The difference in the warmth diffused by the sun to different parts of the earth, or to the same part at differ-

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* The term day, here, implies the time of the sun's continuance above the horizon.

ent seasons of the year, depends principally on the time of its continuance above the horizon, and on its meridian altitude. The change in the sun's distance from the earth must produce some effect; but on account of the small degree of eccentricity of the earth's orbit, the change in distance is only a small part of the whole distance, and consequently the difference in warmth depending on this cause will be inconsiderable. Indeed the sun is in perigee, or nearest the earth, about the first of January; which, in northern latitudes, is the time of our coldest weather.

11. Within the limits of the torrid zone (6.10), when the sun is on the meridian, the direction of its rays is always nearly, and sometimes quite, perpendicular to the surface of the earth. The heat is therefore very great.

On the contrary, within the frigid zones, the sun never rising far above the horizon, the direction of its rays is very oblique, and consequently it produces but little warmth.

Within the temperate zones, the obliquity in the direction of the sun's rays, when it is on the meridian, being mostly neither very great nor very small, a medium temperature is produced.

12. At any place in the temperate zones, the difference between the greatest and least meridian altitudes of the sun, is evidently equal to twice its greatest declination, that is, to nearly 47°. And as the days are longest when the meridian altitude is greatest, and shortest when it is least, the difference in temperature, between the opposite seasons of the year, in which these circumstances have place, must be considerable.

13. In the north temperate zone, *Spring*, *Summer*, *Autumn*, and *Winter*, the four seasons, into which the year is divided, are considered as respectively commencing at the times of the *Vernal Equinox*, *Summer Solstice*, *Autumnal Equinox*, and *Winter Solstice.*

14. Since at the terrestrial equator the poles are in the horizon, the celestial equator, and circles parallel to it,

must cut the horizon at right angles. This position of the sphere is called a *Right Sphere.* At the poles, the equator coincides with the horizon; and hence, circles parallel to it are parallel to the horizon. This is called a *Parallel Sphere.* At any other place on the earth, the equator and its parallels cut the horizon obliquely; and such a position of the sphere is called an *Oblique Sphere.*

15. A circle parallel to the horizon is called an *Almacantar.*

16. The arc of the equator. contained in the order of the signs, between the vernal equinox and the point of the equator, which is in the horizon at the same time with any body, is called the *Oblique Ascension* of the body. Thus, if V be the place of the vernal equinox, VO is the oblique ascension of the body at S.

17. The arc, which is the difference between the right ascension of a body and the oblique ascension, is called the *Ascensional Difference.* Thus, OD is the ascensional difference of the body at S.

18. ***Given the latitude of the place and the sun's declination, to find the time of its rising or setting.***

In the triangle ZPS we have (App. 34),

$\cos ZS = \cos PZ \cos PS + \sin PZ \sin PS \cos SPZ$, or because $ZS = 90°$,

$0 = \cos PZ \cos PS + \sin PZ \sin PS \cos SPZ.$

Hence $\cos SPZ = -\dfrac{\cos PZ \cos PS}{\sin PZ \sin PS} = -\cot PZ \cot PS = -\tan PH \cot PS$,

or cos *semi-diurnal arc* = — tan *latitude* × cot *polar distance.*

Another formula which is frequently used, may be derived from the preceding.

Because, $\cos SPZ = \cos DE = \cos(90°+OD) = -\sin OD$, we have $\sin OD = \tan PH \cot PS = \tan PH \tan DS$,
or sin *Ascensional diff.* = tan *latitude* × tan *declination.*

Now the angle OPE or its measure OE, being equal to 90° or 6 hours, it is evident that when the latitude and declination are both of the same name, that is, both north or both south, the ascensional difference converted into time and added to 6 hours will give the semi-diurnal arc.

When the latitude and declination are of different names, the ascensional difference in time, subtracted from 6 hours, gives the semi-diurnal arc.

The semi-diurnal arc expresses the time of sunset, and subtracted from 12 hours, gives the time of sunrise.

19. ***Given the latitude of the place and the sun's declination, to find its azimuth at rising or setting.***

In the triangle ZPS we have (App. 34).

$$\cos PS = \cos PZ \cos ZS + \sin PZ \sin ZS \cos PZS,$$

or because $ZS = 90°$,

$$\cos PS = \sin PZ \cos PZS.$$

$$\text{Hence } \cos PZS = \frac{\cos PS}{\sin PZ} = \frac{\cos PS}{\cos PH},$$

$$\text{or } \cos \textit{azimuth} = \frac{\cos \textit{polar distance}}{\cos \textit{latitude}}.$$

20. ***Given the latitude of the place, the sun's altitude* and declination, to find the time of day.***

Let S, *Fig.* 18, be the situation of the sun ; and put,

$L = HP =$ the latitude.

$D = PS = 90° \pm$ sun's declination,

$H = SE =$ the sun's altitude,

$P = ZPS =$ the hour angle.

Then in the triangle ZPS, we have (App. 34),

$$\cos ZPS = \frac{\cos ZS - \cos PS \cos ZP}{\sin PS \sin ZP},$$

$$\text{or } \cos P = \frac{\cos (90° - H) - \cos D \cos (90° - L)}{\sin D \sin (90° - L)}$$

$$= \frac{\sin H - \cos D \sin L}{\sin D \cos L}$$

But (App. 8), $\cos P = 1 - 2 \sin^2 \tfrac{1}{2} P$,

and (App. 13), $\cos D \sin L = \sin (D + L) - \sin D \cos L$.

Hence

$$1 - 2 \sin^2 \tfrac{1}{2} P = \frac{\sin H - (\sin D + L) + \sin D \cos L}{\sin D \cos L}$$

$$= \frac{\sin H - \sin (D + L)}{\sin D \cos L} + 1,$$

$$2 \sin^2 \tfrac{1}{2} P = \frac{\sin (D + L) - \sin H}{\sin D \cos L}$$

$$= \frac{2 \cos \tfrac{1}{2} (D + L + H) \sin \tfrac{1}{2} (D + L - H)}{\sin D \cos L} \text{ (App. 21.)}$$

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* The sun's altitude may be obtained with considerable accuracy by means of a *Sextant of Reflection and an Artificial Horizon.*

$$= \frac{2\cos\left(\frac{D+L+H}{2}\right)\sin\left(\frac{D+L+H}{2}-H\right)}{\sin D \cos L},$$

$$\sin \tfrac{1}{2} P = \sqrt{\frac{\cos\left(\frac{D+L+H}{2}\right)\sin\left(\frac{D+L+H}{2}-H\right)}{\sin D \cos L}}$$

To determine the time accurately, the observation of the sun's altitude should not be made when the sun is near the meridian, as its altitude then changes but slowly. Neither should it be made when the sun is very near the horizon, as the correction for refraction cannot then be depended on with certainty. In general the best time is three or four hours before or after noon.

It is the true altitude of the sun's centre that is to be used in the calculation. Hence the altitude of the lower or upper limb, as obtained by observation, must be corrected for refraction, parallax, and semi-diameter.

21. If the sun's declination did not change, it is evident that it would have equal altitudes at equal times before and after apparent noon. Hence if an observation of the sun's altitude was taken in the forenoon, and the time observed by a good clock or watch, and if in the afternoon the time was also observed when the sun had obtained the same altitude; half the interval added to the time of the first observation, would give the time shown by the clock or watch when the sun was on the meridian. The deviation from 12 o'clock would be the error of the clock with respect to apparent time.

But in consequence of the sun's change in declination during the interval between the observations, it is necessary, in order to render this method accurate, to apply a correction to the time thus obtained. This correction is called the *Equation of Equal Altitudes.* Tables have been calculated, from which the equation is easily obtained. With these tables, the method of obtaining the error of the clock by equal altitudes of the sun, is simple, and it is also very accurate.

22. *Given the latitude of the place, the sun's altitude and declination, to find its azimuth.*

Let Z = PZS = the sun's azimuth; then (App. 34),

$$\cos PZS = \frac{\cos PS - \cos PZ \cos ZS}{\sin PZ \sin ZS}$$

$$\text{or } \cos Z = \frac{\cos D - \sin L \sin H}{\cos L \cos H}$$

But (App. 9),

$\cos Z = 2 \cos^2 \tfrac{1}{2} Z - 1,$

and (App. 14), $\sin L \sin H = \cos L \cos H - \cos (L + H)$.

Hence,

$$2 \cos^2 \tfrac{1}{2} Z - 1 = \frac{\cos D + \cos (L + H) - \cos L \cos H}{\cos L \cos H}$$

$$= \frac{\cos D + \cos (L + H)}{\cos L \cos H} - 1,$$

$$2 \cos^2 \tfrac{1}{2} Z = \frac{\cos (L + H) + \cos D}{\cos L \cos H}$$

$$= \frac{2 \cos \tfrac{1}{2} (L + H + D) \cos \tfrac{1}{2} (L + H - D)}{\cos L \cos H} \text{ (App. 22)}$$

$$= \frac{2 \cos \left(\dfrac{D + L + H}{2}\right) \cos \left(\dfrac{D + L + H}{2} - D\right)}{\cos L \cos H}.$$

$$\cos \tfrac{1}{2} Z = \sqrt{\frac{\cos \left(\dfrac{D + L + H}{2}\right) \cos \left(\dfrac{D + L + H}{2} - D\right)}{\cos L \cos H}}.$$

23. If the sun's declination did not change, it is evident that equal azimuths would correspond with equal altitudes. As the change in declination is but litttle, for a few hours, particularly near the time of the solstices, the azimuths corresponding to equal altitudes must be nearly equal. This circumstance furnishes a simple method of drawing a meridian line that will answer for determining the time of apparent noon, when great accuracy is not required. To do this, describe a number of concentric circles or arcs of circles on a smooth board. At the common centre of these arcs, fix a piece of thick, straight wire, and make it exactly perpendicular to the surface of the board. By the aid of a spirit level, or even of a common plumb-line level, fix the board so that its upper surface may be horizontal.

On a clear day, observe, during the forenoon, when the extremity of the shadow cast by the wire exactly coincides with one of the arcs, and mark the place. In the afternoon, observe when the extremity of the shadow coincides with the same arc. A straight line drawn from the place of the wire, through the middle point of the arc contained between the marks, will be a meridian line. When the shadow of the wire coincides with this line, it is apparent noon.

Greater accuracy will be obtained by extending the observation to several of the concentric arcs, and if they do not give the same line, taking, for the meridian line, a mean among them.

24. *To find the time of the sun's apparent rising or setting.*

At the time of the apparent rising or setting of the sun, the zenith distance ZS = 90° + *refraction* — *parallax*. Let R = refraction — parallax; then ZS = 90° + R, and by an investigation nearly similar to that in article 20th, we have,

$$\sin \tfrac{1}{2} P = \sqrt{\frac{\sin \left(\frac{D + L + R}{2}\right) \cos \left(\frac{D + L + R}{2} - R\right)}{\sin D \cos L}}$$

As it is not important to know the precise time of the rising or setting of the heavenly bodies, it is usual to omit the effects of refraction and parallax, and to consider the bodies as rising or setting when they are really in the horizon.

25. *To find the time of the beginning or end of twilight.*

Twilight commences or ends when the sun is about 18° below the horizon. Therefore the zenith distance ZS = 90° + 18°; and by substituting 18° instead of R in the formula in the last article, we have,

$$\sin \tfrac{1}{2} P = \sqrt{\frac{\sin \left(\frac{D + L + 18^\circ}{2}\right) \cos \left(\frac{D + L + 18^\circ}{2} - 18^\circ\right)}{\sin D \cos L}}$$

If the time of the commencement of twilight be subtracted from the time of sunrise, the remainder will be the duration of twilight.

26. The duration of twilight at a given place, changes with the declination of the sun. In northern latitudes, it is longest when the sun has its greatest north declination; and shortest when the declination is a few degrees south. It is not designed to enter into an explanation of the different circumstances, relative to the duration of twilight, as they are of but little practical utility. But the determina-

tion of the time of shortest twilight, being a problem that has claimed considerable attention, may be introduced.

27. *Given the latitude of the place, to determine the duration of the shortest twilight and the sun's declination at the time.*

In the solution of this problem, twilight is supposed to commence when the sun is at a given distance below the horizon, the sun to rise when it is really in the horizon, and its distance from the pole to remain constant, during the continuance of twilight. The sun's distance below the horizon when twilight commences is generally assumed to be 18°.

Put $2a = 18°$, and let S *Fig.* 19, be the situation of the sun when twilight commences, and S′ its situation in the horizon. Then $PS = PS'$, $ZS' = 90$, $ZS = ZD + DS = 90° + 2a$, and the angle SPS′, converted into time, expresses the duration of twilight. Let PCS be a spherical triangle having the sides respectively equal to the sides of the triangle ZPS′, that is, $PS = PS'$, $PC = PZ$ and $CS = ZS' = 90°$. Also let ZC be the arc of a great circle joining Z and C. Then the angle CPS is equal to ZPS′, and consequently $ZPC = SPS'$. Hence when the angle ZPC is the least possible, the twilight will be shortest.

Now in the triangle ZPC, the two sides ZP and PC are constant; and therefore the angle ZPC will be least when the side ZC is least. But as the two sides CS and ZS of the triangle ZSC are constant, the side ZC will be least when the angle $ZSC = 0$; that is when the sides ZC and CS coincide with ZS. Hence when the twilight is shortest, the angle $PS'Z = PSC = PSZ$.

We have, *Fig.* 20, $ZC = ZD - CD = CS - CD = DS = 2a$. And because $PZ = PC$, if PE bisect the angle ZPC, it will also bisect the base ZC, and be perpendicular to it. Hence,

$$\sin ZPE = \frac{\sin ZE}{\sin PZ} = \frac{\sin a}{\cos PH} = \frac{\sin 9°}{\cos \textit{latitude}}.$$

Twice the angle ZPE converted into time, gives the duration of shortest twilight.

Since the two right angled triangles ZPE and SPE have the same perpendicular PE, we have (App. 45.),

$$\cos ZE : \cos ES :: \cos PZ : \cos PS,$$
$$\cos a : \cos(90° + a) :: \sin \textit{latitude} : \sin \textit{declination},$$
$$\text{or } \cos a : -\sin a :: \sin \textit{latitude} : \sin \textit{declination}.$$

$$\text{Hence } \sin \textit{declination} = -\frac{\sin a}{\cos a} \sin \textit{latitude} = -\tan a \sin \textit{latitude}.$$

The value of sin *declin.* being negative, shows that the declination is of a different name from the latitude of the place. Hence in northern latitudes the declination is *south.*

The times of the year at which the shortest twilight has place may be ascertained by observing in a Nautical Almanac*, the days on which the sun has the declination, found by the above formula.

28. The ancients gave considerable attention to the rising or setting of a star or planet under the circumstances noticed in the following definitions. But they are not now considered of much importance.

29. The *Cosmical* rising or setting of a star, is when it rises or sets at sunrise.

The *Achronical* rising or setting of a star, is when it rises or sets at sunset.

The *Heliacal* rising of a star, is when it rises so long before sunrise as just to become visible above the horizon and then immediately to disappear in consequence of the increasing light from the sun; and its *Heliacal* setting, is when it sets so long after the sun as just to become visible before it descends below the horizon.

SUN'S SPOTS, AND ROTATION ON ITS AXIS.

30. The sun presents to us the appearance of a luminous circular disc. But it does not from thence follow that its surface is really flat; for all globular bodies when viewed at a great distance have this appearance. Observations with a telescope show that the sun has a rotatory motion. And it is only a globular body, that in presenting all its sides to us, would always appear under the form of a circle.

31. When the sun is viewed with a telescope, dark *Spots* of an irregular form are often seen on its disc; and continued or repeated observations show that they have a motion from east to west. Their number, positions and magnitudes are extremely variable. Frequently, several may be seen at once; and at some periods, for a year or more there are none visible. Their magnitude is sometimes such as to render them visible to the naked eye,

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* The *Nautical Almanac* is an important astronomical ephemeris, published annually in England, and at present republished in New York. It may generally be obtained two or three years previous to the year for which it is calculated.

when in consequence of a smoky or thick atmosphere, the sun can be thus viewed without injury. Each spot is usually surrounded with a penumbra, beyond which is a border of light more brilliant than the rest of the sun's disc. Sometimes a spot becomes visible on the eastern limb of the sun, traverses the disc in about 14 days, disappears in the west, and after a like interval reappears in the east. But it is not often that this happens, as the spots frequently dissolve and perish before they arrive at the western side; or having disappeared on that limb, do not reappear. The nature of the solar spots and the causes of these changes are to us unknown.

32. When a spot remains so long permanent, as to be seen twice in the same position on the sun's disc, the interval is found to be about 27 days. But this interval does not express the real period of the sun's rotation on its axis. For during this time the sun, by its apparent annual motion, has advanced nearly a sign forward in the ecliptic. The spot has, therefore, made that much more than a complete revolution, before it appears to a spectator on the earth to be in the same position.

33. The apparent position of a spot with respect to the sun's centre may be obtained by observing, when the sun is on the meridian, the right ascensions and declinations, both of the spot and centre. From three or more observations of this kind, the time of the sun's rotation and the situation of its equator with respect to the ecliptic, may be ascertained. The student who wishes a complete investigation of these, may be referred to the astronomy of *Delambre* or *Biot*. The following results have been obtained by *Delambre*.

The time of the sun's rotation on its axis 25 d. 0 h. 17 m. Inclination of the sun's equator to the ecliptic 7° 19′. The north pole of the sun is directed towards a point in the ecliptic, the longitude of which is 11*s*. 20°. Some astronomers make the time of the sun's rotation on its axis 25 d. 12 h.

34. It is also found that the moon and such of the

planets as admit of sufficiently nice observations to determine the fact, have motions on their axes. This forms a strong analogical proof in favour of the earth's diurnal motion.

ZODIACAL LIGHT.

35. A luminous appearance is sometimes seen after sunset or before sunrise, in the form of a cone or pyramid, with its base at that part of the horizon which the sun has just left or at which it is about to appear, and having its axis in the same direction as the plane of the sun's equator. This phenomenon is called the *Zodiacal Light.* From the circumstance of its direction always corresponding with the sun's equator, it seems to have some connection with the sun's rotation.

36. The angle which the plane of the sun's equator makes with the horizon at a given place, at the time of sunset or sunrise, is different for different positions of the sun in the ecliptic. In our northern climates the greatest inclination, at the time of sunset, has place about the 1st of March; and at this season of the year, the zodiacal light is generally most distinct. At other seasons when the inclination is less, the vapours near the horizon conceal it from our view. On account of the different states of the air, at the season most favourable to its appearance, it is much more distinct in some years, than in others.

The extent of the zodiacal light is various, being sometimes more than 100°, and sometimes not more than 40° or 50°.

CHAPTER X.

OF THE MOON.

1. The moon, next to the sun, is the most conspicuous of the heavenly bodies, and is particularly distinguished by the periodical changes to which its figure and light

are subject. The different appearances which it presents, are called the *Phases* of the moon.

2. By repeatedly observing the moon, when on the meridian, it is found that it has a motion among the fixed stars, from west to east, and that it comes to the meridian about 50 minutes later on each succeeding day. This motion is not uniform, but at a mean it is 13° 10′ 35″ in 24 mean solar hours. It is also found, that the moon is sometimes on the north and sometimes on the south side of the ecliptic, continuing about as long on one side as on the other; and that its orbit nearly coincides with the plane of a great circle which intersects the ecliptic in an angle of about 5°.

3. The points in which the moon's orbit cuts the plane of the ecliptic, are called the moon's *Nodes*. That node in which the moon is, when passing from the south to the north side, is called the *Ascending Node*. The other, in which it is when passing from the north to the south side, is called the *Descending Node*. The nodes are distinguished by the following characters.

Ascending node, ☊.
Descending node, ☋.

4. At periods of about a month each, the moon entirely disappears, and continues invisible during two or three days. About the middle of this time, the longitudes of the sun and moon are equal. It is then said to be *New Moon*.

5. When the moon again becomes visible, it is seen soon after sunset, a little above the western part of the horizon, under the form of a circular segment, the exterior boundary being a semicircle, and the interior, a semi-ellipse, having for its greater axis the diameter of the semicircle. This phase of the moon is called a *Crescent*, and is represented in *Fig.* 21. The points A and B are called the *Cusps*, or *Horns*.

6. The convex part of the crescent is turned towards the sun; and if a great circle bisect, at right angles, the line AB, which joins the cusps, it will pass through the sun.

7. From day to day the luminous segment increases in breadth, the boundary becomes less concave, and the moon advances to the east from the sun, till in a little more than seven days from the time of new moon, the difference of their longitudes* is 90°. This situation of the moon with respect to the sun, is called the *First Quarter.* Nearly at the same time the moon appears as a semicircle, the right line, joining the cusps, becoming the boundary on the side opposite the sun. The moon is then said to be *Dichotomized*, or to be a *Half Moon.*

8. After this, the side opposite the sun becomes convex, and the convexity, as well as the breadth of the segment, increases till the longitudes of the sun and moon differ 180°, which takes place about fifteen days from new moon. At this time the moon appears nearly as a complete circle. It is then said to be *Full Moon.*

A phase of the moon, between the first quarter and full moon, is represented in *Fig.* 22. When the moon appears under this shape, it is said to be *Gibbous.*

9. After full moon, the western side of the moon becomes elliptical, and the convexity and breadth decrease. In about twenty-two days from the time of new moon, the longitudes of the sun and moon differ 270°. It is then said to be *Last Quarter.* About this time the moon is again dichotomized.

After this, the western side of the moon becomes concave, and the breadth of the segment continues to decrease till the moon again becomes invisible, a day or two before new moon.

10. The interval from new moon to new moon, or from full moon to full moon, is called a *Lunation* or *Lunar Month.* Its mean length is 29 d. 12 h. 44 m. 3 sec.

11. Any two of the heavenly bodies are said to be in *Conjunction*, when their longitudes are the same; and to be in *Opposition*, when their longitudes differ 180°. The

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* By the difference of their longitudes, is meant the excess of the moon's longitude above that of the sun, the former being increased by 360°, when it is less than the latter.

points in the orbit, at which the moon is when in conjunction or opposition, are called the *Syzigies;* those at which it is, when its longitude exceeds the sun's by 90° or 270°, are called the *Quadratures*; and the middle points, between the syzigies and quadratures, are called the *Octants.* Some of these are designated by characters, as follows:—

Conjunction, ☌,
Opposition, ☍,
Quadrature, □.

12. The time between two consecutive conjunctions or oppositions, of a body with the sun, is called a *Synodic Revolution* of that body.*

13. It follows from the preceding articles, that the new moon has place when the moon is in conjunction with the sun; the full moon, when it is in opposition; and that the first and last quarters have place when it is in quadratures. Also, that the synodic revolution of the moon is the same with a lunation or lunar month.

14. For a few days before and after new moon, we can discern the entire disc, the obscure part appearing with a faint dusky light. Near the time of full moon, this part is entirely invisible.

15. When the moon is viewed with a telescope, the line separating the light part from the dark is seen to be very irregular and serrated; and its form varies even during the time of observation. Bright spots are frequently seen on the dark part, near the line of separation. The light of these spreads till it becomes united to the rest. The whole enlightened surface also appears diversified with spots of different shapes and different degrees of brightness. These spots always retain the same position with

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* As the distances of the planets, Mercury and Venus, from the sun are each less than that of the earth (7.28), they can never be in opposition to the sun. But they may be in conjunction, either by being between the sun and earth, or by being on the opposite side of the sun. The former is called the *Inferior*, and the latter the *Superior* conjunction. For either of these planets, a *Synodic Revolution* is the interval between two consecutive conjunctions of the same kind.

respect to each other, and occupy nearly the same place on the disc. It therefore follows, that *nearly the same face of the moon is always turned towards the earth.*

16. A consideration of the preceding circumstances leads to the conclusion, that the moon is an opaque globular body, having its surface diversified with mountains and valleys; and that it shines by reflecting the light of some other body. And from the relative situations of the sun, moon, and earth, at the times of the different phases, this body appears to be the sun.

At the time of new moon, the sun and moon are in nearly the same direction from the earth, and as the moon's distance is less than the sun's (5.14 and 16), the enlightened side of the moon is turned from the earth, and it is therefore invisible. At the first or last quarter, the difference of longitudes of the sun and moon being 90° or 270°, it is plain that about one half of the enlightened side is turned towards the earth. At the full, the moon being in opposition to the sun, nearly its whole enlightened side is turned towards the earth.

17. If, at the time of new moon, the moon is at or near to one of its nodes, as it is an opaque body, and is then directly between the sun and earth, or very nearly so, it must prevent, at least, some of the sun's light from arriving at the earth. This is sometimes the case, and occasions what is called an *Eclipse of the Sun.* When at the time of full moon, the moon is at, or near the node, it is in the earth's shadow, and an *Eclipse of the Moon* is produced. When the moon passes between the earth, and a star or planet, the circumstance is called an *Occultation* of the star or planet. These phenomena will be considered in the next chapter.

18. The *Elongation* of a body is its angular distance from the sun, as seen from the earth.

19. The apparent breadth of the visible, enlightened part of the moon, is nearly equal to the product of the apparent diameter multiplied by the square of the sine of half the elongation.

Let E, *Fig.* 23, be the centre of the earth, MANB the moon, C its centre, and let the sun be in the direction CS. Also let ACB be perpendicular to CS, AD to EC, and EM, EN, tangents to the moon at M and N. As this is not a problem in which it is important to obtain great accuracy, we may suppose the moon to be in the plane of the ecliptic, and the rays of light from the sun, that arrive at the moon, to be parallel. Then AMB is the enlightened half of the moon, of which only the part AM is visible from the earth. It is seen under the angle AEM.

$$\text{Put } C = \text{angle ECS},$$

$$\delta = \text{angle CEM} = \text{app. semidiam. of moon.}$$

$$\text{Then CD} = \text{AC} \cos \text{ACE} = \text{AC} \sin \text{SCE} = \text{AC} \sin C,$$

$$\text{ED} = \text{EC} - \text{AC} \sin C,$$

$$\text{AD} = \text{AC} \sin \text{ACE} = -\text{AC} \cos \text{SCE} = -\text{AC} \cos C$$

$$\tan \text{AEC} = \frac{\text{AD}}{\text{ED}} = -\frac{\text{AC} \cos C}{\text{EC} - \text{AC} \sin C}.$$

But AC = EC sin δ (7.13).

$$\text{Hence } \tan \text{AEC} = -\frac{\text{EC} \sin \delta \cos C}{\text{EC} - \text{EC} \sin \delta \sin C} = -\frac{\sin \delta \cos C}{1 - \sin \delta \sin C}.$$

Or since AEC and δ are both small,

$$\text{AEC} = -\frac{\delta \cos C}{1 - \sin \delta \sin C}$$

$$\text{AEM} = \text{CEM} - \text{AEC} = \delta + \frac{\delta \cos C}{1 - \sin \delta \sin C}$$

$$= \delta + \delta \cos C - \delta \cos C + \frac{\delta \cos C}{1 - \sin \delta \sin C}$$

$$= \delta + \delta \cos C + \frac{\delta \sin \delta \cos C \sin C}{1 - \sin \delta \sin C}$$

As the last term is very small it may be rejected, and we shall have,

$$\text{AEM} = \delta + \delta \cos C = \delta . (1 + \cos C) = 2 \delta \cos^2 \tfrac{1}{2} C \text{ (App. 9)} \quad \text{- A.}$$

On account of the small distance of the moon from the earth, compared with the sun's distance from both of them (5.14 and 16), we may, without material error in the present case, consider the line ES′, drawn from the earth in the direction of the sun, as parallel to CS. Then,

$$\text{AEM} = \delta + \delta \cos C = \delta + \delta \cos (180^\circ - \text{CES}')$$

$$= \delta - \delta \cos \text{CES}' = \delta . (1 - \cos \text{CES}')$$

$$= 2 \delta \sin^2 \tfrac{1}{2} \text{CES}', \text{ (App. 8)}. \quad \text{- - - - - - B.}$$

20. In like manner as the moon reflects light to the earth, the earth reflects light to the moon, with this difference, that when the moon gives least light to the earth, the earth gives the most to the moon, and the contrary.

It is the light reflected from the earth to the moon, and from the moon back to the earth, that renders the obscure part visible.

There are two causes for the disappearance of the obscure part of the moon's disc some time after new moon (14). One of these is the real diminution in the quantity of light received from that part, in consequence of the decrease in the quantity it receives from the earth; the other is the effect which the increased light of the luminous part has, in preventing the feeble light of the obscure part from making an impression on the eye.

MOON'S MEAN MOTION IN LONGITUDE.

21. If the time at which the moon's centre passes the meridian be observed, its right ascension at that time becomes known. And from the observed altitude, corrected for refraction and parallax, the north polar distance and consequently the declination is easily obtained (6.2). With the right ascension and declination, the longitude may be calculated (6.19). By repeated observations and calculations of this kind the interval from the time at which the moon has any given longitude, till it again arrives at the same longitude, may be ascertained. This interval, which is the *Tropical Revolution* of the moon, is found to vary. From observations, made at distant periods, its mean length is determined to be 27.321582 mean solar days.* Hence 27.321582 d. : 1 d. :: 360° : 13° 10′ 35″ = moon's daily mean motion in longitude.

The mean motion of the moon, here given, corresponds to the commencement of the present century. A comparison of modern observation with those of remote periods proves that the moon's mean motion is subject to a small, but thus far a continual, *Acceleration.* Investigations in physical astronomy have made known the cause of this acceleration, and have shown that it is really a periodical inequality in the moon's motion, which requires a great length of time to go through its different values.

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* It may be observed that astronomers generally express such periods in mean solar time; unless the contrary is specified.

22. The moon's mean longitude, the 1st of January 1801 at mean noon, on the meridian of Greenwich, was 3*s*. 28° 17′ 8″.3. With the *Epoch* of the mean longitude, and the moon's mean motion, the mean longitude may be determined for any other given time.

MOON'S NODES AND INCLINATION OF THE ORBIT.

23. With different longitudes and latitudes of the moon, deduced from the observed right ascensions and declinations, the situations of the nodes and the inclination of the orbit may be found, by methods nearly similar to those employed for determining the place of the vernal equinox, and the obliquity of the ecliptic (7.6 and 6.2).

24. The moon's nodes are not fixed; they have a *retrograde* motion, which is ascertained, by determining their situations at different periods. This motion is subject to several inequalities, and diminishes from century to century. At the commencement of the present century the mean length of a tropical revolution of the nodes was 6798.54019 days, or 18 years and 224.17944 days; and at that time the mean longitude of the ascending node was 0*s*. 13° 53′ 18″. The annual mean motion of the nodes in longitude is 19° 20′ 29″.

25. The inclination of the moon's orbit to the plane of the ecliptic, varies from about 5° to 5° 18′. It is greatest when the moon is in quadratures and least when it is in syzigies. The mean inclination, which is always the same, is 5° 8′ 48.″

MOON'S ORBIT.

26. Observations and investigations, similar to those made in the case of the sun, prove that the moon's orbit is nearly an ellipse, having the centre of the earth in one focus, about which the radius vector describes areas nearly proportional to the times.

27. Considering the mean distance of the moon from the earth, a unit, or 1, the eccentricity of the orbit is .0548553, which gives the greatest equation of the centre 6° 17′ 54″.5

28. The apsides of the moon's orbit have a direct motion, by which they perform a mean tropical revolution in 3231.4751 days, or 8 years and 309.537 days. At the commmencement of the present century, the mean longitude of the perigee was 8*s*. 26° 10′.

29. Besides the equation of the centre, the moon's motion is subject to numerous inequalities, of which the three following are the principal.

The most considerable is called the *Evection*, and was discovered by Ptolemy. It depends on the angular distance of the moon from the sun, and of the moon from the perigee, and amounts, when greatest, to 1° 20′ 30″.

The second is called the *Variation*, and was discovered by Tycho Brahe. It disappears when the moon is in the syzigies and quadratures, and is greatest when it is in the octants. It then amounts to 35′ 42″.

30. The third is called the *Annual Equation*, and depends on the mean anomaly of the sun. When greatest, it amounts to 11′ 12.″

30. The motion of the moon's nodes, the change in the inclination of the orbit, the motion of the apsides, and the preceding inequalities in the moon's motion, are caused by the sun's attraction, and are completely explained by investigations in physical astronomy. These investigations have also led to the discovery of other minute inequalities in the moon's motion, and thence conduced to the accuracy of tables for computing its place at any given time.

31. The most accurate tables of the moon are those calculated by Burckhardt, and by Damoiseau. The former employs 36 equations for the moon's longitude, and the latter 47. The moon's place, calculated by either of these sets of tables, always agrees, within a few seconds, with its place as determined by observation.

DIFFERENT REVOLUTIONS OF THE MOON.

32. From the tropical revolution of the moon (21), by taking into view the motion of the equinoxes, the sun's apparent motion, and the motions of the apsides, and nodes of the moon's orbit, the other revolutions of the moon are easily determined. The following are the lengths of these revolutions, as given by Delambre:—

	DAYS.
Tropical revolution, - - -	27.3215255
Sidereal revolution, - - -	27.3215830
Synodic revolution, - - -	29.5305885
Anomalistic revolution, - - -	27.5545704
Revolution from one node to the same,	27.2122222

MOON'S REVOLUTION ON ITS AXIS.

33. It has been stated (15), that the moon presents nearly the same face to the earth. It must therefore revolve on its axis in the same direction and same time that it revolves in its orbit. But accurate and continued observations show, that the moon's spots do not preserve exactly the same situations on the disc. They are seen alternately to approach and recede from the edge. Those that are very near the edge, successively disappear and again become visible, making periodical oscillations, which are called the *Librations* of the moon.

34. The librations of the moon are not occasioned by an unequal motion on its axis. For, admitting this motion to be uniform, and the axis to be nearly perpendicular to the plane of the orbit, small portions on the east and west sides of the moon ought alternately to come into sight and to disappear, in consequence of its irregular motion in its orbit. This is conformable to observation, and is called the *Libration in Longitude.*

35. Besides the motion of the spots in an easterly and westerly direction, they are observed to have a small alternate motion from north to south. This is called the

Libration in Latitude, and shows that the moon's axis, though nearly, is not exactly, perpendicular to the plane of its orbit.

36. In consequence of the earth's diurnal motion, a spectator at the surface sees portions of the moon a little different, according to its different positions above the horizon. This is called the *Diurnal* or *Parallactic Libration.*

37. From calculations founded on accurate observations of the lunar spots, it has been found, that the equator of the moon is inclined to the plane of the ecliptic in an angle of 1° 30′; and that the line, in which the plane of the equator cuts the plane of the ecliptic, is always parallel to the line of the nodes.

38. If three planes be supposed to pass through the centre of the moon, one representing the equator of the moon, another the plane of its orbit, and the third being parallel to the ecliptic; then the last will lie between the two others, and will intersect them in the same line in which they intersect each other. It will make, with the first, an angle of 1° 30′; and with the second, an angle of 5° 9′. This curious fact was discovered by Cassini; and it has been explained by Lagrange from the theory of gravity.

MOON'S DIAMETER AND MOUNTAINS.

39. The greatest and least horizontal parallaxes of the moon for a place on the equator, are 53′ 52″ and 61′ 32″ (5.14). The corresponding apparent diameters are found to be 29′ 22″ and 33′ 31″. If $d =$ real diameter of the moon and R = equatorial radius of the earth, we have (7.15),

$$d = \text{R}.\frac{29' \, 22''}{53' \, 52''} = \tfrac{1}{2}\text{R}.\frac{1762}{3230} = 2\text{R}.\frac{1762}{6460} = 2\text{R}.\frac{3}{11} \text{ very}$$

nearly.

Hence the diameter of the moon is about $\frac{3}{11}$ of the equatorial diameter of the earth, and consequently its surface is about $\frac{1}{13}$ of the earth's surface, and its volume about $\frac{1}{49}$ of the earth's volume. The moon's diameter in English miles is 2160.

40. It has been observed (16) that the moon's surface is diversified with mountains and valleys. The heights of some of these mountains, in comparison with the diameter of the moon, are found to exceed those of the earth. Though not a subject of much importance, it may be interesting to the student to know a method of ascertaining the heights of the lunar mountains.

41. Let ABO, *Fig.* 24, be the enlightened hemisphere of the moon, E the situation of the earth, ES′ the direction of the sun from the earth, and SM a solar ray, touching the moon in O, which will be one of the points in the curve, separating the enlightened from the dark part of the moon. Also let M be the summit of a mountain, situated near to O, and sufficiently elevated to receive the sun's light. To a spectator at E, the summit M of the mountain will appear as a bright spot on the dark part of the moon.

The angle MEO may be measured by means of a micrometer, attached to the telescope. In this case, as in a former (19), we may consider ES′ as parallel to MS. We may also without material error consider the angle MES′ as equal to the elongation CES′. Let OD be perpendicular on ME, and put,

∂ = apparent diameter of the moon,
d = AO = real diameter of the moon.

We have, $$\mathrm{CO}\tan\mathrm{MCO} = \mathrm{MO} = \frac{\mathrm{OD}}{\sin\mathrm{OME}} = \frac{\mathrm{EO}\sin\mathrm{MEO}}{\sin\mathrm{OME}} = \frac{\mathrm{EO}\sin\mathrm{MEO}}{\sin\mathrm{MES'}} = \frac{\mathrm{EC}\sin\mathrm{MEO}}{\sin\mathrm{CES'}}\ \text{nearly.}$$

Hence, $$\tan\mathrm{MCO} = \frac{\mathrm{EC}}{\mathrm{CO}}.\frac{\sin\mathrm{MEO}}{\sin\mathrm{CES'}} = \frac{1}{\sin\frac{1}{2}\partial}.\frac{\sin\mathrm{MEO}}{\sin\mathrm{CES'}} \qquad (7.13)$$
$$= \frac{2}{\sin\partial}.\frac{\sin\mathrm{MEO}}{\sin\mathrm{CES'}}$$

Height of the mountain $= a\,\mathrm{M} = \mathrm{MC} - \mathrm{OC} = \dfrac{\mathrm{OC}}{\cos\mathrm{MCO}} - \mathrm{OC}$

$$= \mathrm{OC}.\frac{1-\cos\mathrm{MCO}}{\cos\mathrm{MCO}} = \mathrm{OC}.\frac{1-\cos\mathrm{MCO}}{\cos\mathrm{MCO}}.\frac{\sin\mathrm{MCO}}{\sin\mathrm{MCO}}$$

$$= \mathrm{OC}.\frac{1-\cos\mathrm{MCO}}{\sin\mathrm{MCO}}.\tan\mathrm{MCO} = \mathrm{OC}.\frac{1}{\cot\frac{1}{2}\mathrm{MCO}}\tan\mathrm{MCO}$$

(App. 11).

$$= \mathrm{OC}\tan\mathrm{MCO}\tan\tfrac{1}{2}\mathrm{MCO} = \tfrac{1}{2}\mathrm{OC}\tan^2\mathrm{MCO}$$

$$= \tfrac{1}{2}\mathrm{OC}.\frac{4}{\sin^2\partial}.\frac{\sin^2\mathrm{MEO}}{\sin^2\mathrm{CES'}} = d.\left(\frac{\sin\mathrm{MEO}}{\sin\partial\sin\mathrm{CES'}}\right)^2 = d.\left(\frac{\text{ang. MEO}}{\partial\sin\mathrm{CES'}}\right)^2$$

Dr. Herschel has made observations on a number of the lunar mountains. For one of these the data are, the angle MEO $= 40''.625$, apparent diameter of the moon $= 32' \; 5''.2$, and the elongation $= 125° \; 8'$. Hence taking the moon's diameter 2160 miles, we may easily obtain from the preceding formula, the height of the mountain $= 1.45$ miles.

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42. Luminous spots, which are entirely unconnected with the phases, or in other words, are not the reflection of the sun's light, are sometimes seen on the moon's disc. These are supposed to be volcanoes.

43. If the moon was surrounded with an atmosphere, such as appertains to the earth, it would by its action, in changing the rays of light, produce a very sensible effect in the duration of an eclipse of the sun, or an occultation of a star or planet (17). But no such effect has been observed. It is therefore inferred that the *moon has no atmosphere.* If it has one, it must be either of very little extent, or of extremely little density.

MOON'S PASSAGE OVER THE MERIDIAN.

44. In consequence of the moon's daily increase in right ascension, it passes the meridian later on each day than on the preceding (2). The daily retardation varies from about 38 to 66 minutes.

To obtain the time of the moon's passage on any particular day, let R = the excess of the moon's right ascension above the sun's, at noon of the given day,* S = daily motion of the sun in right ascension, M = that of the moon, both being considered as uniform during the day, T = the required time of the moon's passage, and A = the arc of the equator, which passes the meridian, between noon and the moon's passage.

Then $24^h : T :: M : \frac{T.M}{24_h}$ = moon's motion in right ascension during the time T.

$$\text{Hence } A = R + \frac{T.M}{24^h}.$$

$$\text{Also } 24_h : T :: 360° + S : A = \frac{T.\,(360° + S)}{24^h}$$

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* When the moon's ascension is less than the sun's, it must be increased by 360° or 24h.

$$\text{Therefore } \frac{T.(360^\circ + S)}{24_h} = R + \frac{T.M}{24^h}$$

$$T.(360^\circ + S) = 24_h.R + T.M,$$

$$T = \frac{24^h.R}{360^\circ + S - M} = \frac{24_h.R}{24_h + S - M}.$$

45. If M = the daily motion of a planet in right ascension, the preceding formula will give the time of its passage over the meridian, observing that when the motion of the planet is retrograde, the sign of M must be changed and the formula will then become,

$$T = \frac{24_h\ R}{360^\circ + S + M}.$$

For a fixed star, M = 0, and the formula becomes,

$$T = \frac{24_h\ R}{360^\circ + S}.$$

MOON'S RISING AND SETTING.

46. On account of the moon's change in declination, the semi-diurnal arc, found with its declination at the time of its passing the meridian (9.18), is not correct. If however the semi-diurnal arc, thus obtained, be applied to the time of the moon's passage, by subtracting for the rising or adding for the setting, it will give the approximate time of rising or setting.

To obtain the time more correctly, find the declination for the approximate time and again calculate the semi-diurnal arc, which must be corrected on account of the moon's change in right ascension. Thus, $24^h + S - M : 24_h ::$ semi-diurnal arc : corrected semi-diurnal arc, which applied as before to the time of the moon's passage, gives the time of the moon's rising or setting, very nearly.

If D = the difference between the times of the moon's passage on two consecutive days, one of which precedes and the other follows the required time of the moon's rising or setting, the last correction may be made thus: As $24_h : 24^h + D ::$ semi-diurnal arc : corrected semi-diurnal arc.

As the mean length of the semi-diurnal arc is about 6 hours; it is better, in the operation for obtaining the approximate time of rising or setting, to make use of the moon's declination 6 hours before or after its passage over the meridian, according as it is the rising or setting that is required.

MOON'S PARALLAX IN LONGITUDE AND LATITUDE.

47. The effect of parallax in changing the altitudes of the heavenly bodies has been shown in a preceding chapter, and a method given for determining it. But this is not the only effect of parallax. It also changes the right

ascension, declination, longitude, and latitude of a body. It may be proper here to investigate formulæ for calculating the effect of parallax on the moon's longitude and latitude, as they will be useful in our chapter on eclipses of the sun.

48. Let HZR, *Fig.* 25, be the meridian, HR the horizon, Z the zenith, EQ the equator, ECO the ecliptic, P and *p* their poles, E the vernal equinox, A the *true* place of the moon, B its *apparent* place, as depressed by parallax, in the vertical circle ZABK, and *p*A*a* and *p*B*b*, circles of latitude, passing through the true and apparent places. By the effect of parallax, the true longitude E*a* is changed to the apparent longitude E*b*, and the true latitude A*a*, to the apparent latitude B*b*.

49. The difference between the true and apparent longitude of a body, produced by parallax, is called the *Parallax in Longitude ;* and the difference between the true and apparent latitude, is called the *Parallax in Latitude.*

50. The point in the ecliptic, which is above the horizon, and at 90° distance from the intersection of the ecliptic and horizon, is called the *Nonagesimal Degree of the Ecliptic.*

51. The data usually given to calculate the moon's parallax in longitude and latitude, are the moon's true longitude, and its latitude, or distance from the north pole of the ecliptic, its horizontal parallax, the obliquity of the ecliptic, the latitude of the place, and the right ascension of the mid-heaven.

On account of the spheroidal figure of the earth, the horizontal parallax, at any given time, is different at different places (5.12). The parallax for a given place is called the *Reduced Parallax.* If the earth were a sphere, having a radius equal to the straight line joining the given place and the centre, it is evident that the parallax for all parts of it would be the same as the reduced parallax for the given place. It is also plain, from the definition (4.11), that the reduced latitude of any place is its latitude on the supposition of the earth being a sphere. If, therefore, the reduced latitude and parallax be used, the earth may be considered as a sphere, which will simplify the investigations for finding the parallax in longitude and latitude.

52. Let pPCD be a great circle, passing through p and P, and pZnI another, passing through p and Z. Because P is the pole of EQ, the pole of pPD is in EQ; and because p is the pole of EC, the pole of pPD is in EC; the point E, in which EQ and EC intersect each other, is, therefore, the pole of pPD; and consequently, ED and EC are each 90°. In like manner, because Z and p are the poles of HR and EO, the point O, is the pole of pZnI, and On and OI are each 90°. Consequently, n is the nonagesimal degree of the ecliptic. Also, En is the longitude of the nonagesimal degree, and nI is its altitude. These quantities are used in finding the parallax in longitude and latitude, and must first be found.

53. Put

ω = Pp = obliquity of the ecliptic.
H = ZP = complement of the reduced latitude,
M = EM = right ascension of the mid-heaven,
n = En = longitude of the nonagesimal,
h = pZ = 90° — Zn = nI = altitude of the nonagesimal,
L = Ea = moon's true longitude,
Δ = pA = moon's true distance from north pole of the ecliptic,
ϖ = moon's horizontal parallax, reduced,
Π = moon's parallax in longitude,
π = moon's parallax in latitude.

In the triangle pPZ, we have given Pp, PZ, and pPZ = 180° — ZPD = 180° — DM = 180° — (90° — EM) = 90° + EM = 90° + M, to find pZ = the altitude of the nonagesimal, and PpZ = Cn = 90° — En = the complement of the longitude of the nonagesimal.

LONGITUDE OF THE NONAGESIMAL.

Let S = PpZ + PZp, D = PpZ — PZp, E = 180° — ½ S, and F = 180° — ½ D. Then (App. 41),

$$\tan \tfrac{1}{2} S = \frac{\cos \frac{1}{2}(H - \omega)}{\cos \frac{1}{2}(H + \omega)} \cdot \cot \tfrac{1}{2}(90° + M)$$

$$= \frac{\cos \frac{1}{2}(H - \omega)}{\cos \frac{1}{2}(H + \omega)} \cdot \tan \tfrac{1}{2}(90° - M),$$

$$\tan (180° - \tfrac{1}{2} S) = \frac{\cos \frac{1}{2}(H - \omega)}{\cos \frac{1}{2}(H + \omega)} \cdot \tan (180° - 45° + \tfrac{1}{2} M)$$

$$= \frac{\cos \frac{1}{2}(H - \omega)}{\cos \frac{1}{2}(H + \omega)} \cdot \tan \tfrac{1}{2}(270° + M)$$

$$= \frac{\cos \frac{1}{2}(H - \omega)}{\cos \frac{1}{2}(H + \omega)} \cdot \tan \tfrac{1}{2}(M - 90°),$$

$$\text{or } \tan E = \frac{\cos \frac{1}{2}(H - \omega)}{\cos \frac{1}{2}(H + \omega)} . \tan \tfrac{1}{2}(M - 90^\circ).$$

Again (App. 42),

$$\tan \tfrac{1}{2} D = \frac{\sin \frac{1}{2}(H - \omega)}{\sin \frac{1}{2}(H + \omega)} . \cot \tfrac{1}{2}(90^\circ + M).$$

Whence, by transforming as above we have,

$$\tan F = \frac{\sin \frac{1}{2}(H - \omega)}{\sin \frac{1}{2}(H + \omega)} . \tan \tfrac{1}{2}(M - 90^\circ).$$

Now $\frac{1}{2} S + \frac{1}{2} D = PpZ = 90^\circ - n$.
Hence, $n = 90^\circ - (\frac{1}{2} S + \frac{1}{2} D)$,
or $n = 450^\circ - (\frac{1}{2} S + \frac{1}{2} D) = 180^\circ - \frac{1}{2} S + 180^\circ - \frac{1}{2} D + 90^\circ$.

Consequently, $n = E + F + 90^\circ$, rejecting 360°, when the sum exceeds that number.

ALTITUDE OF THE NONAGESIMAL.

We have (App. 44),

$$\tan \tfrac{1}{2} h = \frac{\cos \frac{1}{2} S}{\cos \frac{1}{2} D} . \tan \tfrac{1}{2}(H + \omega)$$

$$= \frac{\cos E}{\cos F} . \tan \tfrac{1}{2}(H + \omega).$$

PARALLAX IN LONGITUDE.

54. The triangle ApB, gives,

$$\sin \Pi = \sin ApB = \frac{\sin AB \sin ZBp}{\sin Ap}$$

$$= \frac{\sin \varpi \sin (N + p) \sin ZBp}{\sin \Delta} \qquad (5.5).$$

In the triangle ZpB,
the angle $ZpB = nb = na + ab = Ea - En + ab = L - n + \Pi$,

$$\text{and } \sin ZBp = \frac{\sin pZ \sin ZpB}{\sin ZB} = \frac{\sin h \sin (L - n + \Pi)}{\sin (N + p)}.$$

$$\text{Hence } \sin \Pi = \frac{\sin \varpi \sin h}{\sin \Delta} \sin (L - n + \Pi) \qquad (C).$$

PARALLAX IN LATITUDE.

55. The triangles pZA and pZB give (App. 34),

$$\cos pZB = \frac{\cos Ap - \cos Zp \cos ZA}{\sin Zp \sin ZA} = \frac{\cos Bp - \cos Zp \cos ZB}{\sin Zp \sin ZB},$$

$$\text{or, } \frac{\cos \Delta - \cos h \cos N}{\sin N} = \frac{\cos (\Delta + \varpi) - \cos h \cos (N + p)}{\sin (N + p)}$$

$\cos \Delta \sin (N + p) - \cos h \cos N \sin (N + p) = \cos (\Delta + \varpi) \sin N - \cos h \sin N \cos (N + p)$

$\cos (\Delta + \varpi) \sin N = \cos \Delta \sin (N + p) - \cos h [\sin (N + p) \cos N - \cos (N + p) \sin N]$

$$= \cos \Delta \sin (N + p) - \cos h \sin p \qquad \text{(App. 13)}$$
$$= \cos \Delta \sin (N + p) - \cos h \sin \varpi \sin (N + p) \qquad (5.5)$$
$$= \sin (N + p). (\cos \Delta - \sin \varpi \cos h.)$$

$$\text{Therefore, } \cos (\Delta + \pi) = \frac{\sin (N + p)}{\sin N}. (\cos \Delta - \sin \varpi \cos h).$$

$$\text{But, } \frac{\sin Ap \sin ZpA}{\sin ZA} = \sin pZB = \frac{\sin Bp \sin ZpB}{\sin ZB}$$

$$\text{or, } \frac{\sin \Delta \sin (L - n)}{\sin N} = \frac{\sin (\Delta + \pi) \sin (L - n + \Pi)}{\sin (N + p)}$$

$$\frac{\sin (N + p)}{\sin N} = \frac{\sin (\Delta + \pi) \sin (L - n + \Pi)}{\sin \Delta \sin (L - n)} \qquad \text{(D).}$$

$$\text{Hence, } \cos (\Delta + \pi) = \frac{\sin (\Delta + \pi) \sin (L - n + \Pi)}{\sin \Delta \sin (L - n)}. (\cos \Delta - \sin \varpi \cos h)$$

$$\cot (\Delta + \pi) = \frac{\sin (L - n + \Pi)}{\sin (L - n)}. \left(\cot \Delta - \frac{\sin \varpi \cos h}{\sin \Delta},\right) \qquad \text{(E)}$$

$$\text{Make } \tan x = \frac{\sin \varpi \cos h}{\sin \Delta}.$$

$$\text{Then, } \cot (\Delta + \pi) = \frac{\sin (L - n + \Pi)}{\sin (L - n)}. (\cot \Delta - \tan x)$$
$$= \frac{\sin (L - n + \Pi)}{\sin (L - n)}. \left(\frac{\cos \Delta}{\sin \Delta} - \frac{\sin x}{\cos x}\right)$$
$$= \frac{\sin (L - n + \Pi)}{\sin (L - n)}. \frac{\cos \Delta \cos x - \sin \Delta \sin x}{\sin \Delta \cos x}$$
$$= \frac{\sin (L - n + \Pi) \cos (\Delta + x)}{\sin (L - n) \sin \Delta \cos x}. \qquad \text{(F).}$$

The apparent polar distance $(\Delta + \pi)$ being calculated by either of the formulæ (E) and (F), we have $\pi = (\Delta + \pi) - \Delta$.

56. We may obtain formulæ that will give the parallax in latitude, without first finding the apparent latitude, and which in some cases may be more convenient than those in the last article.

We have (55.E)

$$\frac{\sin (L - n) \cot (\Delta + \pi)}{\sin (L - n + \Pi)} = \cot \Delta - \frac{\sin \varpi \cos h}{\sin \Delta}$$

$$\cot \Delta = \frac{\sin \varpi \cos h}{\sin \Delta} + \frac{\sin (L - n) \cot (\Delta + \pi)}{\sin (L - n + \Pi)}$$

$$\cot \Delta - \cot (\Delta + \pi) = \frac{\sin \varpi \cos h}{\sin \Delta} - \cot (\Delta + \pi) + \frac{\sin (L - n) \cot (\Delta + \pi)}{\sin (L - n + \Pi)}.$$

Hence (App. 27),

$$\frac{\sin \pi}{\sin \Delta \sin (\Delta + \pi)} = \frac{\sin \varpi \cos h}{\sin \Delta} -$$

$$\frac{\cot(\Delta+\pi)\,[\sin(L-n+\Pi)-\sin(L-n)]}{\sin(L-n+\Pi)}$$

$$=\frac{\sin\varpi\cos h}{\sin\Delta}-\frac{2\sin\frac{1}{2}\Pi\cos(L-n+\frac{1}{2}\Pi)\cot(\Delta+\pi)}{\sin(L-n+\Pi)}$$

(App. 21)

$$\sin\pi=\sin\varpi\cos h\sin(\Delta+\pi)-$$
$$\frac{2\sin\frac{1}{2}\Pi\sin\Delta\cos(L-n+\frac{1}{2}\Pi)\cos(\Delta+\pi)}{\sin(L-n+\Pi)}$$

$$=\sin\varpi\cos h\sin(\Delta+\pi)$$
$$-\frac{\sin\Pi\sin\Delta\cos(L-n+\frac{1}{2}\Pi)\cos(\Delta+\pi)}{\cos\frac{1}{2}\Pi\sin(L-n+\Pi)} \qquad \text{(App. 7)}$$

But (54.C), $\frac{\sin\Pi\sin\Delta}{\sin(L-n+\Pi)}=\sin\varpi\sin h$. Therefore,

$$\sin\pi=\sin\varpi\cos h\sin(\Delta+\pi)$$
$$-\frac{\sin\varpi\sin h\cos(L-n+\frac{1}{2}\Pi)\cos(\Delta+\pi)}{\cos\frac{1}{2}\Pi} \qquad \text{(G)}$$

EFFECT OF PARALLAX ON THE APPARENT DIAMETER OF THE MOON.

57. The moon is nearer to any place on the earth's surface when it is elevated above the horizon, than when it is in the horizon. The angle under which its diameter is seen, will therefore be greater in the former case, than in the latter.

Let δ = moon's horizontal semidiameter,

δ' = moon's apparent semidiameter at a given situation above the horizon. Then (7.13),

$$\frac{\sin\delta'}{\sin\delta}=\frac{D}{D'}=(\textit{Fig. } 6)\ \frac{BC}{AB}=\frac{\sin BAZ}{\sin BCZ}=\frac{\sin(N+p)}{\sin N} \qquad (5.6)$$

$$=\frac{\sin(\Delta+\pi)\sin(L-n+\Pi)}{\sin\Delta\sin(L-n)}\ (55.\ D),$$

Hence $$\sin\delta'=\frac{\sin\delta\sin(\Delta+\pi)\sin(L-n+\Pi)}{\sin\Delta\sin(L-n)} \qquad \text{(H)}$$

CHAPTER XI.

ECLIPSES OF THE SUN AND MOON.—OCCULTATIONS.

1. As the moon is an opaque body, and shines only by reflecting the sun's light, when, at the time of full moon, it enters the earth's shadow, it must become eclipsed. When, at the time of new moon, the moon passes between the sun and a spectator on the earth, it must occasion, to him, an eclipse of the sun.

2. If we suppose the sun and earth to be spheres, as they are, very nearly, the sun being much larger than the earth, the shadow of the earth must have the form of a cone, the length of which depends on the magnitudes of the bodies, and on their distance from each other. The moon's shadow is also conical, but of much less extent than that of the earth.

EARTH'S SHADOW.

3. Let ABG and *abg*, *Fig.* 26, be sections of the sun and earth, by a plane passing through their centres S and E, and A*a*C and B*b*C, tangents to the circles ABG and *abg*. Disregarding, at present, the action of the earth's atmosphere, in changing the direction of those rays of the sun which pass through it, the triangular space *a*C*b* will be a section of the earth's shadow.

The line EC is called the *Axis* of the earth's shadow.

4. With a view to conciseness of expression in some of the succeeding articles, we shall put R = E*b* = radius of the earth, P = moon's horizontal parallax, *p* = sun's horizontal parallax, *d* = moon's apparent semidiameter, and δ = sun's apparent semidiameter.

5. The earth's shadow extends to more than three times the distance of the moon.

Put $n = 206264''.8$ (5.8).

From the triangle EBC, we have,

$$\text{SEB} = \text{EC}b + \text{EB}b, \text{ or } \delta = \text{EC}b + p.$$

Therefore, $\text{EC}b = \delta - p$,

$$\text{and EC} = \frac{\text{R}}{\sin \text{EC}b} = \frac{\text{R}.n}{\text{EC}b} = \frac{\text{R}.n}{\delta - p}.$$

Now (5.8), the moon's distance from the earth $= \frac{R.n}{P}$.

Hence, as $\delta - p$ never amounts to 17′ and P always exceeds 53′, the distance EC is more than three times the moon's distance from the earth.

6. Let hMh' be a circular arc, described with the centre E, and a radius equal to the distance between the centres of the earth and moon; and let Adh and Bch' be tangents to the sections of the sun and earth, crossing each other between them.

When any part of the moon enters the space between the lines dh and bC, that part will, evidently, be deprived of a portion of the sun's light, and will therefore appear less bright. As the moon approaches the line bC, its light continues to be diminished; and when the edge comes in contact with bC, the eclipse commences. Hence there is a gradual diminution of the moon's light, previous to the commencement of an eclipse of the moon. There is also a gradual increase in the light after the eclipse has ended. This is conformable to observation.

7. If we suppose the line dh to revolve about EC, its inclination to EC continuing the same, and thus to form the surface of the frustum of a cone, of which $cdhh'$ is a section, the space included within that surface is called the *Penumbra.* The earth's shadow, of which aCb is a section, is sometimes called the *Umbra.*

8. The moon sometimes enters the penumbra, and again passes out, without any part entering the umbra or real shadow. In such cases, it sustains a diminution of its light, but is not said to be eclipsed.

9. Any section of the earth's shadow or of the penumbra, by a plane perpendicular to the axis of the shadow, is a circle. If we suppose such a section of the shadow and penumbra to be made at the distance of the moon, the apparent semi-diameter of the section of the earth's shadow, as seen from the centre of the earth, is called the *Semi-diameter of the Earth's Shadow;* and the apparent semi-diameter of the section of the penumbra, is called the *Semi-diameter of the Penumbra.*

The angle MEm is the semi-diameter of the earth's shadow, and the angle MEh' is the semi-diameter of the penumbra.

10. The semi-diameter of the earth's shadow is equal to the sum of the moon's and sun's horizontal parallaxes, less the apparent semi-diameter of the sun.

In the triangle EmC, we have,

$$\text{the angle } m\text{EM} = \text{E}mb - \text{EC}m.$$

Now, the angle mEM is the semi-diameter of the earth's shadow, Emb is the moon's horizontal parallax = P, and ECm = $\delta - p$ (5).

Hence,

Semi-diameter of the Earth's Shadow $= \text{P} - (\delta - p) = \text{P} + p - \delta$.

11. The diameter of the earth's shadow, at the moon, is more than double the apparent diameter of the moon, and consequently the moon may be entirely enveloped in the shadow.

If we take $\text{P} = 57'\ 22''$, $p = 8''.6$, and $\delta = 16'\ 1''.3$, we obtain the mean semi-diameter of the earth's shadow $= 41'\ 29''.3$, and consequently the mean diameter $= 82'\ 58''.6$; which is more than twice the apparent diameter of the moon.

12. If, at the time of full moon, the apparent distance of the moon's centre, from the axis of the shadow, does not become less than $\text{P} + p + d - \delta$, there cannot be an eclipse.

When the edge of the moon touches the earth's shadow, its centre is at a distance from it, equal to the moon's semi-diameter. If therefore, to the semi-diameter of the shadow, we add the moon's semi-diameter, we have the distance of the moon's centre from the axis of the shadow, at the beginning or end of an eclipse of the moon, equal $\text{P} + p + d - \delta$.

13. The distance of the moon's centre from the axis of the shadow, at the time of full moon, depends on the moon's distance from the node, and on the inclination of the orbit. We may, for a given inclination of the orbit, and given value of $\text{P} + p + d - \delta$, determine within what distance from the node the moon must be, in order that an eclipse may take place.

By taking the least and greatest inclinations of the orbit, the greatest and least values of $P + p + d - \delta$, and also taking into view the inequalities in the motions of the sun and moon, it has been found, according to Delambre, that when at the time of mean full moon, the difference of the mean longitudes of the moon and node exceeds 12° 36′, there cannot be an eclipse; but when this difference is less than 9° there must be one. These numbers are called the *Lunar Ecliptic Limits.*

14. From tables of the mean motions of the sun, moon, and node, the mean time of any full moon, and the difference of the mean longitudes of the moon and node, at that time, are easily obtained. Then by the lunar ecliptic limits, we know whether or not there will be an eclipse, except when the difference of longitudes of the moon and node, is between 9° and 12° 36′, in which case further calculation is necessary.

15. We have hitherto considered the earth's shadow as limited by those rays, which, passing from the edge of the sun, touch the corresponding side of the earth. But it is found that the observed duration of an eclipse of the moon always exceeds the duration computed on the supposition of the shadow being thus limited. This is accounted for, by supposing the most of those rays of light which pass near the surface of the earth to be absorbed by the earth's atmosphere, as the effect of such an absorption would evidently be an increase in the extent of the earth's shadow. In consequence of the gradual diminution of the moon's light, previous to its entering the earth's shadow, and gradual increase on leaving it (6), the time of the commencement or end of a lunar eclipse cannot be observed with great accuracy. Astronomers therefore differ with regard to the amount of the correction which should be made. It is, however, usual in computing an eclipse of the moon, to increase the semi-diameter of the earth's shadow by a sixtieth part; or which amounts to the same, to add as many seconds as the semi-diameter contains minutes.

16. There is another effect of the earth's atmosphere,

which is perceptible in eclipses of the moon. Those rays of the sun which enter the earth's atmosphere and are not absorbed, have their directions changed, so as to meet at a less distance than they otherwise would do. The rays through the atmosphere near the surface of the earth, are so much converged as to meet at a distance from the earth, less than that of the moon. In this way a sufficient quantity of light is thrown on the moon to render it visible, even when it is in the middle of the shadow. It then appears with a dull reddish light.

17. As an eclipse of the moon is occasioned by a real loss of light on the moon, and not by the interposition of any body between the moon and the spectator on the earth, it must present the same appearance to all those who have the moon above their horizon during the eclipse, and observe it at the same time. It will be shown that the case is different with eclipses of the sun.

18. The semi-diameter of the earth's penumbra is equal to the sum of the moon's horizontal parallax, sun's horizontal parallax, and the apparent semi-diameter of the sun.

From the triangles ELh', and ELB, we have,

$$MEh' = Eh'c + ELc = Eh'c + EBc + SEB.$$

But $Eh'c = P$, $EBc = p$, and $SEB = \partial$. Hence,

$$\textit{Semi-diameter of the penumbra} = P + p + \partial.$$

MOON'S SHADOW.

19. The length of the moon's shadow is about equal to the distance of the moon from the earth. At the time of new moon, the shadow sometimes extends beyond the earth, but more frequently it does not quite reach it. This difference depends in part on a change in the length of the shadow itself, resulting from a change in the moon's distance from the sun, but principally on the change in the distance of the moon from the earth.

Let now *agb* be considered as a section of the moon, and M the centre of the earth, then, the same figure will answer for determining the different circumstances relative to the moon's shadow, that have been found for the earth's shadow. Put,

R = radius of the earth
$R' = Eb$ = moon's radius,
$p' = EBb$ = sun's horizontal parallax for the moon,
$\gamma = SEB$ = sun's apparent semi-diameter at the moon.

We have, (5.8)

$$SM = \frac{R.n}{p}, \text{ and } EM = \frac{R.n}{P}.$$

Now, $ES = SM - EM = \frac{R.n}{p} - \frac{R.n}{P} = \frac{R.n\,(P-p)}{P.p}.$

Hence, (7.13) $\gamma = \frac{SM.\delta}{ES} = \frac{\frac{R.n.\delta}{p}}{\frac{R.n\,(P-p)}{Pp}} = \frac{P.\delta}{P-p}.$

We have also (7.15) $R' = \frac{R.d}{P}$, and $SB = \frac{R.\delta}{p} = \frac{R'.\gamma}{p'}.$

Hence, $p' = \frac{p.R'.\gamma}{R.\delta} = \frac{p}{R.\delta}.\frac{R.d}{P}.\frac{P.\delta}{P-p} = \frac{p.d}{P-p}.$

And, $EC = \frac{R'.n}{ECB} = \frac{R'\,n}{\gamma - p'} = \frac{\frac{R.d.n}{P}}{\frac{P.\delta}{P-p} - \frac{p.d}{P-p}}$

$$= \frac{R.\,d.n\,(P-p)}{(P.\delta - p.d)\,.P} = \frac{R.n}{P}.\frac{P-p}{\frac{\delta}{d}P - p}.$$

In the last expression for the value of EC, the factor $\frac{R.n}{P}$ expresses the moon's distance from the centre of the earth. And it is evident the other factor $\frac{P-p}{\frac{\delta}{d}P - p}$ is greater than, equal to, or less than a unit, according as d is greater than, equal to, or less than δ. Hence the moon's shadow extends beyond the centre of the earth, just to it, or not so far, according as the moon's apparent diameter is greater than, equal to, or less than the sun's apparent diameter.

The mean apparent diameter of the moon is less than that of the sun; and in any given period, the time for which the true apparent diameter of the moon is less than that of the sun is greater, than the time during which it is greater. Hence it will occur that at the time of new moon, the apparent diameter of the moon will more frequently not be so great as that of the sun, than that it will be greater.

20. The apparent semi-diameter, as seen from the

moon, of a section of the moon's shadow at the earth, is equal to,

$$(d-\delta).\frac{P}{P-p}.$$

From the triangle mEC, we have,

$$mEM = Fmb - ECm = d - (\gamma - p') = d + p' - \gamma = (19),\ d + \frac{p.d}{P-p} - \frac{P.\delta}{P-p} = \frac{P.d - P.\delta}{P-p} = (d-\delta).\frac{P}{P-p}$$

21. The greatest breadth of the moon's shadow, at the earth, is about $\frac{1}{61}$ part of the earth's diameter. The expression for the breadth is,

$$\frac{d-\delta}{P-p}.\ 2\,R.$$

To obtain the breadth, we have,

$$Mm = \frac{EM.\ \text{angle}.\ MEm}{n} = \frac{1}{n}.\frac{R.n}{P}.\frac{(d-\delta).P}{P-p} = \frac{d-\delta}{P-p}.\ R$$

$$\text{or } mm' = 2\ Mm = \frac{d-\delta}{P-p}.\ 2\ R.$$

If we take $d = 1005''.5$, $\delta = 945''$, $P = 3692''$, and $p = 8''.5$, which are, the greatest value of d, least of δ, and the corresponding values of P and p, we evidently obtain the greatest value of mm'. These numbers give,

$$mm' = \frac{d-\delta}{P-p}.\ 2\ R \quad \frac{60}{3684}.\ 2\ R = \frac{1}{61}.\ 2\ R \text{ nearly.}$$

22. As the sun can only be entirely eclipsed at those parts of the earth's surface on which the moon's shadow falls, it is evident from the preceding article, that in the most favourable cases, this phenomenon can only have place for a small portion of the earth's surface, at the same instant of time. But in consequence of the moon's motion, the shadow traverses the surface in an easterly direction, and thus passes over successive portions. The greatest breadth of the surface that the shadow can in any eclipse thus pass over, excepting towards the poles, is about $\frac{1}{61}$ part of the earth's diameter, or 130 miles.

23. When for a point on the earth, in the right line passing through the centres of the sun and moon, $d = \delta$,

the breadth of the shadow will be nothing. In this case, the sun will be entirely eclipsed, but it will not continue so for any perceptible time. When d is less than δ, the expression for the breadth of the shadow becomes negative; the rays from the edge of the sun, which pass near the moon, crossing each other, in that case, before they arrive at the earth. In those parts of the earth where this has place, *the edge of the sun will appear as a ring, surrounding the moon.*

The greatest breadth at the earth, of the part of the penumbra intercepted between the extreme crossing rays, is about $\frac{1}{34}$ part of the diameter of the earth, or 233 miles. Consequently the breadth of the space extending across the earth's surface, within which the phenomenon just mentioned will have place, may in some cases extend to about 233 miles; but excepting towards the poles it cannot much exceed this distance.

Changing the expression $\frac{d-\delta}{\mathrm{P}-p}.\,2\,\mathrm{R}$ (21), to $\frac{\delta-d}{\mathrm{P}-p}.\,2\,\mathrm{R}$, and taking the greatest value of δ, least value of d, and corresponding values of P and p, we have,

$$\frac{\delta-d}{\mathrm{P}-p}.\,2\,\mathrm{R} = \frac{977-881}{3232-9}.\,2\,\mathrm{R} = \frac{1}{34}.\,2\,\mathrm{R}.$$

24. The apparent semi-diameter, as seen from the moon, of a section of the moon's penumbra, at the earth, is equal to

$$(d+\delta).\,\frac{\mathrm{P}}{\mathrm{P}-p}$$

From the triangles ELh' and ELB, we have,
$\mathrm{ME}h' = \mathrm{E}h'c + \mathrm{EL}c = \mathrm{E}h'c + \mathrm{EB}c + \mathrm{SEB} = d + p' + \delta' = d + \frac{p.d}{\mathrm{P}-p} + \frac{\mathrm{P}.\delta}{\mathrm{P}-p} = \frac{\mathrm{P}.d + \mathrm{P}.\delta}{\mathrm{P}-p} = (d+\delta).\,\frac{\mathrm{P}}{\mathrm{P}-p}.$

25. The greatest breadth of the moon's penumbra, at the earth, is a little more than half the earth's diameter. The expression for the breadth is,

$$\frac{d+\delta}{\mathrm{P}-p}.\,2\,\mathrm{R}.$$

To obtain the breadth, we have,

$$Mh' = \frac{\text{EM. ang. ME}h'}{n} = \frac{1}{n} \cdot \frac{\text{R.}n}{\text{P}} \cdot \frac{(d+\delta).\text{P}}{\text{P}-p} = \frac{d+\delta}{\text{P}-p} \cdot \text{R.}$$

$$\text{and } hh' = 2\,\text{M}h' = \frac{d+\delta}{\text{P}-p} \cdot 2\,\text{R.}$$

Taking $d = 881''$, $\delta = 977''.3$, $\text{P} = 3232''$ and $p = 8''.7$, we obtain the greatest value of,

$$hh' = \frac{d+\delta}{\text{P}-p} \cdot 2\,\text{R} = \frac{1858}{3223} \cdot 2\,\text{R} = \frac{23}{40} \cdot 2\,\text{R nearly.}$$

26. As no part of the sun can be hid by the moon, at those parts of the earth which are without the penumbra, the sun may be wholly visible for a large portion of the earth, while it is eclipsed either in part or entirely, in other parts.

27. If, at the time of new moon, the apparent distance of the sun and moon does not become less than $\text{P} - p + d + \delta$, there cannot be an eclipse of the sun to any part of the earth.

Again, considering *agb* as a section of the earth, let M′ be the place of the moon's centre, when in the conical surface, which circumscribes the sun and earth. Then the angle,

$$\text{M}'\text{ES} = \text{EM}'b + \text{EC}b = \text{P} - p + \delta.$$

If to the value of M′ES, we add the apparent semi-diameter of the moon, we shall have, for the apparent distance of the centres of the sun and moon, at the beginning or end of an eclipse of the sun, the expression $\text{P} - p + d + \delta$.

28. From the expression $\text{P} - p + d + \delta$, by taking into view the inclination of the orbit, and the inequalities in the motions of the sun and moon, it has been found, according to Delambre, that when at the time of mean new moon, the difference of the mean longitudes of the moon and node exceeds 19° 2′, there cannot be an eclipse of the sun; but when this difference is less than 13° 14′, there must be one. These numbers are called the *Solar Ecliptic Limits.*

29. As the solar ecliptic limits exceed the lunar, eclipses of the sun must occur more frequently than those of the moon. But as the former are only visible to some parts

of that portion of the earth, which has the sun above the horizon during the eclipse (26), and the latter to the whole of that portion which has the moon above the horizon (17), there are, for any given place, more *visible* eclipses of the moon than of the sun.

NUMBER OF ECLIPSES IN A YEAR.

30. From the solar and lunar ecliptic limits, and the motions of the sun, moon, and node, it is found that the greatest number of eclipses that can take place in a year is seven; and that the least number is two.

When there are seven eclipses in a year, five are of the sun, and two of the moon. When there are only two, they are both of the sun. In every year there are at least two eclipses of the sun.

DIFFERENT KINDS OF ECLIPSES.

31. When the moon just touches the earth's shadow, or approaches very near, without entering it, the circumstance is called an *Appulse*. When a part, but not the whole of the moon, enters the earth's shadow, the phenomenon is called a *Partial* eclipse of the moon; when the moon enters wholly into the shadow, it is called a *Total* eclipse; and when the centre of the moon passes through the axis of the shadow, the eclipse is said to be *Central*. An exactly central eclipse of the moon seldom if ever occurs.

With regard to the sun, when the disc of the moon just touches, or approaches very near, to the disc of the sun, the circumstance is called an *Appulse*. When the moon obscures a part, and only a part, of the sun, the eclipse is said to be *Partial;* and when the moon obscures the whole of the sun, the eclipse is said to be *Total*. When the moon's disc is entirely interposed between the spectator and the sun, but in consequence of the apparent diameter of the moon being less than that of the sun, the edge of the sun is seen as a ring surrounding the moon (23), the eclipse is called *Annular*. Lastly, when the straight line passing through the centres of the sun and moon, passes also through the place of the spectator, the eclipse is said to be *Central.*

A total or an annular eclipse of the sun is only visible for a small portion of the earth at the same time (19, 22 and 23). The continuance of either for any given place, is also short. According to the determinations of Duséjour, the greatest possible duration of a total eclipse is about 8 minutes, and of an annular eclipse, about 12 minutes. In general, the durations are much less.

In a total eclipse, the obscurity is such as to render the principal stars and most conspicuous planets distinctly visible.

ECLIPSES OF THE MOON.

32. The apparent distance of the centre of the moon from the axis of the earth's shadow, and the arcs of the moon's orbit and of the ecliptic passed through by these, during an eclipse of the moon, being necessarily small, may without material error, be considered as right lines. We may also consider the apparent motion of the sun in longitude and the motions of the moon, in longitude and latitude, as uniform during the eclipse. These suppositions being made the calculation of the circumstances of an eclipse of the moon, is very simple.

33. Let NF, *Fig.* 27, be a part of the ecliptic, NL a part of the moon's orbit, C the centre of a section of the earth's shadow at the moon, CD perpendicular to NF, a circle of latitude, and M the centre of the moon at the instant of opposition. Then CM, which is the latitude of the moon in opposition, is the distance of the centres of the shadow and moon at that time.

Let t be some short interval of time expressed in hours, and parts of an hour, and let C′ and M′ be the situation of the entres of the shadow and moon at the time t before or after opposition. Then C′M′ will be the distance of the centres at that time. Draw M′F perpendicular, and AMB parallel to NF. Then CC′ is the motion of the centre of the shadow in the time t, CF is the moon's motion in longitude, and HM′ its motion in latitude. Now as the longitude of the centre of the earth's shadow must always differ by 180°, from the longitude of the sun, the apparent motion of the sun is the same as that of the centre of the shadow. Therefore CC′ expresses the sun's motion in longitude in the time t. And consequently $C'F = CF - CC' =$ the difference of the moon's and sun's motions in longitude, in the time t.

34. Make CG equal to $C'F$, and GM'' perpendicular to NF, and equal to FM'. Then $CM'' = C'M' =$ the distance of the centres of the moon, and earth's shadow at the time t, from opposition. We therefore obtain the distance of the centres of the moon, and shadow, the same, if instead of allowing to each its proper motion we suppose the centre of the shadow to remain at rest at C, and the moon's motion in longitude to be equal to the difference of the motions of the moon and sun, in longitude.

35. From Astronomical Tables we can get the hourly motions of the sun and moon, in longitude, and the moon's hourly motion in latitude. Then supposing the motions uniform, we easily obtain their values for any other short interval of time. Put

$T =$ time of opposition,
$t =$ time of moon's centre passing from M to M',
$m =$ moon's hourly motion in longitude,
$n =$ moon's hourly motion in latitude,
$r =$ sun's hourly motion in longitude,
$\lambda =$ moon's latitude at opposition,
$I =$ angle $M''MR$,
$s = P + p - \delta + \frac{1}{60}(P + p - \delta) =$ semidiam. of earth's shadow (10 and 15).

Then $CF = m.t$, $CC' = r.t$ and $RM'' = HM' = n.t$,
$MR = CG = C'F = CF - CC' = m.t - r.t = t.(m - r)$, tan. $I =$

$$\text{tan } M''MR = \frac{RM''}{RM} = \frac{t.n}{t.(m-r)} = \frac{n}{m-r}.$$

As the expression for the tangent of the angle $M''MR$ does not involve t, it is evident the angle itself will continue the same, whatever be the value of t. Hence the point M'' moves in the line PMQ, which is therefore called the moon's *Relative Orbit*.

36. In the triangle $M''MR$, we have,

$$MM'' = \frac{MR}{\cos M''MR} = \frac{t.(m-r)}{\cos I}.$$

The distance MM'' is the moon's motion on the relative orbit, in the time t. If we take $t = 1$ hour, we have,

The moon's hourly motion on relative orbit $= \dfrac{m - r}{\cos I}$.

37. Let AB, *Fig.* 28, be the ecliptic, C the centre of the earth's shadow at the time of opposition, and CK perpendicular to AB, a circle of latitude. Make $CM = \lambda$, Mb parallel to AB, and $= m - r$, and bc parallel to CK, and $= n$. Through M and c, draw $DMcH$, which will be the moon's relative orbit. With the centre C and a radius $= s$, describe the circle KLPI, which will represent the section of the earth's shadow at the moon. With the same centre and a radius $= s + d$, describe arcs

cutting DH in D and H; and with a radius $= s - d$, describe other arcs, cutting DH in E and G. From C, draw CF perpendicular to DH. Then supposing the moon to move in the direction DH, it is evident that D is the place of the moon's centre at the beginning of the eclipse; E, its place at the beginning of the total eclipse; F, its place when nearest the centre of the shadow; G, its place at the end of the total eclipse; and H, its place at the end of the eclipse. When $s - d$ is less than CF, the eclipse cannot be total.

38. Because CD = CH, and CF is perpendicular to DH, we have FD = FH. The point F, therefore, designates the moon's place at the middle of the eclipse.

In the triangles MFC and M*bc*, the angles F and *b* are right angles; and because *bc* is parallel to CM, the angle FMC = M*cb*. Therefore, MCF = *b*M*c* = I (35).

MIDDLE OF THE ECLIPSE.

39. In the triangle MCF,

$$\text{MF} = \text{CM} \sin \text{MCF} = \lambda \sin \text{I}.$$

But, taking x = interval of time between the middle of the eclipse and the time of opposition, we have (36),

$$\text{MF} = \frac{x.(m-r)}{\cos \text{I}}.$$

$$\text{Hence, } \frac{x.(m-r)}{\cos \text{I}} = \lambda \sin \text{I}, \text{ or } x = \frac{\lambda \sin \text{I} \cos \text{I}}{m-r}.$$

Now if M = the time of the middle, we obtain,

$$\text{M} = \text{T} \mp x = \text{T} \mp \frac{\lambda \cos \text{I} \sin \text{I}}{m-r}.$$

The *upper* sign must be used when the latitude is *increasing;* and the *lower*, when it is *decreasing*.

The nearest distance of the centre is CF = $\lambda \cos \text{I}$.

BEGINNING AND END OF THE ECLIPSE.

40. Let B = the time of beginning, E = the time of the end, and x = the interval between the middle and either of these. Then,

$$\frac{x.(m-r)}{\cos \text{I}} = \text{DF} = \sqrt{\text{DC}^2 - \text{FC}^2}$$

$$= \sqrt{(s+d)^2 - \lambda^2 \cos^2 \text{I}}$$

$$= \sqrt{(s+d-\lambda \cos \text{I}).(s+d+\lambda \cos \text{I})}$$

$$\text{or } x = \frac{\cos \text{I} \sqrt{(s+d-\lambda \cos \text{I}).(s+d+\lambda \cos \text{I})}}{m-r}.$$

Hence, B = M $- x$, and E = M $+ x$, become known.

BEGINNING AND END OF THE TOTAL ECLIPSE.

41. Put B′ = the time of the beginning of the total eclipse, E′ = the time of the end, and x' = the interval between each of these and the middle. Then,

$$x' = \frac{\cos I \sqrt{(s - d - \lambda \cos I).(s - d + \cos I)}}{m - r},$$

$$B' = M - x', \text{ and } E' = M + x'.$$

QUANTITY OF THE ECLIPSE.

42. In an eclipse of the moon, it is usual to suppose that diameter of the moon, which, produced if necessary, passes through the centre of the shadow, to be divided into twelve equal parts, called *Digits*, and to express the quantity of the eclipse by the number of those parts that is contained within the shadow, at the time when the centres of the moon and shadow are nearest. When the moon is entirely within the shadow as in total eclipses, the quantity of the eclipse is still expressed by the number of digits of the moon's diameter which is contained in that part of a radius of the shadow, passing through the moon's centre, which is intercepted between the edge of the shadow and the inner edge of the moon. Thus, the number of digits contained in SN, expresses the quantity of the eclipse, represented in the figure. Hence, if Q = the quantity of the eclipse, we have,

$$Q = \frac{NS}{\frac{1}{12} NV} = \frac{CS - CN}{\frac{1}{12} NV} = \frac{CS - (CF - FN)}{\frac{1}{12} NV}$$

$$= \frac{CS + FN - CF}{\frac{1}{12} NV} = \frac{12\,(CS + FN - CF)}{NV}$$

$$= \frac{12\,(s + d - \lambda \cos I)}{2\,d} = \frac{(s + d - \lambda \cos I).\,6}{d}$$

CONSTRUCTION OF AN ECLIPSE OF THE MOON.

43. The times of the different circumstances of an eclipse of the moon may easily be determined by a geometrical construction, within a minute or two of the truth. To render the construction explicit, suppose the time of opposition to be 8 h. 35 m. 20 sec. on some given day. Then, as 60 minutes : 35 m. 20 sec. : : moon's hourly motion on relative orbit : moon's distance from the point M at 8 o'clock. If this distance be taken in the dividers, and laid on the relative orbit, from M backwards to the point 8, it will give the moon's place at that hour. Then taking in the dividers, the moon's hourly motion on the relative orbit, and laying it on the orbit from 8 to 9, 9 to 10, 10 to 11, and backwards, from 8 to 7, and 7 to 6, we have the places of the moon's centre at those hours respectively. By dividing the hour spaces into quarters, and subdividing these into 5 minute spaces or minute spaces, we easily perceive the times at which the moon's centre is at the points D, E, F, G, and H.

ECLIPSES OF THE SUN.

44. It has been shown in a preceding article (27), that when the angular distance of the centres of the sun and moon is equal to $P - p + \delta + d$, the edge of the moon just touches the conical surface which circumscribes the sun and earth. When the distance of the centres becomes less than $P - p + \delta + d$, and as long as it continues less, it is evident the moon must obscure a portion or the whole of the sun's light to some part of the earth's surface. The eclipse, thus considered in reference to the earth in general, is called the *General Eclipse*. The times of the beginning and end of the general eclipse may be obtained either by calculation or construction, nearly in the same manner as the beginning or end of an eclipse of the moon.

45. Although the calculation of the general eclipse of the sun is equally simple with that of a lunar eclipse, it is very different when the object is to determine the circumstances of the eclipse for any particular place. Then, it is necessary to take into view the situation of the place on the illuminated surface of the earth, or, which amounts to the same, to consider the effects of parallax. This circumstance renders the calculation tedious, at least when it is desired to give to the results all the accuracy of which the problem is susceptible.

46. We shall first give a method of obtaining results nearly true, by means of a geometrical construction. When the construction is carefully performed, on a large scale, the error in the time of beginning or end will not exceed one or two minutes. This method will therefore suffice, except considerable accuracy is required. We shall afterwards give a method of obtaining by caculation, from the results of the construction, others that will be more accurate.

The earth will still be considered as a sphere, and accordingly the reduced latitude of the place, and the reduced parallax, must be used (10.51).

47. A section of the earth made by a plane passing through its centre, perpendicular to the line joining the centres of the earth and sun, is called the *Circle of Illumination*. It forms nearly the boundary between the enlightened and dark parts of the earth's surface. As the sun is larger than the earth, it evidently enlightens a small portion more than one half

of the earth. The enlightened part is still further increased by the effect of the earth's atmosphere in refracting the rays of light.

48. A plane between the earth and sun, perpendicular to the straight line joining their centres, and at a distance from the earth's centre, equal to the distance of the centres of the earth and moon, is called the *Plane of Projection.*

49. If from the sun's centre to every point in the circumference of the circle of illumination, straight lines be conceived to be drawn, they will form the surface of a right cone, a section of which, by the plane of projection is a circle, and is called the *Circle of Projection.*

50. A plane passing through the centre of the sun and the poles of the earth, and which consequently passes through the earth's centre, is called the *Universal Meridian.* When, by the diurnal rotation of the earth on its axis, any place on its surface is brought to coincide with this plane, the sun must be on the meridian of that place.

51. Let S, *Fig.* 29, be the centre of the sun, E, the centre of the earth, TSU the plane of the universal meridian, and let AUWT and PRQV, each conceived to be perpendicular to the plane of the paper, be, respectively, the circles of illumination and projection. If D be a place on the earth's surface, a spectator at D will see the sun's centre, in the direction of the line DS, which intersects the circle of projection in L. The point L is called the *Projection of the Sun's Centre,* for the spectator at D.

52. Let DF and LM be each perpendicular to TUS, the plane of the universal meridian, and FG and MC each perpendicular to ES, the line joining the centres of the earth and sun. Then DF and LM are the distances of the points D and L from the universal meridian; and FG and MC are the distances of the same points from a plane perpendicular to the universal meridian, and passing through the centres of the sun and earth.

53. From the triangle EQS, we have,

$$\mathrm{CEQ} = \mathrm{EQU} - \mathrm{ESU}$$

or *Apparent semi-diameter of the circle of Projection* $= \mathrm{P} - p$.

54. By similar triangles, EU : CQ : : ES : CS : : EN : CM. But $\mathrm{CQ} = \mathrm{EC} \tan \mathrm{CEQ} = \mathrm{EC} \tan (\mathrm{P} - p)$; and we may without sensible error consider $\mathrm{EN} = \mathrm{FG}$. Hence,

$$\mathrm{EU} : \mathrm{EC} \tan (\mathrm{P} - p) :: \mathrm{FG} : \mathrm{CM} = \frac{\mathrm{FG}.\ \mathrm{EC} \tan (\mathrm{P} - p)}{\mathrm{EU}}.$$

$$\text{Therefore, } \tan \mathrm{CEM} = \frac{\mathrm{CM}}{\mathrm{EC}} = \frac{\mathrm{FG}}{\mathrm{EU}}.\ \tan (P - p),$$

$$\text{or, } \mathrm{CEM} = \frac{\mathrm{FG}}{\mathrm{EU}}.\ (\mathrm{P} - p).$$

$$\text{In like manner, } \mathrm{MEL} = \frac{\mathrm{FD}}{\mathrm{EU}}.\ (\mathrm{P} - p).$$

If $\mathrm{X} = \mathrm{MEL} =$ the apparent distance of the projection of the sun's centre from the universal meridian, Y = CEM = the apparent distance of the projection of the centre from the plane passing through the centres of the earth and sun, perpendicular to the universal meridian, and R = EU = radius of the earth, we have,

$$X = \frac{FD}{R} . (P - p), \text{ and } Y = \frac{FG}{R} . (P - p).$$

55. Let AUBT, *Fig.* 30, be a section of the earth by the plane of the universal meridian, PP′ the earth's axis, P the north pole, P′ the south pole, TU a diameter of the circle of illumination, CQ a diameter of the equator, ES the line joining the centres of the earth and sun, and D a given place on the earth. Also let DLH be a plane perpendicular to PP′ the earth's axis, DF a straight line perpendicular to LH, and consequently to the universal meridian, LM and FR, each parallel to AB, and LN and FG, each parallel to TU. Then, as in a preceding article (52), DF is the distance of the place from the universal meridian, and FG is its distance from the plane passing through the centres of the earth and sun, perpendicular to the universal meridian. Also PH = PD = the complement of the latitude of the place, DLH = DPH = the hour angle from noon, and TP = AC = the sun's declination. Put,

D = TP = the sun's declination,
U = DLH = the hour angle from noon,
H = CH = the reduced latitude of the place,

Then HL = EH sin HEP = R cos H,
and DF = DL sin DLH = HL sin U = R cos H sin U.

$$\text{Hence (54), } X = \frac{DF}{R} . (P - p) = \cos H \sin U . (P - p) \qquad \text{(A).}$$

Also,

$$EL = EH \cos HEP = R \sin H,$$
$$EM = EL \cos TEP = R \sin H \cos D$$
$$= R . \frac{\sin (H + D) + \sin (H - D)}{2} \qquad \text{(App. 16)}$$
$$ML = EL \sin TEP = R \sin H \sin D,$$
$$LF = DL \cos DLH = HL \cos U = R \cos H \cos U.$$

But from the similar triangles FNL and EML, we have,

EL : LM : : FL : LN,
or R sin H : R sin H sin D : : R cos H cos U : MR
1 : sin D : : R cos H cos U : MR.

Hence MR = R cos U sin D cos H

$$= R \cos U . \frac{\sin (H + D) - \sin (H - D)}{2} \qquad \text{(App. 17)}$$

When the latitude of the place and declination of the sun are both north, as represented in the figure, ER = EM — MR. It will be the same when the latitude and declination are both south. But when one is north and the other south, ER = EM + MR. Therefore,

$$FG = ER = EM \mp MR = R.\frac{\sin(H+D)+\sin(H-D)}{2}$$

$$\mp R\ \cos U.\frac{\sin(H+D)-\sin(H-D)}{2}$$

Consequently (54),

$$Y = \frac{\sin(H+D)+\sin(H-D)}{2}.(P-p)$$

$$\mp\frac{\sin(H+D)-\sin(H-D)}{2}\cos U.(P-p). \qquad \text{(B)}$$

PROJECTION OF THE SUN'S CENTRE ON THE CIRCLE OF PROJECTION.

56. Let AB, *Fig.* 31, be the line in which the plane, through the centres of the sun and earth, and perpendicular to the universal meridian, intersects the circle of projection, and CD the intersection of the universal meridian with the same circle. With the centre C, and a radius equal to P—*p*, describe the semicircle ADB, to represent the northern half of the circle of projection. With a sector,* make AE and BF, each equal to H, the reduced latitude of the place, and make EG, EI, FH and FK, each equal to D, the sun's declination. Join GH, EF, and IK, and bisect R*v* in N. Through N, draw LNM parallel to EF. Make NO equal to *w*E, and with the centre N, and radii NO and NR, describe the semicircles, PYO and *t*RQ. With a sector, make YW equal to U, the hour angle from noon. Join WN, draw WV parallel to CD, and through T, draw STU parallel to PO, meeting WV in U.

In this construction, the sun's declination is supposed to be north, and the time in the afternoon. When the declination is south, the semicircle *t*RQ must be on the upper side of PO, and WN must be produced to meet it in T. When the time is before noon, the arc YW must be laid off from Y to the right hand.

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* The *Sector* is an instrument, generally made of ivory or boxwood, about a foot in length, with a joint in the middle. There are several lines on each side of it. But the only one we shall notice is the line of chords, which is used to lay off a given number of degrees on the arc of a circle. This line is marked with the letter C. It consists of two lines, running each way from the centre of the joint to near the ends of the instrument; each line being divided into 60 parts or degrees, and each degree subdivided into halves. At the 60 on each line, there is a brass pin with a small puncture.

To lay off any number of degrees on the arc of a given circle, take the radius of the circle in the dividers, and setting one foot in the puncture at the end of one line, open the sector till the other foot of the dividers just reaches to the puncture at the end of the other line. Then setting one foot of the dividers to the given number of degrees on one line, open them till the other foot reaches to the same number of degrees on the other line. This distance applied as a chord to the arc, will intercept the given number of degrees.

To measure a given arc open the sector as before. Then, taking the chord of the arc in the dividers, apply them to the line of chords, moving them without changing their opening, till each foot is at the same number of degrees on each line. This number of degrees will be the measure of the given arc.

$$\text{Now, } NO = wE = wF = CF \sin FCw = \cos H.\,(P - p),$$

$$SU = NV = NW \cos WNL = NO \sin U$$

$$= \cos H \sin U.\,(P - p).$$

Hence, (55. A), $SU = X$.

$$\text{Also, } Cv = CH \cos v\,CH = \sin HCB.\,(P - p)$$

$$= \sin (H + D).\,(P - p),$$

$$CR = CK \cos RCK = \sin KCB.\,(P - p)$$

$$= \sin (H - D).\,(P - p),$$

$$CN = \frac{Cv + CR}{2} = \frac{\sin (H + D) + \sin (H - D)}{2}.\,(P - p),$$

$$RN = \frac{Cv - CR}{2} = \frac{\sin (H + D) - \sin (H - D)}{2}.\,(P - p),$$

$$NS = NT \cos YNW = NR \cos U$$

$$= \frac{\sin (H + D) - \sin (H - D)}{2}.\,\cos U\,(P - p),$$

$$CS = CN \mp NS = \frac{\sin (H + D) + \sin (H - D)}{2}.\,(P - p)$$

$$\mp \frac{\sin (H + D) - \sin (H - D)}{2} \cos U.\,(P - p).$$

Hence (55. B), $CS = Y$.

Since US, the distance of the point U, from the universal meridian, is equal to X, and CS, its distance from the plane, perpendicular to the universal meridian, and passing through the centres of the earth and sun, is equal to Y, the point U is the projection of the sun's centre, for the spectator at the given time and place (54).

57. It is evident from the construction, that the semicircles PYO and *t*RQ, depend only on $P - p$, the difference of the horizontal parallaxes of the moon and sun, D the sun's declination, and H the latitude of the place. Therefore, since $P - p$ and D may be considered as constant during the continuance of the eclipse, the same semicircles will answer for finding the point U at any other time during that interval.

58. When it is required to find the projection of the sun's centre for several different times, on the same figure, it is better to omit drawing the lines WN and UTS, and instead, to lay the edge of a ruler from W to N, make a mark at T on the semicircle *t*RQ, and draw T*s* parallel to CD. The distance T*s*, applied on VW, from V to U, will give the point U the same as before.

59. From the similar triangles WNV and TNS, we have WN : TN :: WV : NS. But WN = NO, TN = NR, and NS = UV. Therefore, NO : NR :: WV : UV. Hence (Conic Sections), the point U is in an ellipse, of which PO is the transverse axis, and R*v* the conjugate. On account of the earth's diurnal motion, the position of U is continually

changing. But it is evident, that as long as $P - p$ and D may be considered constant, its ***Path will be the ellipse described about the axes*** **PO *and* R*v*.**

POSITION OF THE MOON'S RELATIVE ORBIT ON THE CIRCLE OF PROJECTION.

60. Make D*a* and D*b* each equal 23° 28′, the obliquity of the ecliptic; join *ab* and on it describe the semicircle *adb*. From *b* lay off the arc *bd* equal to the sun's longitude. When the longitude exceeds 6 signs, its excess above 6 signs must be laid off from *a* to *d*. Draw *dm* perpendicular to *ab*, and through *m* draw C*m*Z.

Since the universal meridian passes through the poles of the earth (50), it coincides with a circle of declination. Hence CD, its intersection with the circle of projection, may be considered as the arc of a declination circle. In like manner, the line in which a circle of latitude passing through the centre of the sun, intersects the circle of projection, may be considered as the arc of a circle of latitude. The angle contained between these lines will be the angle of position for the sun (6.20). Put,

$L = bd =$ sun's longitude,
$\omega = Db =$ obliquity of the ecliptic,
$S =$ the angle of position.

$$\text{Then } bn = Cb \sin b\,CD = Cb \sin \omega,$$
$$mn = nd \cos bnd = bn \cos L = Cb \sin \omega \cos L,$$
$$Cn = Cb \cos b\,CD = Cb \cos \omega,$$
$$\tan DCZ = \frac{mn}{Cn} = \frac{Cb \sin \omega \cos L}{Cb \cos \omega} = \tan \omega \cos L.$$

But (6.21.C), $\tan S = \tan \omega \cos L$.
Hence, $\tan DCZ = \tan S$; or $DCZ = S$.

Therefore CZ makes with CD, the angle DCZ equal to S, the angle of position. It is also evident, that by the construction, CZ will fall to the right hand, or west of CD, when the longitude is less than 90° or more than 270°, and to the left hand or east, when the longitude is more than 90° and less than 270°. Hence CZ is a circle of latitude, the plane of which passes through the sun's centre.

61. Having CZ the circle of latitude, the moon's relative orbit may be drawn, and the places of the moon's centre at different hours, be determined in the same manner as in an eclipse of the moon (37 and 43), only using the time of conjunction instead of the time of opposition. Let *pq* be the moon's relative orbit; and let *u* be the place of the moon's centre at the same time the projection of the sun's centre is at U. If the distance U*u* is less than the apparent semidiameters of the sun and moon, a part of the sun, at least, is then eclipsed.

62. Considering the earth a sphere, a vertical line (4.5) and consequently a vertical circle, at a given place, will pass through the centre.

Hence, since C is in the straight line joining the centres of the sun and earth, CU is the intersection of the circle of projection with a vertical circle passing through the centre of the sun. The position of *u*, the moon's centre, with regard to this circle, is therefore determined.

CONSTRUCTION OF AN ECLIPSE OF THE SUN.

63. The construction, *Fig.* 32, is for an eclipse of the sun on the 27th of August, 1821, in the morning, and it is adapted to the meridian and latitude of Philadelphia. The points 7, 8, 9, 10, and 11 on the line *pq*, represent the situation of the moon's centre on the plane of projection, at those hours respectively. The other points 7, 8, 9, 10 and 11, represent the projections of the sun's centre at the same times. The lines 7*c*, 8*e*, 9*g*, 10*h*, and 11*k*, are drawn parallel to AB, meeting the lines VII*c*, VIII*s*, IX*v*, X*w*, and XI*x*, in the points *c*, *e*, *g*, *h*, and *k*.

Draw a line MN, *Fig.* 33, and in it take any point S, to represent a fixed position of the sun's centre. Then taking 7*c*, the distance from the moon's centre at 7 o'clock, to the line VII*c*, lay it from S to the right hand to *c*. In like manner, make the distances S*e*, S*g*, S*h*, and S*k*, in the line MN, respectively equal to the distances 8*e*, 9*g*, 10*h*, and 11*k*, observing that each distance is to be laid off to the right or left of the point S, according as the moon's centre is to the right or left of the projection of the sun's centre. Draw the lines *c*7, *e*8, *g*9, *h*10 and *k*11, perpendicular to MN. Make *c*7 equal to the distance *c*7 on the line passing through the projection of the sun's centre at 7 o'clock. Also make *e*8, *g*9, *h*10, and *k*11, respectively equal to the corresponding distances in *Fig.* 32, observing that each must be placed above or below the line MN, according as the moon's centre is higher or lower than the projection of the sun's centre. Then the points 7, 8, 9, 10, and 11, *Fig.* 33, will represent the positions of the moon's centre, at those hours, with regard to S, the sun's centre. Join the points 7,8, 8,9, 9,10, and 10,11, and the broken line thus formed, which will deviate but little from one right line, will represent very nearly the apparent relative orbit of the moon. With the centre S, and a radius equal to the sum of the apparent semi-diameters of the sun and moon, describe arcs cutting the moon's path in *b* and *r*, which will be the positions of the moon's centre at the beginning and end of the eclipse. From S, draw S*q* perpendicular to the part 8,9, of the moon's path; then *q* will be the position of the moon's centre, when it is nearest to the centre of the sun, and consequently when the eclipse is greatest. If the hour spaces 7,8, 8,9, and 10,11 be each divided into quarters and these subdivided into three equal parts, or spaces of five minutes, the times of beginning, greatest obscuration, and end, can be easily estimated.

64. The hour spaces on the moon's apparent path are not equal, and therefore the times obtained from equal divisions of them, are not quite accurate. The error from this cause will not, however, exceed one or

two minutes; and if the construction be made for each half hour, it will be much less.

With the centre S, and a radius equal to the sun's apparent semi-diameter, describe a circle to represent the sun's disc; and with the centre q, and a radius equal to the moon's apparent semidiameter, describe another circle to represent the moon's disc. The part of the sun's disc, that is intercepted by the moon's, shows the part eclipsed. If Sq be produced to m and n, and mn be measured by the scale used in the construction, the quantity of the eclipse will evidently be obtained by this proportion. *As the sun's apparent diameter* : *mn* : : 12 *digits* : *the digits eclipsed.*

Let u, *Fig.* 32, be the projection of the sun's centre at the time the eclipse commences. Then Cu will be a vertical circle passing through the sun's centre at that time (62). Draw SF, *Fig.* 33, making the angle FSM equal to the angle uCB; then V is the sun's vertex. Now as the eclipse commences at the point a of the sun's disc, the angle VSa expresses the angular distance from the sun's vertex, of the point at which the eclipse commences. The knowledge of this angle is important to the astronomer, who wishes to observe with accuracy the commencement of an eclipse of the sun. Without it he would not know at what part of the edge to fix his attention, while waiting to see the first impression.

CALCULATION OF AN ECLIPSE OF THE SUN.

67. Let B designate the approximate time of the beginning of the eclipse, found by construction (63), and t some short interval of 4 or 5 minutes. Calculate for the time $B - t$, by means of astronomical tables, the sun's longitude, hourly motion, and semi-diameter; also the moon's longitude, latitude, horizontal parallax, semi-diameter, and hourly motions in longitude and latitude. Then as our object is to obtain the difference of the apparent longitudes of the sun and moon, and the moon's apparent latitude, in order to obtain their apparent distance, subtract the sun's horizontal parallax, from the reduced horizontal parallax of the moon, and considering the remainder as the moon's parallax, calculate the parallax in longitude and latitude (10. 54 and 56), using the reduced latitude of the place.* To the moon's true longitude and latitude, apply, respectively, the parallaxes in longitude and latitude according to their signs, and the apparent longitude and latitude are obtained. Take the difference between the true longitude of the sun and the apparent longitude of the moon, which will be the moon's apparent distance from the sun, in longitude.

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* This is equivalent to considering the apparent place of the sun the same as the true, and referring the whole effect of parallax to the moon. It is not rigidly exact. For we virtually calculate the sun's parallax in longitude and latitude by making use of the moon's longitude and latitude, instead of the sun's. But on account of the small quantity of the sun's parallax, and the little difference in the longitudes at the time of an eclipse, the error is quite insensible.

68. With the sun and moon's longitudes, the moon's latitude, and their hourly motions at the time $B - t$, find the longitudes and the moon's latitude at the time $B + t$. For this latter time, calculate the parallaxes in longitude and latitude, and thence deduce the apparent distance of the moon from the sun in longitude, and the moon's apparent latitude.

69. Let EC, *Fig.* 34, be a part of the ecliptic and S the place of the sun's centre, which we shall consider fixed. Let SA be the apparent distance of the moon from the sun in longitude, and AD, perpendicular to EC, the apparent latitude, at the time $B - t$. Then D will be the moon's apparent place. In like manner let L be the moon's apparent place at the time $B + t$. The line DL, which does not sensibly differ from a straight line, represents the part of the moon's apparent relative orbit, passed through during $2t$, the interval between the times $B - t$ and $B + t$.

70. Let the point G in the line DL, be the place of the moon's centre at the true time of beginning. Put,

a = SA = appar. dist. of moon from sun in long. at the time $B - t$,
c = AD = moon's app. latitude at the time $B - t$,
m = AH = diff. of moon's app. distances from the sun in long. at $B - t$ and $B + t$,
n = wL = diff. of moon's apparent latitudes at $B - t$ and $B + t$,
s = SG = sum of app. semi-diameters of sun and moon,
I = angle wDL = inclination of moon's app. rel. orbit,
x = AF.

$$\text{Then } \tan I = \tan wDL = \frac{wL}{Dw} = \frac{n}{m},$$

$$Gv = Dv \tan vDG = x \tan I,$$

$$SF = a - x, \text{ and } FG = AD + Gv = c + x \tan I,$$

$$SF^2 + FG^2 = SG^2, \text{ or } (a - x)^2 + (c + x \tan I)^2 = s^2$$

$$a^2 - 2ax + x^2 + c^2 + 2cx \tan I + x^2 \tan^2 I = s^2 \qquad (B)$$

$$(1 + \tan^2 I).x^2 - 2(a - c \tan I)\,x = s^2 - a^2 - c^2,$$

$$x^2 - \frac{2.(a - c \tan I)}{1 + \tan^2 I}.x = \frac{s^2 - a^2 - c^2}{1 + \tan^2 I}$$

$$x^2 - \frac{2.(a - c \tan I)}{1 + \tan^2 I}.x + \frac{(a - c \tan I)^2}{(1 + \tan^2 I)^2} = \frac{s^2 - a^2 - c^2}{1 + \tan^2 I} + \frac{(a - c \tan I)^2}{(1 + \tan^2 I)^2}$$

$$= \frac{(s^2 - a^2 - c^2).(1 + \tan^2 I) + (a - c \tan I)^2}{(1 + \tan^2 I)^2}$$

$$= \frac{s^2 + s^2 \tan^2 I - (c^2 + 2\,a\,c \tan I + a^2 \tan^2 I)}{(1 + \tan^2 I)^2}$$

$$= \frac{s^2.(1 + \tan^2 I) - (c + a \tan I)^2}{(1 + \tan^2 I)^2}$$

$$= \frac{s^2}{1 + \tan^2 I} \cdot \left(1 - \frac{(c + a \tan I)^2}{s^2.(1 + \tan^2 I)}\right)$$

$$= s^2. \cos^2 I. \left(1 - \frac{(c + a \tan I)^2 \cos^2 I}{s^2}\right)$$

$$x - \frac{a - c \tan I}{1 + \tan^2 I} = \pm s. \cos I \sqrt{\left(1 - \frac{(c + a \tan I)^2 \cos^2 I}{s^2}\right)}$$

$$x = (a - c \tan I) \cos^2 I \pm s. \cos I \sqrt{\left(1 - \frac{(c + a \tan I)^2 \cos^2 I}{s^2}\right)}$$

$$\text{Make } \sin \theta = \frac{(c + a \tan I) .\cos I}{s}.$$

$$\text{Then, } \sin^2\theta = \frac{(c + a \tan I)^2. \cos^2 I}{s^2}$$

$$\text{And } \sqrt{\left(1 - \frac{(c + a \tan I)^2 \cos^2 I}{s^2}\right)} = \sqrt{\left(1 - \sin^2\theta\right)} = \cos \theta.$$

$$\text{Hence, } x = (a - c \tan I) \cos^2 I \pm s. \cos I \cos \theta.$$

If the line DL were produced, it is evident there would be another point in it, such that its distance from S would be the same with that of the point G. The two values of x in the formula, correspond to these two points. Hence the value of x corresponding to G, is,

$$x = (a - c \tan I) \cos^2 I - s. \cos I \cos \theta.$$

When the moon's apparent latitude at the time $B + t$ is less than at the time $B - t$, we must make,

$$\sin \theta = \frac{(c - a \tan I) .\cos I}{s}, \text{ and we shall have}$$

$$x = (a + c \tan I) .\cos^2 I - s. \cos I \cos \theta.$$

Now, as $m : x :: 2t : \frac{2tx}{m}$ = interval from $B - t$ to the true time of the beginning. Hence if B' = the true time of beginning, we have,

$$\tan I = \frac{n}{m}, \sin \theta = \frac{(c \pm a \tan I) \cos I}{s},$$

$$B' = B - t + \left((a \mp c \tan I) .\cos^2 I - s \cos I \cos \theta\right). \frac{2t}{m}.$$

The *upper* signs must be used when the apparent latitude is *increasing*, and the *under*, when it is *decreasing*. The same is to be observed in the three following articles.

71. The end of the eclipse may be found in a similar manner. Let E be the approximate time of the end, and make the same calculations for $E - t$ and $E + t$, as for $B - t$ and $B - t$. Then, if a, c, m, n, s, I and θ, designate the same quantities as in the last article, but having the values appertaining to the end of the eclipse, and E' = the true time of end, we have,

$$\tan I = \frac{n}{m}, \sin \theta = \frac{(c \mp a \tan I) \cos I}{s},$$

$$E' = E - t + \left(s.\cos I\cos\theta - (a \pm c\tan I)\cos^2 I\right).\frac{2t}{m}.$$

72. To find the true time of greatest obscuration, let G be the approximate time, and calculate as before, for the times $G - t$ and $G + t$, the apparent distances of the moon from the sun, in longitude, and the moon's apparent latitudes. Let SK be the apparent distance of the moon from the in longitude at the time $G - t$, SN at the time $G + t$, and KI, NP perpendiculars to EC, the corresponding latitudes. Then if Sq be drawn perpendicular to IP, q will be the place of the moon, when the apparent distance of the centres is least. Let $x = KV$, and a, c; m, n, s and I, designate the same quantities as before, but having the values appertaining to the middle.

From the similar triangles PIQ and SqV, we have,

$$IQ : PQ :: Vq : SV.$$

But $IQ = m$, $PQ = n$, $Vq = c + x\tan I$, and $SV = a - x$.

$$\text{Hence, } m : n :: c + x\tan I :: a - x,$$

$$\frac{a - x}{c + x\tan I} = \frac{n}{m} = \tan I,$$

$$x = \frac{a - c\tan I}{1 + \tan^2 I} = (a - c\tan I).\cos^2 I.$$

When the apparent latitude is decreasing,

$$x = (a + c\tan I)\cos^2 I.$$

Hence, if G' = the true time of greatest obscuration, we have,

$$\tan I = \frac{n}{m},\ G' = G - t + (a \pm c\tan I).\cos^2 I.\frac{2t}{m}.$$

73. To find Sq, the nearest distance of the centres, we have (72),

$$Vq = c \pm x\tan I = c \pm \tan I.(a \mp c\tan I)\cos^2 I$$
$$= c \pm \sin I\cos I.(a \mp c\tan I).$$

$$\text{Hence, } Sq = \frac{Vq}{\cos I} = \frac{c}{\cos I} \pm \sin I.(a \mp c\tan I).$$

Now to find the quantity of the eclipse, if δ = the sun's apparent semidiameter, and d = the moon's apparent semi-diameter at the time of the greatest obscuration, we have, *Fig.* 33,

$mn = Sq + qn + Sm = Sq + \delta - Sq + d - Sq = \delta + d - Sq = s -$

$$Sq = s - \frac{c}{\cos I} \mp \sin I.(a \mp c\tan I). \quad \text{Hence,}$$

$$\textit{Digits Eclipsed} = \frac{6.mn}{\delta} = \frac{6.\left(s - \dfrac{c}{\cos I} \mp \sin I.(a \mp c\tan I)\right)}{\delta}$$

74. When Sq, the nearest apparent distance of the centres, is less than the difference between δ and d, the eclipse will be either annular or total. It will be annular if δ be greater than d, and total if d be greater than δ. The time when an eclipse commences or ceases to be total or annular may be found by the same formulæ as the beginning or end of the eclipse,

only making $s =$ to the difference of δ and d, and giving to a, c, m, &c. the values which they have in finding the time of greatest obscuration.

75. Let S*b*, *Fig.* 34, be a circle of latitude, S*d* a declination circle, and S*h* a vertical circle, all passing through the sun's centre. Then *b*S*d* will be the angle of position, and *d*S*h* the angle contained by the declination circle and vertical circle, passing through the sun. Put $L =$ the sun's longitude at the beginning of the eclipse, $\Delta =$ the sun's distance from the north pole of the equator, $a = SF =$ the apparent difference of the sun and moon's longitudes, $c =$ the moon's apparent latitude, $H =$ the latitude of the place, $\omega =$ the obliquity of the ecliptic, and $U =$ the hour angle from noon. Then (6.19 and 20)

$$\cos \Delta = \sin L \sin \omega, \text{ and } \tan bSd = \cos L \tan \omega \qquad \text{(C)}$$

In the triangle PSZ, *Fig.* 18, we have (App. 37),

$$\cot S = \frac{\cot PZ \sin PS - \cos ZPS \cos PS}{\sin ZPS}$$

$$= \frac{\tan H \sin \Delta - \cos U \cos \Delta}{\sin U}$$

Hence, *Fig.* 34, $\tan dSh = \dfrac{\tan H \sin \Delta - \cos U \cos \Delta}{\sin U}$ (D)

$$\text{Also } \tan FSG = \frac{FG}{FS} = \frac{c}{d} \qquad \text{(E)}$$

The angle *b*S*d* will be to the left of S*b*, when the sun's longitude is less than 90°, or more than 270°, and to the right when it is between 90° and 270°. The angle *d*S*h* will be to the right of S*d*, in the forenoon, and to the left in the afternoon. By attending to these circumstances, and adding or subtracting accordingly, the angle *b*S*h*, and consequently ES*h*, becomes known. Thence, by applying the angle FSG, we have the angle *h*SG, contained between the vertical circle passing through the sun's centre, and the line joining the centres of the sun and moon, at the beginning of the eclipse.

76. Supposing the latitude of the place for which the calculation is made, to be truly known, and also its longitude from the place for which the solar and lunar tables are computed, the results obtained from the preceding formulæ, when the calculations are carefully performed, will have an accuracy corresponding with that of the tables themselves, or very nearly so. But the best of these tables are liable to errors of a few seconds. Consequently the times obtained will be liable to small errors, depending on the former.

77. Except in cases when the greatest precision is required in the results, it will only be necessary to calculate the longitudes of sun and moon, and the moon's latitude, from the tables, for the time $G - t$. The longitudes and latitude at the time $B - t$ and $E - t$, may be found from the former by means of their hourly motions. It will also be sufficient to calculate the moon's apparent longitudes and latitudes for the times $B - t$, $G - t$ and $E - t$. Then for the beginning of the eclipse we may take

m = moon's apparent relative motion in longitude during the time $(G - t) - (B - t)$, or which is the same, $G - B$, and n = moon's apparent motion in latitude during the same time. For the end we may give to m and n, the values of the same quantities, for the time $E - G$. And for the greatest obscuration we may give to them the values of those quantities for the time $E - B$. With these values of m and n, we obtain very nearly the inclinations of the apparent relative orbit, at the times of beginning, end, and grearest obscuration.

As the value of x, in the equation B (70), must be small, its square may be omitted with but little error. We shall then have,

$$a^2 - 2ax + c^2 + 2cx \tan I = s^2,$$

$$2.(a - c \tan I).x = a^2 + c^2 - s^2,$$

$$x = \frac{a^2 + c^2 - s^2}{2.(a - c \tan I)},$$

$$\text{Hence } B' = B - t + \frac{(a^2 + c^2 - s^2).(G - B)}{2m\,(a \mp c \tan I)}$$

$$E' = E - t + \frac{[(s^2 - a^2 + c^2)]\ .(E - G)}{2m\,(a \pm c \tan I)}$$

$$G' = G - t + \frac{[(a \mp c \tan I) \cos^2 I]\ .\ E - B)}{m}$$

In each formulæ the *upper* sign is to be used when the apparent latitude is *increasing*, and the *lower* when it is *decreasing*.

78. Instead of finding the approximate times of beginning and end by construction, we may, though with more labour, perform the whole by calculation. Thus, let the sun's longitude and the moon's apparent longitude and latitude be calculated for the time of new moon. From these longitudes we know, whether the *Apparent Ecliptic Conjunction*, that is, the circumstance of the apparent longitude of the moon being the same with the sun's longitude, has place before or after new moon. For a time an hour or two, earlier or later, than the new moon, according as the apparent conjunction is before or after, again calculate the sun's longitude and moon's apparent longitude and latitude. Then considering the apparent relative orbit of the moon as a straight line passing through the apparent positions of the moon with regard to the sun, at those times, we can obtain the approximate times of beginning, greatest obscuration and end, nearly in the same manner as the beginning, middle, and end of a lunar eclipse.

PATH OF A CENTRAL ECLIPSE OF THE SUN.

79. The latitude and longitude of the place, to which the sun is centrally eclipsed, at a given time during the continuance of the central eclipse, may be easily determined with considerable accuracy, by means of a geometrical construction.

If we suppose the circle ATBU, *Fig*, 30, to be described with a radius equal to $P - p$, it is evident (54 and 55), that $FD = X$, and $ER = FG$

$= Y$. Then in the right angled triangle EFD, we have $EF = \sqrt{(ED^2 - FD^2)} = \sqrt{[(P - p)^2 - X^2]}$. Now if X and Y are known for any given time, EF and ER are known, and consequently the position of the point F. The position of PE is determined by the sun's declination, and therefore that of the line HFL, which passes through F, perpendicular to PE. In the right angled triangle ELH, EL and EH being known, the angle PEH, the complement of the latitude of the place, is also known. In the right angled triangle, DLF, FL and FD being known, the angle DLH, which is the hour angle from noon, is known.

80. Let AUBT, *Fig.* 35, be the circle of projection, described with the radius $P - p$, TU the intersection of the universal meridian with this circle, AEB perpendicular to TU, and *pq* the moon's relative orbit (60). If D be the place of the moon's centre at a given time, on a given meridian, it is manifest the place which would have the projection of the sun's centre also at D, would then have a central eclipse. Make TP = the sun's declination, laying it to the *left* of T, when the declination is *north*, but to the *right*, when the declination is *south*, and draw PEP′. Through D, draw MDG parallel to TU, and DF parallel to AB. Then $KM = \sqrt{(EM^2 - EK^2)} = \sqrt{[(P - p)^2 - X^2]}$. With the centre E, and a radius equal to KM, describe an arc, cutting DF, in F, and through F, draw LFH, perpendicular to PP′. Then PEH is the complement of the latitude of the place (80.).

Make KG = FL, and through G, draw EGI. Then because NG = EK = X, and EN = GK = FL, UEI is the hour angle from the universal meridian at the required place (80). The place will be to the *west* or *east* of the universal meridian, according as the point D is to the *right* or *left* of TU. Now, for the place for which the construction is made, the time that the moon's centre is at D, and consequently the distance of the place from the universal meridian is known. Hence for the same instant we know the distance, in time, of the given place and required place, from the universal meridian; and by taking the difference, or sum of these distances, according as they are on the same or different sides of the universal meridian, the longitude, in time, of the required place from the given place, becomes known.

By making the construction for every 15 or 20 minutes, during the continuance of the central eclipse, we shall have the latitudes and longitudes of a series of places, at which the eclipse will be central. A curve line, drawn on a map or globe, through those places, will represent what is called the *Path of the Central Eclipse.*

81. By a process but little different from the preceding, the longitudes and latitudes of those places that will have the eclipse of a given magnitude, for instance 6 or 9 digits, may be obtained.

OCCULTATIONS.

82. If at the time of mean conjunction of the moon and

a star, that is, when the moon's mean longitude is the same with the longitude of the star, their difference of latitude exceed 1° 37′ there cannot be an occultation; but if the difference be less than 51′, there must be an occultation somewhere on the earth. Between these limits there is a doubt, which can only be removed by the calculation of the moon's true place.

83. The construction for an occultation, is nearly the same as for an eclipse of the sun. There is however a difference in some parts. The radius CB, *Fig.* 31, must be equal, P, for a star, and $P - p$ for a planet, p being the horizontal parallax of the planet. The arcs FH, FK, EG, and EI must be equal to the declination, and bd to the longitude, of the star or planet. In determining the projected place of the star, for a given time, we must use the hour angle corresponding to the difference between the given time, and the time the star is on the meridian.

To get the position of the circle of latitude, lay off the declination of the star or planet, from D to b', and draw $b'c$ parallel to ab. With the centre C and radius Cc, describe the arc cm', cutting dm, produced if necessary, in m'. Join Cm', which will be the circle of latitude, as is easily deduced from the expression for the angle of position (6.21. B.)

The distance Cr' on the circle of latitude, must be equal to the difference or sum, of the latitude of the moon, and that of the star or planet, according as they are of the same, or of different names. It must be placed above C, when the moon is to the north of the other body, but below, when it is to the south.

For a star, the moon's motion in longitude, will be its relative motion in longitude. For a planet the moon's relative motion in longitude is obtained by subtracting the motion of the planet when direct, and adding it when retrograde.*

In *Fig.* 33, it is evident, that for a star we must take Sb and Sr, each equal to the apparent semi-diameter of the moon; and for a planet, equal to the sum of the apparent semi-diameters of the moon and planet.

84. When considerable accuracy is required, the moon's relative motion on the ecliptic must be reduced to its motion on a parallel to the ecliptic, passing through the star or planet. This is done by multiplying the relative motion in longitude by the cosine of the latitude of the star or planet. For if AB, *Fig.* 36, be an arc of the ecliptic, and DE the corresponding arc of a circle, parallel to it, we have,

$$BC : EF :: AB : DE = \frac{AB.\ EF}{BC}.$$

$$\text{But, } EF = Ca = BC \cos BCE = BC \cos BE.$$

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* The apparent motion of a planet is sometimes retrograde. This circumstance will be more particularly noticed in the next chapter.

$$\text{Hence, DE} = \frac{\text{AB. BC} \cos \text{BE}}{\text{BC}} = \text{AB} \cos \text{BE}.$$

85. The difference between the calculation of an occultation and that of an eclipse of the sun, is easily deduced from what has been said in the two preceding articles.

86. Observations of an eclipse of the sun or of an occultation of a star, made at places, whose longitudes and latitudes are correctly known, furnish means of determining the errors of the tables at the time; and they are frequently used for that purpose, particularly those of an occultation. The positions of many of the stars are determined with great precision, and the moon's parallax and apparent diameter are very accurately known. But the moon's longitude and latitude, computed from the best lunar tables, are liable to errors of several seconds. Hence if the observed time of beginning or end of an occultation, of a star whose position is well determined, does not agree with the time obtained by calculation, the difference must depend on errors in the computed longitude and latitude of the moon.

87. It is evident from the formulæ for computing the parallaxes in longitude and latitude, that those parallaxes are not sensibly affected by small errors in the longitude and latitude. The errors in the apparent longitude and latitude may therefore be considered the same as those in the true.

88. Let a and c be the apparent distances of the moon from the star in longitude and latitude, respectively, as obtained by calculation for the observed time of beginning, and l be the latitude of the star. Also let $x =$ the error in the moon's longitude, and $y =$ the error in the latitude. Then $a + x$ and $c + y$ will be the true apparent distances of the moon from the star in longitude and latitude, at the observed time of beginning.* Consequently, if $s =$ the moon's apparent semi-diameter, we have,

$$(a + x)^2. \cos^2 l + (c + y)^2 = s^2,$$

$$\text{or, } a^2 \cos^2 l + 2\, ax \cos^2 l + x^2. \cos^2 l + c^2 + 2\, cy + y^2 = s^2.$$

Now as x and y are small quantities, the terms involving their squares may be omitted. Hence, if $\frac{1}{2}(s^2 - a^2 \cos^2 l - c^2) = e$, we have,

$$a \cos^2 l.\, x + cy = e.$$

As the errors x and y will not sensibly change during the continuance of an occultation, another similar equation may be obtained from an observation of the end, and a calculation for that time. Thus,

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* The signs of x and y, are both put affirmative. If either or both of them ought to be negative, it will be determined by the calculation.

$$a' . \cos^2 l . x + c'y = e'.$$

From these two equations the values of x and y are easily found.

89. The errors of the lunar tables, in longitude and latitude, may also be determined, by observing the moon's right ascension and declination, either at the place for which the tables are constructed, or at any other whose longitude and latitude are accurately known. From the observed right ascension and declination, the moon's longitude and latitude may be calculated (6.18.) and thence the errors ascertained.

90. In calculating the moon's longitude and latitude for any instant of time, as reckoned at a given place on a meridian, different from that for which the tables are constructed, we must reduce the given time to the time that is reckoned at the same instant at the latter place. This reduction depends on the difference of longitude of the two places, which for astronomical purposes is generally expressed in time; one mean solar hour corresponding to 15°. An error in the difference of longitudes will produce errors in the computed longitude and latitude of the moon.

91. An observation of an occultation, at a place whose longitude is not correctly known, furnishes one of the most accurate means of determining it. The accuracy will be increased if, on the day of occultation, the errors in the tables have been ascertained, by observations at a known meridian.

92. Supposing the tables accurate, or that the errors have been ascertained and allowed, the difference between the observed and calculated time of the beginning or end of an occultation, or of an eclipse of the sun, at any place whose latitude is accurately known, must depend on errors in the computed longitude and latitude of the moon, produced by an error in the longitude of the place.

93. A small error in the longitude of the place, or which is the same,

in the difference of time as reckoned at the two meridians, will very little affect the parallaxes in longitude and latitude, as is evident from the formulæ for computing these quantities. Consequently the errors produced in the apparent longitude and latitude will be sensibly the same as in the true.

94. Let m and n be the moon's hourly motions in longitude and latitude respectively, and $x =$ the error in the difference of meridians. Then mx and nx will be the errors in the moon's computed apparent longitude and latitude. Hence, a and c being, as before, the computed, apparent distances of the moon from the star in longitude and latitude, and s the moon's apparent semi-diameter, at the observed time of the beginning or end, we have,

$$(a + mx)^2 . \cos^2 l + (c + nx)^2 = s^2,$$

or, $a^2 \cos^2 l + 2\, amx \cos^2 l + m^2 x^2 \cos^2 l + c^2 + 2\, cnx + n^2 x^2 = s^2$

Now as the longitude of the place is supposed to be nearly known, x must be a small quantity, and the terms involving its square may be neglected. Hence we obtain,

$$x = \frac{s^2 - c^2 - a^2 \cos^2 l}{2\, am \cos^2 l + 2\, cn}.$$

In a similar manner we may determine the longitude of a place, from an observation of an eclipse of the sun.

95. Some astronomers think the apparent diameter of the sun, obtained from observation and given in the solar tables, is too great. They infer this from a comparison of the observed time of the beginning or end of a solar eclipse, at a known meridian, with the time obtained by calculation, after making allowance for the errors in the tables in other respects. To account for it, they suppose the apparent diameter of the sun is amplified by the very lively impression so luminous an object makes on the organ of sight. This amplification is called *Irradiation.* Duséjour thinks that, in the calculation of solar eclipses, the semi-diameter of the sun, as given by the tables, ought to be diminished by $3''\frac{1}{2}$. He also supposes the moon's atmosphere inflects the rays of light, so as to produce an effect on the beginning or end of a solar eclipse, or of an occultation, equal to a diminution of $2''$ in the semi-diameter of the moon. This is called the *Inflexion* of the moon.

The subject of irradiation and inflexion is involved in considerable uncertainty. Delambre considered the existence of either as very doubtful. Professor Bessel, in observing the transit* of Mercury in 1832, had for a prominent object the determination of the question of irradiation. From his observations, it appears that the disc of the sun, as seen through the telescope used by him, was not affected by irradiation. The observations of Professor Argelander, who observed with him, but with an instrument of much less power, indicated some irradiation; though of considerably less extent than was assumed by Duséjour.

CHAPTER XII.

OF THE PLANETS.

1. Hitherto our attention has been chiefly directed to the Earth, and to those two conspicuous luminaries, the Sun and Moon. We shall now take some notice of the bodies called Planets. These bodies, like the moon, are observed to be sometimes on one side of the ecliptic, and sometimes on the other. Their paths, therefore, cross the ecliptic. Their apparent motions are very irregular; sometimes *Direct*, that is from west to east, or according to the order of the signs; and sometimes *Retrograde*, or from east to west. There are also times, at which a planet appears to be *Stationary*, or to have but very little motion for several days.

2. The points in which the path of a planet cuts the plane of the ecliptic, are called the *Nodes*. The node through which the planet passes, from the south to the north side of the ecliptic, is called the *Ascending* node. The other is called the *Descending* node.

3. The *Geocentric* place of a body, is its place as seen from the earth. The *Heliocentric* place, is its place as it would be seen from the sun.

4. If a straight line be conceived to be drawn from the

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* The phenomenon called a transit will be noticed in the next chapter.

centre of a planet, perpendicular to the plane of the ecliptic, the distance from the point, in which it meets the ecliptic, to the centre of the sun, is called the *Curtate Distance* of the planet. The point is called the *Reduced Place* of the planet.

5. If the reduced place of a planet, the centre of the sun, and the centre of the earth, be joined by three straight lines, they will form a plane triangle, lying in the plane of the ecliptic. In this triangle, the angle at the centre of the earth is called the *Elongation;* the angle at the centre of the sun, the *Commutation;* and the angle at the reduced place of the planet, the *Annual Parallax.*

6. The sun, earth, moon, and planets, are frequently designated by characters, as follows:—

Sun	☉	Juno	⚵
Mercury	☿	Ceres	⚳
Venus	♀	Pallas	⚴
Earth	⊖ or ♁	Jupiter	♃
Moon	☾	Saturn	♄
Mars	♂	Uranus	♅
Vesta	⚶		

VENUS.

7. We commence with Venus, it being the most brilliant of the planets, and one whose phenomena are easily observed. This planet always accompanies the sun, being seen alternately on the east and west side, and never receding from it more than about 45°. When it is to the east of the sun, it is seen in the evening and is called the *Evening Star;* and when to the west, it is seen in the morning and is called the *Morning Star.*

The evening and morning star, called, by the ancient Greeks, *Hesperus* and *Phosphorus*, were at first thought to be different stars. The discovery that they are the same, is ascribed to Pythagoras.

8. When Venus is the evening star, and is at its greatest elongation from the sun, it appears through the telescope to have a semicircular disc, like the moon in quadratures,

with its convexity turned to the west.* From that time, as it approaches the sun, its splendour increases for a while, though the breadth of the illuminated disc, diminishes, like the moon in the wane. At the same time, the diameter, measured by the distance of the horns, increases.

9. Venus continues to approach the sun, till at length it becomes invisible, in consequence of the sun's superior light. After some time it appears on the west side, and is seen in the morning before the sun rises.

10. Though Venus is not, in general, visible at the time of its conjunction with the sun, it has sometimes been seen as a dark spot, passing over the sun's disc. This phenomenon is called a *Transit* of Venus. When Venus is thus seen on the disc of the sun, its apparent diameter is easily measured, and is found to be nearly one minute.

11. As Venus proceeds to the westward of the sun, its disc is seen as a crescent, continually increasing at the same time the diameter is diminishing. The convexity is then turned towards the east. When the planet is at its greatest western elongation, the disc is again a semicircle. From that time, as it again approaches the sun, the visible disc, like the moon after the first quarter, approaches nearer to a circle, and just previous to its being lost in the sun's rays, at the superior conjunction (10.12. Note), the disc does not sensibly differ from a complete circle. Its diameter is then only about 10 seconds.

12. From the superior conjunction the diameter of Venus increases, but the apparent disc changes from a full orb, till, at the greatest eastern elongation, it again becomes a semicircle. The period, that circumscribes all these changes, is the same as the time from one conjunction to another of the same kind. This period, which is the synodic revolution (10.12) of Venus, is, at a mean, about 584 days.

13. The different phases of Venus are readily accounted

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* In observing Venus with a telescope, it is better to have the object end partly covered in order to diminish the light.

for, by supposing it to be an opaque, spherical body, revolving round the sun from west to east, at a distance less than that of the earth, and shining by reflecting the sun's light.

14. Venus never deviates more than a few degrees from the plane of the ecliptic. By supposing it to move in a circular orbit in the plane of the ecliptic, we can easily obtain an approximation to its distance from the sun, in parts of the earth's distance. Calling the earth's distance from the sun 1, let x be the distance of Venus from the sun. Then its distance from the earth at inferior conjunction will be $1 - x$; and at superior, $1 + x$. Hence, (7.13), $1 + x : 1 - x :: 60'' : 10'' :: 6 : 1$, and consequently $7x = 5$, or $x = .714$.

15. Since the visible disc of Venus when at its greatest elongation, either eastern or western, is a semicircle, it is evident that the annual parallax (5) of the planet is then a right angle. Hence taking 45° as the greatest elongation, the distance of Venus from the sun, found by trigonometry, is .707. As the distance of Venus from the sun, in different posititions is determined to be nearly the same, it appears the orbit is nearly a circle.

16. From the synodic revolution (12), the periodic revolution may be determined. For, from the time Venus is in conjunction with the sun, till it is again in conjunction of the same kind, its angular motion about the sun must exceed the earth's by a complete revolution, or 360°. The periodic revolution of Venus is 224 d. 17 h. nearly.

If $t = 584 =$ the synodic revolution in days, $m = 59'\ 8'' =$ the earth's motion in one day, $x =$ the diurnal motion of Venus, and $p =$ the periodic time, we have,

$$tx = 360^\circ + tm,$$

$$\text{or, } x = m + \frac{360^\circ}{t} = 1^\circ\ 36',$$

$$\text{and } p = \frac{360^\circ}{x} = \frac{360^\circ \times t}{mt + 360^\circ} = 224 \text{ d. } 17 \text{ h. nearly.}$$

Another method of finding the periodic time will be given in a succeeding article.

17. A while after the greatest eastern elongation, Venus becomes nearly stationary with respect to the fixed stars, having for a short time no sensible motion in longitude. After that its motion becomes retrograde, and continues so till near the greatest western elongation, when being again a short time stationary, it afterwards becomes direct. The motion then continues direct through the remaining part of the synodic revolution. These are necessary consequences of the respective distances of the earth and Venus from the sun and of their respective motions in their orbits.

As it will simplify the explanation and will produce no material error, we may still suppose the orbit of Venus to coincide with the plane of the ecliptic. Let S, *Fig.* 37, be the sun, BDP*p* the orbit of Venus, and E*e* a part of the earth's orbit. Also, let E and P be the positions of the earth and Venus at the time of inferior conjunction, and *e* and *p*, their places one day after. Draw *e*C parallel to ES. Now in the triangle *e*S*p*, we know *e*S = 1, S*p* = .71 nearly, *e*S*p* = *p*SP — *e*SE = 1° 36′ — 59′ = 37′. Whence the angle S*ep* is found = 1° 31′. But the angle S*e*C = ES*e* = 59′. Since, therefore, the angle S*ep* is greater than the angle S*e*C, the position of *p* is to the right hand or west of *e*C, and consequently the apparent motion of Venus is retrograde, in this part of the orbit. It is evident, without computation, that the motion of Venus must be direct, in the part of the orbit opposite to the earth. It is also plain that when the motion is changing from direct to retrograde or the contrary, the planet must appear stationary for a time.

18. Admitting the truth of Kepler's third law, (7.30), and supposing the planets and earth to move in circular orbits in the plane of the ecliptic, it may be proved by analytical investigation, that the apparent motion of each planet must be retrograde in the part of the orbit next the earth, and direct in the opposite part. We may also, by such investigations, determine the *Stationary* points, that is, the points at which the planet appears stationary; and

likewise the times during which the motions of each planet must appear retrograde. Now the planets do not move in the plane of the ecliptic, nor in circular orbits; but none of them, or at least none except Pallas, deviate so far in either respect, as much to affect the results obtained on those suppositions.

The computed durations of the retrograde motions of the planets are found to agree very nearly with the durations obtained by observation. This near agreement forms a strong proof in favour of the earth's motion and of the Copernican System.

19. When Venus is in or near to its superior conjunction, nearly the whole of its enlightened disc is turned towards the earth. On account, however, of its greater distance, when thus situated, the light received from it at the earth is then by no means so great as in some other positions. The position which Venus must have when it yields the greatest quantity of light to the earth is easily determined by an ivestigation in differential calculus. This is found to take place when the planet is in the inferior part of its orbit, and has an elongation of about 39° 43′; which occurs about 36 days before and after inferior conjunction. At these times and for several preceding and following days, Venus is a very conspicuous object; its brilliancy being such as to render it *distinctly visible with the naked eye, during the day time.* This is especially the case when being in the position mentioned, it has at the same time considerable north declination; as it then acquires a greater elevation above the horizon, and consequently its light is less dissipated in traversing the atmosphere, than when farther south.

POSITION OF THE NODES.

20. When a planet is at either of its nodes, it is in the plane of the ecliptic, and consequently its latitude is nothing. Now from the observed right ascension and declination of a planet at any time, its geocentric longitude

and latitude may be calculated (6.19). If several longitudes and latitudes be thus obtained, about the time the planet is passing from one side of the ecliptic to the other, the exact time at which its latitude is nothing, may be obtained by proportion, and also its longitude at that time. This longitude of the planet will evidently be the geocentric longitude of the node. If similar observations and calculations be made when the planet returns to the same node, we shall again have the geocentric longitude of the node, which on account of the different position of the earth in its orbit, will be different from the former. From these two longitudes, supposing the node to have no motion, its heliocentric longitude may be determined.

Let S, *Fig*. 38, be the sun, N the node, E, the place of the earth at the time the planet was found to be in the node from the first set of observations, and E′ its place at the second time. Also let EQ, E′Q and SQ, all parallel to each other, represent the direction of the vernal equinox. Put $V = SE =$ radius vector of the earth at the first time, the mean radius vector being $= 1$, $S = QES =$ sun's longitude, $G = QEN =$ the geocentric longitude of the node, and V', S' and G' the same quantities for the second time. Then if $v = SN =$ radius vector of the planet when at the node, and $N = QSN =$ the heliocentric longitude of the node, we have,

$$SEN = QES - QEN = S - G,$$

$$SNE = QAN - QSN = QEN - QSN = G - N,$$

$$\sin SNE : \sin SEN :: SE : SN,$$

$$\sin (G - N) : \sin (S - G) :: V : v,$$

$$V. \sin (S - G) = v. \sin (G - N).$$

In like manner, $V'. \sin (S' - G') = v. \sin (G' - N)$.

Therefore,

$$\frac{V \sin (S - G)}{V'. \sin (S' - G')} = \frac{\sin (G - N)}{\sin (G' - N)} = \frac{\sin G \cos N - \cos G \sin N}{\sin G' \cos N - \cos G' \sin N}$$

$$= \frac{\sin G - \cos G \tan N}{\sin G' - \cos G' \tan N}$$

Hence,

$$\tan N = \frac{V. \sin (S - G) \sin G' - V'. \sin (S' - G') \sin G}{V \sin (S - G) \cos G' - V'. \sin (S' - G') \cos G}.$$

We have also $v = \dfrac{V. \sin (S - G)}{\sin (G - N)}$.

21. From observations made at distant periods, it is found that the nodes of the planets have *slow retrograde motions.*

22. The heliocentric longitude of either node being determined from observations on the planet, when in that node, and the motion of the node being also ascertained, its heliocentric longitude may be found for any given time. When the heliocentric longitudes of the two nodes of a planet are thus determined for the same time, they are found to differ 180°. Hence it follows, that the line of the nodes, and consequently the plane of the orbit, passes through the centre of the sun.

INCLINATION OF THE ORBIT.

23. The place of the node of a planet being known, the inclination of the orbit may be easily determined.

To do this, find the geocentric longitude and latitude of the planet, at the time the longitude of the sun is the same with that of the node. Let ENp, *Fig.* 39, be the plane of the ecliptic, E, the earth, S, the sun, N, the node, P, the planet, Pp, perpendicular to the ecliptic, and pD, perpendicular to EN, the line of the nodes.' Then PDp is the inclination of the orbit. Put $E = p\text{EN}$ = the difference between the geocentric longitude of the planet and the longitude of the sun, λ = the geocentric latitude of the planet, and $I = \text{PD}p$ = the inclination of the orbit. Then,

$$\text{P}p = \text{E}p.\tan\lambda,\ \text{and}\ \text{D}p = \text{E}p.\sin E.$$

$$\text{Hence, } \tan I = \frac{\text{P}p}{\text{D}p} = \frac{\tan\lambda}{\sin E}.$$

PERIODIC TIME.

24. The interval from the time the planet is in one of the nodes, till its return to the same, making allowance for the motion of the node, gives the sidereal revolution of the planet. The sidereal revolution of Venus is 224 d. 16 h. 49 m. 8 sec.

HELIOCENTRIC LONGITUE AND LATITUDE, AND RADIUS VECTOR.

25. The place of the node and the inclination of the orbit being known, we may deduce the heliocentric longitude and latitude of a planet, from the geocentric longitude and latitude obtained from observation.

Let $pESN$, *Fig.* 40, be the plane of the ecliptic, E, the earth, S, the sun, P, the planet, N the node, Pp perpendicular to the ecliptic, pD perpendicular to SN, and EQ and SQ the direction of the vernal equinox.

Put N = the heliocentric longitude of the node, S = the sun's longitude, E = the earth's longitude = S + 180°, G = the geocentric longitude of the planet, λ = the geocentric latitude, L = the heliocentric longitude, l = heliocentric latitude, I = the inclination of the orbit, V = the radius vector of the earth, and v = the radius vector of the planet. Then,

$NSp = L - N$, $SEp = G - S$, $ESp = E - L$, $EpS = 180° - SEp - ESp = 180° - G + S - E + L = 180 - G + S - 180° - S + L = L - G$.

$$\text{Now, } Ep.\ \tan \lambda = Pp = Sp.\ \tan.\ l,$$

$$\frac{\tan.\ \lambda}{\tan.\ l} = \frac{Sp}{Ep} = \frac{\sin SEp}{\sin ESp} = \frac{\sin (G - S)}{\sin (E - L)},$$

$$\tan.\ \lambda \sin (E - L) = \tan.\ l \sin (G - S) \qquad (A).$$

Also,

$$Dp = Sp.\ \sin (L - N),$$

$$Sp.\ \tan.\ l = Pp = Dp.\ \tan.\ I = Sp.\ \sin (L - N) \tan.\ I,$$

$$\tan.\ l = \sin (L - N) \tan I.$$

Hence,

$$\tan.\ \lambda \sin (E - L) = \sin (L - N) \sin (G - S).\ \tan.\ I,$$

or, $\tan.\ \lambda \sin [(E - N) - (L - N)] = \sin (L - N) \sin (G - S) \tan I$,

$\tan \lambda \sin (E - N) \cos (L - N) - \tan \lambda \cos (E - N) \sin (L - N) = \sin (L - N) \sin (G - S) \tan.\ I$,

$\tan \lambda \sin (E - N) - \tan \lambda \cos (E - N) \tan (L - N) = \tan (L - N) \sin (G - S) \tan.\ I$,

$$\tan (L - N) = \frac{\tan \lambda \sin (E - N)}{\tan \lambda \cos (E - N) + \sin (G - S) \tan I}.$$

Hence, $L = (L - N) + N$, becomes known.

We have also (A),

$$\tan.\ l = \frac{\tan \lambda \sin (E - L)}{\sin (G - S)}.$$

From the triangle EpS, we have,

$$Sp = \frac{ES \sin SEp}{\sin EpS} = \frac{V \sin (G - S)}{\sin (L - G)}.$$

$$\text{Hence, } v = \frac{Sp}{\cos PSp} = \frac{V \sin (G - S)}{\cos.\ l.\ \sin (L - G)}.$$

26. The sum of the longitude of the node, and of the angle contained in the order of the signs, between the right line, joining the sun and node, and the radius vector of the planet, is called the *Orbit Longitude* of the planet.

Thus the orbit longitude of the planet at P, *Fig.* 40, is QSN + NSP.

If L′ = the orbit longitude, then,

$$Dp = SD.\tan(L - N), \text{ and } SD.\tan(L' - N) = DP = \frac{Dp}{\cos I}$$

$$= \frac{SD.\tan(L - N)}{\cos I}.$$

$$\text{Hence, } \tan(L' - N) = \frac{\tan(L - N)}{\cos. I}.$$

LONGITUDE OF THE PERIHELION, ECCENTRICITY, AND SEMI-TRANSVERSE AXIS.

27. Assuming the orbit of the planet to be an ellipse, we may, from the heliocentric, orbit longitude, and the radius vector, found for three different times, determine the longitude of the perihelion, the eccentricity, and the semi-transverse axis.

Let APD, *Fig.* 41, be the orbit, A the perihelion, and P, P′ and P″, the three positions of the planet. Then SP, SP′ and SP″, the three radius vectors are known, and also from the longitudes, the angles PSP′ and PSP″.

Put $v = SP$, $v' = SP'$, $v'' = SP''$, $x = ASP$, $\theta = PSP'$, $\varphi = PSP''$, a = semi-transverse axis, and ae = the eccentricicity. Then, (Conic Sections*)

$$v = \frac{a.(1 - e^2)}{1 + e.\cos. x} \quad \text{- - - - - -} \quad B$$

$$v' = \frac{a.(1 - e^2)}{1 + e.\cos(x + \theta)} \quad \text{- - - - - -} \quad C$$

$$v'' = \frac{a.(1 - e^2)}{1 + e.\cos(x + \varphi)} \quad \text{- - - - -} \quad D$$

From B and C,

$$v + v\,e.\cos. x = v' + v'\,e.\cos(x + \theta)$$

$$\text{or, } e = \frac{v' - v}{v.\cos. x - v'.\cos(x + \theta)} \quad \text{- -} \quad E$$

In like manner from B and D,

$$e = \frac{v'' - v}{v.\cos x - v''.\cos(x + \varphi)} \quad \text{- -} \quad F$$

Put $v' - v = m$, and $v'' - v = n$, then from E and F, we have,

$$\frac{m}{n} = \frac{v.\cos x - v'.\cos.(x + \theta)}{v.\cos. x - v''.\cos.(x + \varphi)}$$

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* See Appendix, Article 51.

$$= \frac{v.\ \cos.\ x - v'.\ \cos.\ \theta \cos.\ x - v' \sin.\ \theta \sin.\ x}{v.\ \cos.\ x - v''.\ \cos.\ \varphi \cos.\ x - v''.\ \sin \varphi \sin.\ x}$$

$$= \frac{v - v'.\ \cos.\ \theta - v'.\ \sin.\ \theta \tan.\ x}{v - v''.\ \cos.\ \varphi - v''.\ \sin.\ \varphi \tan.\ x}.$$

$$\text{Hence, } \tan.\ x = \frac{n\,(v - v'.\ \cos.\ \theta) - m.\ (v - v''.\ \cos.\ \varphi)}{n v'.\ \sin.\ \theta - m v''.\ \sin.\ \varphi}.$$

The value of x being determined, if it be subtracted from the orbit longitude of the planet in the first position, the remainder will be the orbit longitude of the perihelion. If L be the ecliptic longitude, and L′ the orbit longitude of the perihelion, we have, (26),

$$\tan\,(\text{L} - \text{N}) = \tan\,(\text{L}' - \text{N}) \cos \text{I}.$$

The value of e, the ratio of the eccentricity to the semi-transverse axis may be found from either of the expressions, E and F; and a, the semi-axis, from either B, C, or D.

28. When the longitude of the perihelion, the eccentricity, and the semi-transverse axis, of the orbit of any planet, are determined from several sets of observations, not very remote from each other, they are found, respectively, to be very nearly the same. Hence it appears, the assumption, that the orbit is an ellipse, is true, or at least nearly so.

29. From observations made on the different planets, at remote periods, it is found that the perihelions have slow motions. The motion of the perihelion of Venus is retrograde. Those of the other planets are direct.

The eccentricities of the orbits are also subject to continued, but very minute changes. Some of them are at present increasing, others diminishing.

The semi-transverse axes of the orbits do not change. This was first discovered by La Grange, from investigations in Physical Astronomy, and it is found to be conformable to observation.

EPOCH OF A PLANET BEING AT THE PERIHELION OF ITS ORBIT.

30. From several observations of the planet about the time it has the same longitude as the perihelion, the correct time of its being at the perihelion, may be easily determined by proportion.

ELEMENTS OF THE ORBIT OF A PLANET.

31. The longitude of the ascending node of the orbit, the inclination of the plane of the orbit to the ecliptic, the mean motion of the planet round the sun, the mean distance of the planet from the sun, or which is the same, the semi-transverse axis of its orbit, the eccentricity, the longitude of the perihelion, and the time when the planet is in the perihelion, are called the *Elements of the Orbit.*

32. There are various other methods for determining the elements of the orbit, besides those which have been given in the preceding articles. Those which are founded on observations of the planet, when in conjunction, opposition, and in the nodes, are among the most accurate.

The elements of the orbit, may also be determined with tolerable accuracy, by certain methods of estimation and computation, without extending the observations to the time of the planet's passage through the node. These methods were applied on the discovery of the new planets.

33. When the elements of the planet's orbit have been accurately determined, from a great number of observations, the equation of the centre may be calculated, and tables may be formed, which will give the heliocentric longitude and latitude, and the radius vector for any given time. But the most of the planets are sensibly affected by the mutual attractions among one another. The perturbations thus produced, have been calculated for several of the planets, and form a part of complete tables of the planets.*

GEOCENTRIC LONGITUDE AND LATITUDE.

34. *From the heliocentric longitude and latitude of a planet, as obtained from the tables, to find the geocentric longitude and latitude.*

Put p = EpS = annual parallax, *Fig.* 40, E = SEp = the elongation,

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* The best tables of the planets are those by *Lindenau*, for Mercury, Venus and Mars, with the explanations in Latin; and those by *Bouvard*, for Jupiter, Saturn and Uranus, with the explanations in French.

$S = ESp$ = the commutation, $l = PSp$ = the heliocentric latitude, $\lambda = PEp$ = the geocentric latitude, $V = ES$ = radius vector of the earth, and $v = SP$ = radius vector of the planet. Then $Sp = v. \cos. l$.

Now, by trigonometry,

$$ES + Sp : ES - Sp :: \tan \tfrac{1}{2} (EpS + SEp) : \tan \tfrac{1}{2} (EpS - SEp),$$

$$\text{or, } V + v. \cos. l : V - v. \cos. l :: \tan \tfrac{1}{2} (p + E) : \tan \tfrac{1}{2} (p - E).$$

$$\text{Hence, } \tan \tfrac{1}{2} (p - E) = \frac{V - v. \cos. l}{V + v. \cos. l}. \tan \tfrac{1}{2} (p + E)$$

$$= \frac{V - v. \cos. l}{V + v. \cos. l}. \tan \tfrac{1}{2} (180° - S)$$

$$= \frac{1 - \dfrac{v. \cos. l}{V}}{1 + \dfrac{v. \cos. l}{V}}. \tan (90° - \tfrac{1}{2} S).$$

Put, $\tan. \theta = \dfrac{v. \cos. l}{V}$. Then (App. 15),

$$\tan \tfrac{1}{2} (p - E) = \frac{1 - \tan. \theta}{1 + \tan. \theta}. \tan. (90 - \tfrac{1}{2} S)$$

$$= \tan (45° - \theta). \tan (90° - \tfrac{1}{2} S).$$

And $E = \tfrac{1}{2} (p + E) - \tfrac{1}{2} (p - E) = 90° - \tfrac{1}{2} S - \tfrac{1}{2} (p - E)$.

If $\odot$ = longitude of the sun, then

Geocen. long. of the Planet $= \odot + E = \odot + 90 - \tfrac{1}{2} S - \tfrac{1}{2} (p - E)$.

For the geocentric latitude, we have (25. A),

$$\tan. \lambda = \frac{\sin. E. \tan. l}{\sin S}.$$

35. When a planet is in conjunction or opposition, the angles of elongation and commutation, and consequently their sines are each nothing. In these cases the geocentric latitude cannot be found by the preceding formula. It may however be easily determined in a different manner. Let E, *Fig.* 42, be the earth, S the sun, and P the planet, in inferior conjunction. Then,

$$Sp = v. \cos. l, \text{ and } Pp = v. \sin. l.$$

$$\text{Also } Pp = Ep. \tan \lambda = (V - v. \cos. l). \tan. \lambda.$$

$$\text{Hence, } (V - v. \cos. l). \tan \lambda = v. \sin. l,$$

$$\text{or } \tan \lambda = \frac{v. \sin l}{V - v. \cos. l}.$$

36. From the triangles SpP and EpP, *Fig.* 40, we easily obtain the distance of the planet from the earth. Thus,

$$EP. \sin. \lambda = Pp = v. \sin. l.$$

$$\text{or } EP = \frac{v. \sin. l}{\sin \lambda}.$$

37. Let ϖ = sun's mean horizontal parallax, that is the parallax when the earth is at its mean distance from the sun, designated by a unit or 1,

$p =$ the horizontal parallax of the planet, in a given position, and $R =$ the radius of the earth. Then (5.7),

$$1.\ \varpi = R = EP.\, p,$$

$$\text{or, } p = \frac{\varpi}{EP} = \frac{\varpi.\ \sin \lambda}{v.\ \sin\ l}.$$

The parallax of Venus when in inferior conjunction is about 30″.

38. It has been found from observations, that the apparent semi-diameter of Venus, when at a distance from the earth, equal to the earth's mean distance from the sun, is 8″.25. Hence if $\delta =$ the apparent semi-diameter of Venus, when at a given point P, we have (7.13),

$$\delta = \frac{8''.25}{EP} = \frac{8''.25\ \sin.\ \lambda}{v.\ \sin.\ l}.$$

TRANSIT OF VENUS.

39. When Venus is in inferior conjunction, if it is so near either node, that its geocentric latitude is less than the apparent semi-diameter of the sun, it must pass between the earth and sun, and produce the phenomenon, called a transit (10). This phenomenon is of great importance on account of its use in determining the parallax of the sun, and thence the real distances and magnitudes of the bodies in the solar system. Its use for this purpose was first made known by Dr. Halley.

40. A synodic revolution of Venus, being about 584 days, 5 synodic revolutions will be 2920 days, or 8 years nearly. It follows, therefore, that 8 years after Venus has been in inferior conjunction, it is again in inferior conjunction, nearly on the same day in the year, and consequently nearly in the same part of the heavens. Hence if it was near the node, at the former conjunction, it is also near the same node at the latter. If there was a transit in the first instance, the planet may perhaps be near enough to the node, in the second, to occasion another transit. A more particular calculation shows that whenever there is one transit of Venus, there must, generally, be another at the same node, 8 years before or after. At the end of a second period of 8 years, the planet is too far from the node, at conjunction, for a transit to take place. When there have been two transits at one node, the next two

take place at the other node, but not till more than a century after the former.

The last transits of Venus were in the years 1761 and 1769. The next transit will occur on the 8th of December, 1874; but it will not be visible in the United States. On the 6th of December, 1882, there will be another, the beginning and end of which will both be visible in this country.

41. The time of inferior conjunction of Venus, may be found from tables of the sun and Venus. From the geocentric longitude and latitude of Venus, calculated for the time of conjunction, and also for a time, an hour before or after conjunction, the geocentric hourly motions of Venus in longitude and latitude will be known. The geocentric hourly motion in longitude, being retrograde must be added to the sun's hourly motion, to obtain the relative hourly motion, which will also be retrograde. From the hourly motion in latitude, and the relative hourly motion in longitude, the inclination of the relative orbit may be found in the same manner as the inclination of the moon's relative orbit, in eclipses of the moon. The apparent semi-diameter of Venus may be found by the formula in a preceding article (38).

42. Let LMNQ, *Fig.* 43, represent the disc of the sun, C its centre, AB a part of the ecliptic, the order of the signs being from A to B, CL a circle of latitude, and *vw* the relative orbit of Venus. The apparent motion of Venus being retrograde, will be in the direction from *v* to *w*.

The position of the relative orbit is adapted to the transit of 1769, at which time the latitude of Venus was north, decreasing.

With the centre C and a radius equal to the sum of the semi-diameters of the sun and Venus, let arcs be described, cutting *vw*, in *v* and *w*, and with a radius equal to the difference of the semi-diameters, let other arcs be described, cutting *vw* in *a* and *e*. The situations of Venus with respect to the sun, when at the points *v*, *a*, *e*, and *w*, are respectively called the *First External Contact, First Internal Contact, Second Internal Contact, and Second External Contact.*

43. The times of the different contacts, as seen from the centre of the earth, may be calculated in the same manner as the beginning and end of an eclipse of the moon, using the geocentric latitude of Venus, instead of the moon's latitude, the sum or difference of the semi-diameters of the sun and Venus, instead of the sum of those of the earth's shadow and moon, the geocentric hourly motion of Venus in latitude, instead of the moon's hourly motion in latitude, and the relative hourly motion of Venus in longitude, instead of that of the moon.

PARALLAXES OF THE SUN AND VENUS.

44. Let E, *Fig.* 44, be the earth, V, Venus, and *aa'b'b*, the disc of the sun. Also let AGB be the path of Venus over the sun's disc as seen from E, the earth's centre, the latitude of Venus being north. Then to a spectator at I, towards the north pole, the path will be *a*H*b*, which being nearer the centre is greater than the former. Consequently the duration of the transit as seen from I, is longer than as seen from the centre of the earth. To a spectator at K, towards the south pole, the path will be *a'*F*b'*, which is less than AB. Hence the duration is shorter than as seen from the earth's centre. The difference between the durations as seen from I and K, depends on the parallaxes of Venus and the sun, or rather on the difference between the parallax of Venus and that of the sun. This difference is called the *Relative Parallax* of Venus.

From the observed durations of the transit at the two places, the relative parallax, and thence the parallax of the sun, may be determined.

45. Let a', *Fig.* 43, be the place of the centre of Venus, at the 1st internal contact as seen from a place A in north latitude, where the commencement is accelerated, and end retarded by the effects of parallax, and let aS and $a'R$ be each perpendicular to AB. Put,

T = time of 1st internal contact for the earth's centre,
θ = the same for the place A,
$t = T - \theta$,
$E = CS$ = the elongation of Venus at the time T,
$G = aS$ = the geocentric latitude at the same time,
m = relative hourly motion of Venus in longitude,
r = hourly motion of Venus in latitude,
P = relative parallax of Venus,
Π = relative parallax in longitude at the time θ,
ϖ = relative parallax in latitude at the same time.

$$\text{Then } CR = E + mt + \Pi^{*}, \text{ and } a'R = G + rt + \varpi^{*},$$
$$CR^2 + a'R^2 = a'C^2 = aC^2 = CS^2 + aS^2,$$

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[*] The unknown quantities Π and ϖ, are applied with affirmative signs, although one of them at least, in the present case, must be negative. This produces no error in the investigation. In the numerical computation, these quantities must be used with the signs, which they will be determined to have in particular cases.

or, $(E + mt + \Pi)^2 + (G + rt + \varpi)^2 = E^2 + G^2$, $2Emt + 2E\Pi + 2mt\Pi + m^2t^2 + \Pi^2 + 2Grt + 2G\varpi + 2rt\varpi + r^2t^2 + \varpi^2 = 0$, $(m^2 + r^2).\,t^2 + 2.\,(Em + m\Pi + Gr + r\varpi).\,t = -2.\,(E\Pi + G\varpi + \frac{1}{2}\Pi^2 + \frac{1}{2}\varpi^2)$

$$t^2 + \frac{2.\,(Em + m\Pi + Gr + r\varpi)}{m^2 + r^2}.\,t$$

$$= -\frac{2.\,(E\Pi + G\varpi + \frac{1}{2}\Pi^2 + \frac{1}{2}\varpi^2)}{m^2 + r^2}.$$

If $a = \dfrac{Em + m\Pi + Gr + r\varpi}{m^2 + r^2}$

and $b = \dfrac{2.\,(E\Pi + G\varpi + \frac{1}{2}\Pi^2 + \frac{1}{2}\varpi^2)}{m^2 + r^2}$, we have,

$$t^2 + 2at = -b$$

$$t^2 + 2at + a^2 = a^2 - b$$

$$t + a = \sqrt{(a^2 - b)} = a - \frac{b}{2a} - \frac{b^2}{8a^3} - \&c.$$

$$t = -\frac{b}{2a} - \frac{b^2}{8a^3} - \&c.$$

But $\frac{b^2}{8a^3}$, and all the following terms of the value of t, are extremely small and may be neglected. Hence,

$$t = -\frac{b}{2a} = -\frac{E\Pi + G\varpi + \frac{1}{2}\Pi^2 + \frac{1}{2}\varpi^2}{Em + m\Pi + Gr + r\varpi}$$

$$= -\frac{E\Pi + G\varpi + \frac{1}{2}\Pi^2 + \frac{1}{2}\varpi^2}{(E + \Pi).\,m + (G + \varpi).\,r}.$$

Put $\Pi = uP$, and $\varpi = vP$, then,

$$t = -\frac{Eu + Gv + \frac{1}{2}u\Pi + \frac{1}{2}v\varpi}{(E + \Pi).\,m + (G + \varpi).r}.\,P, \text{ and,}$$

$$T = \theta + t = \theta - \frac{Eu + Gv + \frac{1}{2}u\Pi + \frac{1}{2}v\varpi}{(E + \Pi).\,m + (G + \varpi).\,r}.\,P.$$

46. Let T′ be the time of the second internal contact for the earth's centre, and θ', for the place A. And instead of E, G, m, r, Π, ϖ, u and v, let E′, G′, m', r', Π', ϖ', u', and v' be the values of the corresponding quantities at the 2d internal contact. As the latitude is decreasing and the 2d internal contact for the place A is later than for the earth's centre, the value of r must be taken, negative, in obtaining the latitude at the latter time from that at the former. Attending to these changes, we have in like manner as before,

$$T' = \theta + \frac{E'u' + G'v' + \frac{1}{2}u'\Pi' + \frac{1}{2}v'\varpi'}{(E' + \Pi').\,m' - (G' + \varpi').\,r'}.\,P.$$

Put $\beta = \dfrac{Eu + Gv + \frac{1}{2}u\Pi + \frac{1}{2}v\varpi}{(E + \Pi).\,m + (G + \varpi).\,r}$ and

$$\beta' = \frac{E'u' + G'v' + \frac{1}{2}u'\Pi' + \frac{1}{2}v'\pi'}{(E' + \Pi').\, m' - (G' + \pi').\, r'}.$$

Then $T = \theta - \beta P$, and $T' = \theta' + \beta' P$,

$$T' - T = \theta' - \theta + (\beta + \beta').\, P.$$

If now $d = \theta' - \theta$, and $s = \beta + \beta'$, we have,

$$T' - T = d + sP.$$

47. If, for some other place in a latitude considerably different from that of A, d' and s' be the values of the expressions designated by d and s, we shall have in like manner,

$$T' - T = d' + s'P.$$

$$\text{Hence, } d + sP = d' + s'P,$$

$$sP - s'P = d' - d,$$

$$P = \frac{d' - d}{s - s'}.$$

48. In this expression for the value of P, the two quantities d and d' are known from observation. The quantities s and s' depend on the values of β and β' for the two different places of observation. In the expression represented by β, the values of E, G, m and r, are known. To obtain u and v, let L = the geocentric longitude of Venus, and Δ = its distance from the north pole of the ecliptic. Then (10.54. C. and 56. G),

$$\sin \Pi = \frac{\sin P \sin.\, h \sin (L - n + \Pi)}{\sin.\, \Delta}, \text{ and,}$$

$$\sin \pi = \sin P \cos h \sin (\Delta + \pi)$$

$$- \frac{\sin P \sin h \cos (L - n + \frac{1}{2}\Pi) \cos (\Delta + \pi)}{\cos \frac{1}{2}\Pi}.$$

Or because Π and π, are very small,

$$\sin \Pi = \frac{\sin P \sin h \sin L - n)}{\sin \Delta}, \qquad \text{(Q)}$$

$$\sin \pi = \sin P \left(\cos h \sin \Delta - \sin h \cos \Delta \cos (L - n)\right) \qquad \text{(R)}$$

$$\text{Hence (45), } u = \frac{\Pi}{P} = \frac{\sin \Pi}{\sin P} = \frac{\sin h \sin (L - n)}{\sin \Delta},$$

$$v = \frac{\pi}{P} = \frac{\sin \pi}{\sin P} = \cos h \sin \Delta - \sin h \cos \Delta \cos (L - n).$$

The other two quantities Π and π, contained in the expression represented by β, depend on P, the quantity sought. But as Π and π are very small, it is evident from examination of the expression, that we obtain a near value of β, if the terms in which they enter, are omitted. Hence by taking,

$$\beta = \frac{Eu + Gv}{Em + Gr}, \text{ and } \beta' = \frac{E'u' + G'v'}{E'm' - G'r'},$$

we obtain a very near approximate value of P. Then, taking $\Pi = uP$, $\varpi = vP$, $\Pi' = u'P'$, and $\varpi' = v'P$, and again calculating the values of β and β', we obtain the correct value of P.

49. If ϖ' = the sun's horizontal parallax at the time of the transit, ϖ, being the mean parallax, and V the radius vector, we have $\varpi' = \frac{\varpi}{V}$.

At the time of a transit, the latitude of Venus is so small that we may consider its sine as equal to its tangent, and its cosine = 1. Hence (35),

$$\sin \lambda = \frac{v.\sin.\, l}{V - v}, \text{ and } (37),$$

$$p = \frac{\varpi \sin \lambda}{v.\, \sin l} = \frac{\varpi}{V - v}.$$

$$\text{Therefore } P = p - \varpi' = \frac{\varpi}{V - v} - \frac{\varpi}{V} = \frac{\varpi v}{V.\,(V - v)}.$$

$$\text{Hence, } \varpi = P.\frac{V}{v}\,(V - v).$$

50. The transit of 1769 was observed at Wardhus, a small island on the north coast of Europe, and at Otaheite in the South Sea, and the duration was found to be longer at the former place, than at the latter, by 23 m. 10 sec. The sun's mean horizontal parallax, determined from the observations made at those places is 8″.7. From observations made at other places, results a little different were obtained. From the observations of the two transits in 1761 and 1769, Professor Bessel finds for the equatorial horizontal parallax, at the earth's mean distance, 8″.5776 or 8″.6 nearly. Taking the mean parallax 8″.6, its greatest value will be 8″.7 and its least 8″.5.

51. Taking the sun's parallax 8″.6, the earth's mean distance from the sun is 23984 semi-diameters of the earth, (5.8), or 95,000,000 English miles, nearly. Thence from the sidereal revolutions of the earth and Venus, the mean distance of Venus from the sun, found by Kepler's third law (7.30), is 69 millions of miles. From the observed diameter of Venus, when at a known distance from the earth, its real diameter is easily found. It is about 7600 miles.

62. It is found that Venus revolves on its axis, from west to east, in 23 h. 21 m., and that its axis is inclined to the ecliptic in an angle of about 15°.

MERCURY.

53. Mercury, like Venus, always accompanies the sun. Its greatest elongation is about 23°. The phenomena of Mercury correspond in almost every part with those of Venus, only that it is farther from the earth and nearer to the sun, and consequently more difficult to be observed. Its greatest and least apparent diameters are 11″.2 and 5″. It can only be seen by the naked eye when in favourable positions.

Mercury makes a sidereal revolution round the sun, in about 88 days, at a mean distance of 37 millions of miles. Its diameter is little more than 3,000 miles.

On account of the proximity of Mercury to the sun, it is difficult to determine whether it revolves on its axis. Shroeter thought he ascertained that it made a revolution, like the earth and Venus, from west to east, in 24 h. 5 m.; and that its axis makes but a small angle with the ecliptic.

54. When at the time of inferior conjunction, Mercury is in either node, or very near to it, a transit of Mercury takes place. Transits of Mercury occur much more frequently than those of Venus. Those that will take place between the present time, 1836, and the end of the century, will be on the 8th of May, 1845, the 9th of November, 1848, the 11th of November, 1861, the 4th of November, 1868, the 6th of May, 1878, the 7th November, 1881, the 9th of May, 1891, and the 10th of November, 1894. Of these, the first, second, fifth, and last, will be visible in this country.

The calculation of a transit of Mercury is altogether similar to one of Venus.

55. Mercury and Venus are called *Inferior* planets, because their orbits are included within the earth's. The others are called *Superior*, because their orbits are without the earth's.

MARS.

56. Mars and all the other superior planets, differ from Mercury and Venus, in being seen in opposition as well as

in conjunction. The disc of Mars does not present the phases of the two inferior planets, but it is observed, in particular situations, to deviate very sensibly from a circle. The apparent diameter of Mars undergoes considerable change. When greatest it is 17″, and when least, 3″.5.

The sidereal revolution of Mars is nearly 687 days, and its mean distance from the sun is 144 millions of miles. Its diameter is about 4000 miles.

Mars revolves on its axis from west to east in 24 h. 39 m., and its axis is inclined to the ecliptic in an angle of 59° 42′. Its polar diameter is less than the equatorial. According to the measures of Arago, these diameters are to each other in the ratio of 189 to 194.

JUPITER AND ITS SATELLITES.

57. Jupiter is the most brilliant of the planets, except Venus. Its apparent diameter when greatest is 44″.5, and when least, 30″.

The sidereal revolution of Jupiter is about 4333 days, or nearly 12 years, and its mean distance from the sun is 494 millions of miles. The diameter of Jupiter is 89,000 miles, which is more than 11 times the diameter of the earth. The magnitude of Jupiter is therefore more than 1300 times that of the earth.

Jupiter revolves from west to east, on an axis nearly perpendicular to the ecliptic, in 9 h. 56 m. Its polar diameter is to its equatorial diameter, in the ratio of 167 to 177.

58. When Jupiter is examined with a good telescope, its disc is observed to be crossed near the centre by several obscure spaces which are nearly parallel to each other, and to the plane of the equator. These are called the *Belts* of Jupiter.

59. When Jupiter is viewed with a telescope, even of moderate power, it is seen accompanied by four small stars, nearly in a straight line parallel to the ecliptic. These always accompany the planet, and are called its *Satellites*. They are continually changing their positions with respect to one another, and to the planet, being some-

times all to the right, and sometimes all to the left; but more frequently some on each side. The greatest distances to which they recede from the planet on each side are different for the different satellites, and they are thus distinguished: that being called the *First* satellite, which recedes to the least distance; that the *Second*, which recedes to the next greater distance, and so on. The satellites of Jupiter were discovered by Galileo, in 1610.

60. Sometimes a satellite is observed to pass between the sun and Jupiter, and to cast a shadow which describes a chord across the disc. This produces an eclipse of the sun, to Jupiter, analogous to those which the moon produces on the earth. It follows that Jupiter and its satellites are opaque bodies, which shine by reflecting the sun's light.

Jupiter being an opaque and nearly spherical body, must project a conical shadow in a direction opposite to the sun. When either of the satellites enters this shadow, it must suffer an eclipse and consequently become invisible. Observations show that this is the case. The satellites are frequently seen, even when considerably distant from the planet, to grow faint, and in a little time entirely to disappear. The third and fourth satellites are sometimes observed, after having been eclipsed, again to become visible on the same side of the disc. These phenomena indicate that the satellites of Jupiter are moons which revolve round that planet in like manner as the moon does round the earth.

61. The satellites are sometimes on the opposite side of Jupiter from the earth, and consequently become invisible. Sometimes they are between the earth and Jupiter, in which case they are not easily distinguished from the planet itself.

When a satellite is invisible in consequence of entering into the shadow of Jupiter, the phenomenon is called an *Eclipse of the satellite.*

62. Careful and repeated observations show that the motions of the satellites are from west to east, in orbits

nearly circular, and making small angles with the plane of Jupiter's orbit. Observations on the eclipses of the satellites make known their synodic revolutions, from which their sidereal revolutions are easily deduced. From measurements of the greatest apparent distances of the satellites from the planet, their real distances are determined.

63. A comparison of the mean distances of the satellites, with their sidereal revolutions, proves that Kepler's third law with respect to the planets, applies also to the satellites of Jupiter. The squares of their sidereal revolutions are as the cubes of their mean distances from the planet.

The planets Saturn and Uranus are also attended by satellites, and the same law has place with them.

64. The orbits of the third and fourth satellites are elliptical. Those of the other two, have not been ascertained to differ sensibly from circles.

65. The mutual attractions of the satellites on one another, produce inequalities in their motions, which must be taken into view, when it is designed to determine from calculation their positions at any given time with accuracy. In investigating this subject, La Place discovered two very remarkable conditions, that connect the mean motions of the first three satellites.

He found, *That the mean motion of the first satellite, added to twice the mean motion of the third, is exactly equal to three times the mean motion of the second.*

He also found, *That the mean longitude of the first satellite, less three times that of the second, more twice that of the third, must always be equal to* 180°. It follows from this circumstance that the longitudes of these three satellites can never be the same at the same time, and consequently that *they can never be all eclipsed at once.*

66. The satellites of Jupiter undergo periodical changes in brightness. From very attentive observations of these changes, Dr. Herschel inferred that each satellite revolves on its own axis in the same time that it makes a sidereal revolution round the planet.

67. Observations on the eclipses of Jupiter's satellites

have led to the discovery of a very important fact; which is, that the *Transmission of light is Successive.*

When Jupiter is in opposition, the eclipses of the satellites happen earlier than they ought to do, according to the known durations of their revolutions, and on the supposition that the transmission of light is instantaneous. On the contrary, when Jupiter is near conjunction, they happen later than they ought to do on this supposition. The variations are the same for all the satellites, and are found evidently to be connected with the distance of Jupiter from the earth; the eclipses happening later as the distance is greater. These circumstances are easily explained and the amount of the retardation accurately accounted for, by allowing light to occupy 16 m. 26 sec. in traversing, with a uniform motion, a distance equal to the transverse axis of the earth's orbit. Hence it was inferred by Roemer, a Danish astronomer who, in 1674, first discovered the variations in the times of the eclipses, that the propagation of light, instead of being instantaneous, is actually performed with a measureable, though immensely great velocity. The correctness of this inference has since been fully established by another phenomenon called the aberration of light, which will be noticed in a succeeding chapter.

68. Since light is 16 m. 26 sec. in passing over a distance equal to the diameter of the earth's orbit, it must be 8 m. 13 sec. in passing from the sun to the earth, when these bodies are at their mean distance. Its velocity is therefore 192,000 miles per second, which is greater than any other with which we are acquainted.

69. The eclipses of Jupiter's satellites furnish a simple means of determining the longitudes of places on land, with considerable accuracy.

The tables for calculating these eclipses, constructed by Delambre, and founded on the theory of La Place, give the times of beginning or end of the eclipses, with very little error. These times are calculated and inserted in the Nautical Almanac, for the meridian of Greenwich, in the *Astronomisches Jahrbuch* for the meridian of Berlin, and in the *Connaisance Des Tems*, for the meridian of Paris.

As a satellite really loses its light by entering into the shadow of Jupiter, the commencement of an eclipse must be seen at the same instant by all observers, however distant from one another.* If, therefore, an eclipse of one of the satellites be observed at a place whose longitude is required, the difference between the observed time, and the time computed for the meridian of Greenwich, will give the difference of meridians, supposing the tables to be accurate.

This method of finding the longitude cannot be employed at sea, because the motion of the vessel prevents the use of telescopes of sufficient power for observing the eclipses.

SATURN WITH ITS SATELLITES AND RING.

70. Saturn revolves round the sun in about 10759 days, or nearly 29½ years, at the distance of 905 millions of miles. Its diameter is 79,000 miles. The greatest and least apparent diameters of Saturn, are 20″.1 and 16″.3.

Saturn revolves on its axis from west to east, in 10 h. 29 m. Its axis is inclined to the ecliptic in an angle of about 60°. The polar diameter is to the equatorial, in the ratio of 10 to 11.

71. The planet Saturn is distinguished from all the other planets, in being surrounded by a broad, thin ring, which is entirely detached from the body of the planet. It is ascertained to be opaque and to shine by reflecting the sun's light. This ring was discovered by Huygens, and is discernible, when in favourable positions, with telescopes of small power.

The plane of the ring is inclined to the ecliptic in an angle of 31° 24′. Consequently, the face of the ring can never be turned directly towards the earth. It is generally seen under the form of an eccentric ellipse. The ring becomes invisible when the enlightened face is turned from the earth. On account of its little thickness, it is also invisible in two other cases. These are, when the plane of the ring, produced, passes through the centre of the earth, and when it passes through the centre of the sun.

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* This supposes the telescopes used by the observers to be of equal goodness. For, since the diminution of light is gradual (60), two observers, by the side of each other, but using telescopes of different power, will not lose sight of the satellite at the same instant. The observation also depends on the state of the air, and in some measure on the eye of the observer.

The ring revolves round an axis, perpendicular to its plane, and passing through the centre of Saturn, in 10 h. 29 m.

Observations with telescopes of high power, show that the ring of Saturn really consists of two concentric rings, entirely separate from each other. The breadth of the interior ring is 17,000 miles; of the exterior 10,000 and of the space between them 1800. The distance from the centre of the planet to the inside of the interior ring is 59,000 miles. The thickness of the rings is less than 100 miles.

72. Saturn is accompanied by seven satellites, which move round it from west to east in orbits that are nearly circular. The orbits of the first six nearly coincide with the plane of the ring; that of the seventh makes a less angle with the ecliptic.

URANUS AND ITS SATELLITES.

73. The planet Uranus was discovered by Dr. Herschel in the year 1781. It revolves round the sun in 30687 days, or a little more than 84 years, at the distance of 1800 millions of miles. Its diameter is 35000 miles. The greatest and least apparent diameters are 4″.1 and 3″.7. The distance of Uranus is so great that its revolution on its axis has not been ascertained.

According to the observations of Dr. Herschel, Uranus is accompanied by six satellites, which revolve in orbits nearly perpendicular to the plane of the ecliptic.

VESTA, JUNO, CERES AND PALLAS.

74. These four planets, although less distant than seve-

ral of the others, are so extremely small that they can only be seen with telescopes of considerable power. Ceres was discovered by Piazzi on the first day of the present century; Pallas, by Olbers in 1802; Juno, by Harding in 1803; and Vesta, by Olbers in 1807. They revolve from west to east, in orbits not very different in extent, and contained between those of Mars and Jupiter. The orbit of Pallas differs from those of all the other planets in the greatness of the angle it makes with the ecliptic. This angle is nearly 35°.

75. The following tables contain the elements of the orbits of the planets and the periodic revolutions of the satellites. The elements of the four new planets are to be considered only as approximations. The epoch taken is the 1st of January, 1801, at mean noon on the meridian of Greenwich; except for the four small planets, the epoch for which is Jan. 1st, 1820.

Sidereal Revolutions of the Planets.

	Days.
Mercury	87.969258
Venus	224.700787
The Earth	365.256374
Mars	686.979646
Vesta	1325.743
Juno	1592.661
Ceres	1681.393
Pallas	1686.539
Jupiter	4332.584821
Saturn	10759.219817
Uranus	30686.820830

Mean distances from the Sun, or Semi-axes of the Orbits, the Earth's mean distance being = 1.

Mercury	0.387098
Venus	0.723332
The Earth	1.000000
Mars	1.523692
Vesta	2.36787
Juno	2.66901
Ceres	2.76724
Pallas	2.77289
Jupiter	5.202776
Saturn	9.538786
Uranus	19.182390

Ratio of the Eccentricity to the Semi-transverse axis.

		Ratio of the Eccentricity.	Secular Variation.
Mercury		0.205515	0.00000387
Venus		0.006861	—0.00006275
The Earth		0.016792	—0.00004359
Mars		0.093307	0.00009019
Vesta	1820	0.089130	
Juno		0.257848	
Ceres		0.078439	
Pallas		0.241648	
Jupiter		0.048162	0.00016036
Saturn		0.056150	—0.00031240
Uranus		0.046611	—0.00002521

Mean Longitudes, reckoned from the Mean Equinox.

Mercury		166°	0′	44″
Venus		11	33	3
The Earth		100	39	33
Mars		64	22	56
Vesta	1820	278	30	7
Juno		200	16	24
Ceres		123	16	17
Pallas		108	25	3
Jupiter		112	15	23
Saturn		135	20	6
Uranus		177	48	23

Mean Longitudes of the Perihelia, with the Sidereal Secular Variations.

		Long. Perihel.			Sec. Var.	
Mercury		74°	21′	47″	9′	44″
Venus		128	43	53	—4	28
The Earth		99	30	5	19	41
Mars		332	23	57	26	22
Vesta	1820	249	33	24		
Juno		53	33	46		
Ceres		147	7	31		
Pallas		121	7	4		
Jupiter		11	8	34	11	5
Saturn		89	9	29	32	17
Uranus		167	30	24	4	0

Inclinations of the Orbits to the Ecliptic, with the Secular Variations of the Inclinations to the true Ecliptic.

	Inclination.			Sec. Var.
Mercury	7°	0′	9″	18″.1
Venus	3	23	29	— 4.5
The Earth	0	0	0	0
Mars	1	51	6	— 0.3

		Inclination.			Sec. Var.
Vesta	1820	7	8	9	
Juno		13	4	10	
Ceres		10	37	26	
Pallas		34	34	55	
Jupiter		1	18	52	—22.6
Saturn		2	29	36	—15.5
Uranus		0	46	28	3.1

Longitudes of the Ascending Nodes, with the Sidereal, Secular Motions.

		Long. of ☊.			Sec. Mot.	
Mercury		45°	57′	31″	—13′	2″
Venus		74	54	13	—31	11
The Earth		0	0	0	0	0
Mars		48	0	3	—38	49
Vesta	1820	103	13	18		
Juno		171	7	40		
Ceres		80	41	24		
Pallas		172	39	27		
Jupiter		98	26	18	—26	21
Saturn		111	56	37	—32	22
Uranus		72	59	35	—59	59

Sidereal Revolutions of the Satellites, and their Mean Distances from the Planets about which they revolve. The distances are expressed in terms of the Equatorial Radius of the Planet.

JUPITER.

	Mean Dist.	Sider. Revol.
1st Satellite	6.04853	1.7691378 Days.
2d	9.62347	3.5511810
3d	15.35024	7.1545528
4th	26.99835	16.6887697

SATURN.

	Mean Dist.	Sider. Revol.
1st Satellite	3.351	0.94271 Days.
2d	4.300	1.37024
3d	5.284	1.88780
4th	6.819	2.73948
5th	9.524	4.51749
6th	22.081	15.94530
7th	64.359	79.32960

URANUS.

	Mean Dist.	Sider. Revol.
1st Satellite	13.120	5.8926 Days.
2d	17.022	8.7068
3d	19.845	10.9611
4th	22.752	13.4559
5th	45.507	38.0750
6th	91.008	107.6944

CHAPTER XIII.

ON COMETS.

1. It has already been said (1.9) that a Comet is a body which occasionally appears in the heavens, has a motion among the fixed stars, and only continues visible for a short period. The appearance of a comet is usually that of a collection of vapour, in the centre of which is a nucleus, that is, in general, not very distinctly defined.

2. The motions of some comets are direct, and of others retrograde. In the same comet the motion continues nearly in one plane, passing through the sun's centre; but for different comets the planes make very different angles with the ecliptic. It is found that when a comet first becomes visible, its distance from the sun is diminishing; and that when it ceases to be visible, the distance is increasing.

3. When a comet first appears, its nucleus is usually surrounded by a faintly luminous vapour, to which the name of *Coma* has been given. As the comet approaches the sun, the coma becomes more bright, and frequently shoots out into a long train of luminous transparent vapour in a direction opposite to the sun. This forms the *Tail* of the comet.

As the comet recedes from the sun, the tail precedes it, being still in a direction opposite to the sun, and grows less, till at length the comet resumes nearly its first appearance.

4. The length and form of the tail are very various. In some, the length is only a few degrees, and in others it is more than a quadrant. In the great comet which appeared in 1680, the tail extended to a distance of 70°; and in that of 1618, to the distance of 104°.

5. The tail is not, however, by any means an invariable appendage of the smaller comets, and some of the larger ones do not exhibit the least appearance of a tail. The smaller comets generally appear merely as round or somewhat oval masses of vapour, increasing in density towards the centre, but frequently without any distinct nucleus. Stars of small magnitude are distinctly visible through what appears to be the densest part of their substance.

6. Comets have sometimes been known to pass near to some of the heavenly bodies, and to have had their motions considerably affected by the consequent attractions; but without producing any sensible influence on the motions of those bodies. In one instance a comet passed among the satellites of Jupiter and was thrown by the attraction of the planet entirely out of the orbit it had been describing, and forced into another of much greater extent; yet not the least perceptible derangement of the motions of the satellites was produced. Hence it evidently follows that the *quantity of matter forming the mass of a comet must be very small.*

7. A comet remains so short a time in sight, and describes so small a part of its course within our view, that, from observation alone, without the assistance of hypothesis, it would be impossible to determine the nature of its path. The hypothesis most conformable to analogy is, that the comet moves in an ellipse, having the sun in one of the foci, and that the radius vector describes areas, proportional to the times.

The ellipse in which a comet moves is very eccentric, and consequently the part of it in the vicinity of the vertex or perihelion, and through which the comet passes while it continues visible, must coincide very nearly with a parabola.

8. The elements of a comet's orbit are, the inclination of the orbit, the position of the line of the nodes, the longitude of the perihelion, the perihelion distance from the sun, and the time when the comet is in the perihelion. These are less in number than those of the orbit of a planet (12.31), because the observations that can be made during one appearance of a comet are not sufficient to determine with any degree of accuracy the transverse axis of the orbit and the periodic time.

9. Assuming the orbit of a comet to be an ellipse or parabola, and that the radius vector describes areas proportional to the times, the elements may be determined from *three* observed geocentric places of the comet. This, though a problem of considerable difficulty, may be performed in a great variety of ways; almost every noted astronomer of latter time having given a method of his own. Dr. Bowditch, in an appendix to the third volume of his translation of Laplace's Mécanique Céleste, has introduced several of the best methods, and added tables which facilitate the computation.

10. The orbits of several observed comets appear, from calculations which have been made, to be *Hyperbolas;* and consequently the bodies moving in them must, after having passed their perihelions, move on in space, never to return. But the orbits of the larger number appear to be ellipses, generally of great eccentricity. The bodies moving in them must therefore, unless thrown out of their paths by the actions of other bodies, return to their perihelions, and thus perform revolutions round the sun in periods of various length depending on the extent of the orbit.

11. One of the most celebrated comets is that called *Halley's* comet, in consequence of its return to its perihelion in 1759, having been predicted in the early part of the century by Edmund Halley, an eminent astronomer of that time. He calculated, from recorded observations, the elements of a number of comets: and on comparing them, he found that those of the comets of 1531 and 1607 very nearly agreed with those of the comet of 1682, which

he had himself observed; the interval being about 76 and 75 years. This near agreement led him to suppose that instead of three different comets, it might be the *same* comet which had appeared in these different years. Making further researches, he became satisfied of the correctness of the supposition he had made, and concluded that the variation in period must have been produced by the attractions of the other heavenly bodies. Having, therefore, made a rough calculation of the effect which the attraction of Jupiter would produce on the revolution the comet was then performing, he ventured to predict its return in the latter part of 1758 or early part of 1759. Subsequently, Clairaut calculated the effects of the attractions of both Jupiter and Saturn, and determined the time of the return to the perihelion to be in the middle of April, 1759. It arrived there about a month prior to that time. With more ample means for correct computation, furnished by the observations during the comet's appearance in 1759, and, by the improvements in analysis, its return last year, 1835, was very accurately predicted.

12. Besides Halley's comet, there are two others, which are known to have made repeated revolutions round the sun. They are both comets of short periods. The first of these is called *Encke's* comet, from Professor Encke, of Berlin, who first ascertained its periodical character in 1819. He found its period to be 1207 days, or nearly 3¼ years, and predicted its return in 1822, which was verified by observation. Its subsequent returns have been predicted and observed.

13. From calculations and observations, made on the successive returns of this comet, Professor Encke infers that the length of its period is *continually diminishing*. To account for this he supposes the existence of very rare ethereal matter pervading space, the resistance of which, producing no perceptible influence on the motions of the denser bodies of our system, becomes sensible in its effects on those of such vaporous masses as the comets. The effect of such a resistance would evidently be to diminish

the velocity of the body; and the known consequence of a diminished velocity would be a diminution in the mean distance from the sun, to an extent that would render the revolution shorter, though performed with this diminished velocity. But Pontécoulant, an able French astronomer, considers the observations hitherto made, as being by no means sufficient to establish the existence of a resisting medium.

14. The other comet of short period was identified as a periodical one in 1825, by M. Biela, of Josephstadt, and is called *Biela's* comet. It is very small, and without the least appearance of a nucleus. It moves in a moderately eccentric ellipse, performing its revolution in about 6¾ years. The return in 1832, calculated by Biela, took place according to prediction. Its orbit very nearly intersects that of the earth.

15. The number of recorded comets, visible to the naked eye, amounts to several hundreds; though many of these, it is probable, are only reappearances of the same comet. The actual number of comets is not known; but including those that become visible by the aid of a telescope, it is believed to amount to many thousands.

CHAPTER XIV.

ABERRATION OF LIGHT, AND NUTATION OF THE EARTH'S AXIS.

1. Dr. Bradley, in the course of some accurate observations on the fixed stars, found that their apparent places were subject to small changes, amounting when greatest to about 40″. He also ascertained that those changes were annual, as their magnitudes were the same at the same time in each year. These observations were commenced in the year 1725, and continued for several years.

After several unsuccessful attempts to explain the cause of these periodical changes, it occurred to him that in consequence of the progressive motion of light, and of the earth's motion in its orbit, the apparent place of a star ought generally to be different from the true place.

2. Let OB, *Fig.* 45, be a portion of the earth's orbit, so small that it may be considered as a right line, and the earth's motion in it, uniform; and let ES be the direction of a fixed star, from the point E. Also let AE be the distance through which the earth moves in some short portion of time, and aE the distance through which a particle of light moves in the same time. Then a particle of light, which, coming from the star in the direction SE, is at a, at the same time that the earth is at A, will arrive at E, at the same time that the earth does. Let $A'a'$, $A''a''$ and ES', be each parallel to Aa. Then aa' is to AA' and $a'a''$ is to $A'A''$ in the ratio of Ea to EA. Consequently when the earth is at A', the particle of light is at a', and when the earth is at A'', the particle of light is at a''. The particle of light, therefore, continues in the same direction from the earth, that is, in the direction Aa or ES'. Hence it meets the earth at E, in the direction $S'E$. To an eye at E, the particle of light entering it in the direction $S'E$, appears to come from a star in the direction ES'. What has been said of a single particle, will apply to all the particles coming from the star, and entering the eye. Consequently the star appears to be in the direction ES'.

3. The angle which expresses the change produced in the apparent place of a body, by the motion of light combined with the motion of the spectator, is called the *Aberration.* Thus $S'ES$ is the aberration of the star S.

4. Various formulæ have been investigated for computing the effect of aberration on the longitudes, latitudes, right ascensions and declinations of the heavenly bodies, and particularly of the fixed stars. Of those that apply to the fixed stars, the following are some of the most simple.

5. Let L be the longitude of the sun at the time for

which the aberration is required, and L′ and λ, the longitude and latitude of the star. Then,

$$\textit{Aber. in Long.} = \frac{-20'' 253 \cos (L - L')}{\cos \lambda}.$$

$$\textit{Aber. in Lat.} = 20'' .253 \sin (L' - L) \sin \lambda.$$

6. Let A be the right ascension and D the declination of the star, L being the sun's longitude as before. Then,

$$\textit{Aber. in Right Ascen.} = \frac{0''.837 \cos (A + L) - 19''.416 \cos (A - L)}{\cos. D.}$$

$$\textit{Aber. in Decl.} = \sin D \,[19''.416 \sin (A - L) - 0''.837 \sin (A + L) - 8''.066 \cos L \cos D].$$

7. In consequence of the aberration, each star appears to describe an ellipse in the heavens, of which the true place is the centre; the semi-transverse axis is 20″.253 and the semi-conjugate is 20″.253 sin λ.

8. The supposition of the earth's annual motion serves fully to explain the phenomena of the aberration. On the contrary, the amounts of the aberrations for different stars, and at different times, computed on that supposition, are found exactly to agree with observation. *These circumstances form a sufficient proof of the reality of the earth's annual motion.*

9. The aberration of the sun, which has place only in longitude, is — 20″.253. Thus the sun's apparent place is always about 20″.25, behind its true place. Solar tables give the apparent place of the sun, as affected by aberration, and it is this which is generally wanted.

10. For a planet, the aberration is different from what it is for a fixed star, because the planet changes its place during the time that light is passing from it to the earth. The aberration is therefore increased or diminished by the geocentric motion of the planet during this time.

For the moon, the aberration is always very small, only amounting to a fraction of a second.

11. Besides the aberration produced by the annual

motion of the earth, there is another, called the *Diurnal* aberration, which is produced by the earth's motion on its axis. This is however so small as to be nearly insensible.

NUTATION.

12. Small inequalities, which have been observed in the precession of the equinoxes, and in the mean obliquity of the ecliptic, are called *Nutations*. These inequalities were discovered by Dr. Bradley, while employed in verifying his theory of the aberration.

13. The period of the changes of these inequalities was observed to be about the same as the period of the revolution of the moon's nodes; and it was found that the quantities of the inequalities depended on the place of the node.

14. The phenomena of the nutation may be represented by supposing, that while a point, which may be considered as defining the mean place of the pole of the equator, describes a circle in the heavens, round the pole of the ecliptic, at a distance from it, equal to the mean obliquity of the ecliptic, and with a retrograde motion of $50''.1$ annually, another point representing the true pole of the equator, moves round the former at the distance of $9''$, so as to be always 90° more easterly than the moon's ascending node. The inequalities thus produced in the precession of the equinoxes, and in the obliquity of the ecliptic, will very nearly agree with the observed inequalities. The agreement becomes more exact, if, instead of supposing the true pole to describe a circle about its mean place, it be supposed to describe an ellipse, having its semi-transverse axis equal $9''.6$ and its semi-conjugate $7''.5$.

15. If N be the longitude of the moon's ascending node, the variation in the obliquity of the ecliptic, produced by the nutation is $+ 9''.6 \cos N$; the inequality in the motion of the equinoxes in longitude, sometimes called the *Equation of the Equinoxes in Longitude*, is $17''.946 \sin N$; and the inequality in their motion in right ascension, called the

Equation of the Equinoxes in Right Ascension, is $-16''.462 \sin N$.

16. The equation of the equinoxes in longitude equally affects the longitudes of all the stars. The equation of the equinoxes in right ascension also affects the right ascensions of all the stars, but it only forms a part of the nutation in right ascension.

17. If A be the right ascension of a star, and D its declination, then,

Nutation in Right Ascen. =

$$-16''.462 \sin N - 8''.373 \cos (A - N) \tan D$$
$$- 1''.227 \cos (A + N) \tan D.$$

Nutation in Decl. =

$$+ 8''.373 \sin (A - N) + 1''.227 \sin (A - N).$$

18. In catalogues of the fixed stars, the mean places and annual variations are given. By the preceding formulæ the aberrations and nutations may be calculated; and thence the true apparent places may be found.

19. The exact apparent positions of the fixed stars are so continually wanted by the practical astronomer in adjusting and examining the adjustments of his instruments, and as points of reference, that much attention has been devoted to the obtaining concise and accurate methods of deducing these from the mean places given in the catalogues. In the catalogue of 2881 principal fixed stars, published a few years since under the direction of the Astronomical Society of London, besides the mean places and annual precessions, there are given certain constant logarithms for each star, by means of which, with other logarithms depending on the longitudes of the sun, moon, and moon's node, given in the Nautical Almanac for each day in the year, the true apparent place of any of these stars may be found with great facility.

Professor Bessel, in his *Tabulæ Regiomontanæ*, has given general formulæ and tables for reducing the mean places of the stars to apparent places. He has also given with great precision the mean and apparent places of 36 principal fixed stars, regarded as *fundamental* stars, for a period of 100 years, commencing in 1750 and ending in 1850.

The Nautical Almanac contains a table of the values for every fifth day in the year, of certain quantities computed from Bessel's formulæ of reduction, which facilitate the computation of the apparent places of stars not given in the Astronomical Society's Catalogue. The apparent places of 100 principal fixed stars, are likewise given; two of them, α and δ, Ursæ Minoris, for each day, and the others for every tenth day.

20. Some of the bodies termed fixed stars have been ascertained to have slow motions, called their *proper motions*, in consequence of which, their positions with regard to the other stars become actually changed. The proper motions of several have been determined, and are noticed in the best catalogues.

CHAPTER XV.

CLASSIFICATION OF THE FIXED STARS.—ANNUAL PARALLAX.—VARIABLE STARS.—DOUBLE STARS.—NEBULÆ.

1. The classification of the stars according to their apparent magnitudes has already been noticed (6.23). It may be added that the magnitudes of stars which seem to be intermediate in brightness between those of two classes, so as to render it doubtful in which they would be more properly placed, are usually denoted by two numbers with a point between them. Thus, the expression 1.2 denotes a star intermediate between those of the first and second magnitudes.

2. In the catalogue published by the Astronomical Society (14.20), which contains all the stars to the fifth magnitude inclusive, and many of the sixth and seventh magnitudes, there are 18 stars designated as being of the *first magnitude*, 29 of the *second*, and over 100 of the *third*. There are also 3 stars, marked as intermediate between the first and second magnitudes, and 29 between the

second and third. The numbers in the other magnitudes increase rapidly as we descend in the scale of brightness.

3. With regard to the relative light afforded by stars of the different magnitudes, it is considered that, taking a star of the medium brightness in each magnitude, we do not greatly err in regarding the light of one of the second magnitude as being half that of one of the first, the light of one of the third as half that of one of the second, and thus on through the different classes.

ANNUAL PARALLAX.

4. The *Annual Parallax* of a fixed star is the angle contained between two straight lines, conceived to be drawn from the star, one to the sun and the other to the earth, when the earth is in such a part of its orbit that its radius vector is perpendicular to the latter line; or, in other words, it is the greatest angle at the star that can be subtended by the semi-diameter of the earth's orbit.

5. If the annual parallax of a star was known, we might, by using it and the semi-diameter of the earth's orbit, instead of the sun's parallax and the radius of the earth, find the star's distance in the same manner that the sun's distance was found (5.16). The attention, therefore, of astronomers furnished with the most perfect instruments, has been perseveringly directed to observations on some of those stars which from their splendour may be regarded as the nearest, in order to determine their annual parallax. But, though those observations have been made with the utmost care, and when the earth was in the most favourable positions, they have not been able to unite in assigning a measurable parallax to any of them. Now the accuracy to which astronomical observations with the best instruments is at present brought, is such, that if the parallax amounted to a single second, it would have been determined. We must therefore conclude that *the annual parallax of the nearest fixed star is less than one second.*

6. On the supposition that the annual parallax of a star was 1″, the distance of the star would be 206265 times

the distance of the sun from the earth. This is a distance so immense, that light, which moves with the amazing velocity of 192,000 miles per second (12.68), would require more than three years to come from the star to the earth. Yet inconceivably great as it is, the distance of the nearest fixed star must be still greater.

7. The light from the same body, at different distances, varies inversely as the square of the distance. Hence, assuming the light of a star of each magnitude to be half that of one of the next greater magnitude (3), it is easy to ascertain that a star of the first magnitude would have to be placed at 181 times its distance, to appear as one of the sixteenth magnitude. We may therefore conclude, that among the innumerable stars of the latter magnitude there must be many, the light of which, though moving with a velocity that would suffice to make the circuit of our globe nearly 8 times in a second, has required more than 500, probably more than 1000 years, to traverse the space by which they are separated from the earth.

VARIABLE STARS.

8. One of the most remarkable of the variable stars (6.30) is the star *Omicron* in the constellation *Cetus*. When brightest, it sometimes appears as a large star of the second magnitude, and it preserves this appearance about two weeks. It then decreases during about three months, till it becomes entirely invisible; and remaining thus about five months, it again becomes visible and gradually increases till it acquires its greatest brightness. The period of these changes is 334 days. The star does not, however, always acquire the same degree of brightness, nor increase and diminish by the same gradations. And it is represented as having in one instance continued invisible for a period of four years, commencing in 1672 and ending in 1676.

9. Another remarkable variable star is that called *Algol*, or *β Persei*. It usually appears as a star of the second magnitude, and continues as such for a period of 2 d. 14 h.,

when it suddenly begins to diminish in splendour, and in about 3½ hours is reduced to the fourth magnitude. It then begins to increase, and in about 3½ hours more regains its usual brightness; going through all its changes in 2 d. 20 h. 48 m.

There are several other stars which undergo periodical changes in their brightness, but they are less conspicuous than the two which have been noticed.

10. There are also some instances recorded of stars suddenly appearing in great splendour, retaining their positions unchanged among the other stars, and after a time entirely disappearing. One of the most noted of these, is the star which broke forth with great brilliancy on the 11th November, 1572, in the constellation *Cassiopeia*, and was attentively observed by Tycho Brahe, a celebrated Danish astronomer. It was then as bright as Sirius, the brightest of the permanent stars, and it increased till its splendour was such as to render it visible at mid-day. It began to diminish in December of the same year, and in March 1574, it entirely disappeared.

In the years 945 and 1264, a brilliant star appeared in the same region of the heavens, in which that of 1572 was seen. The accounts of the situation or situations are very imperfect; but from the tolerably near agreement in the intervals of time, some have supposed that it must have been the same star that appeared at each of those times, and consequently that it has a period of a little over 300 years.

DOUBLE STARS.

11. From the time of the invention of the telescope, double stars (6.28) had been occasionally observed by astronomers, but they were not much noticed till Dr. Herschel directed attention to them by the publication, in 1782 and 1785, in the Philosophical Transactions, of catalogues of a large number which he had observed. He arranged them in classes according to their apparent proximity; placing in the first class those whose closeness was such

that they could only be separated by the most powerful telescopes, and whose distance from each other did not exceed 1″ or 2″; in the second class, those that were less difficult to separate, but yet required a very superior instrument, the distance between the stars not exceeding 5″, in the third class those whose distance was from 5″ to 15″, and in the fourth, fifth, and sixth classes, those whose distances were from 15″ to 30″, from 30″ to 1′, and from 1′ to 2′, or more. A good 3½ feet achromatic telescope is sufficient to separate most of the stars in the third and lower classes.

12. Dr. Herschel, continuing his observations, found that regular though slow progressive changes were taking place in a number of the double stars, with regard to distance, or position from each other, or both. In the course of about twenty-five years from the commencement of his observations he satisfactorily ascertained that the changes, in several of them at least, were exactly such as would be produced by the revolution of one of the stars around the other, or of both around their common centre of gravity. And in two papers, published in the Philosophical Transactions, he announced the existence of sidereal systems, composed of two stars, revolving about each other in regular orbits. These stars have received the appellation of *binary stars*, to distinguish them from other double stars, whose apparent proximity may probably proceed merely from the one being situated beyond the other, as seen from the earth, without their having any physical connection.

13. From the changes which have taken place in their proximity, several of the double stars belong now to different classes from those in which they were placed by Dr. Herschel. Some of them, which, when his catalogues were formed, were extremely difficult to separate, may now be separated with a telescope of moderate power; and others, which were then easily separated, now require the most powerful instruments.

14. Within a few years past, the subject of double stars

has claimed great attention, and they have been extensively observed, particularly by J. South, Professor Struve, and J. F. W. Herschel. The last named observer has carefully examined those contained in the two catalogues published by his father, and both he and Professor Struve have added largely to the list.

The determination of the angle of position of double stars has been greatly facilitated of latter time by the use of a micrometer of a particular construction, called the *position micrometer.*

15. The observations on several of the binary stars have been sufficient to enable astronomers to determine the form of their orbits, and, at least approximately, the lengths of their periods. The orbits are elliptical. The periods are very various; some extending to several centuries and others only to a few years. The following are the periods of some that are considered as the best ascertained: *γ Leonis*, 1200 years; *γ Virginis*, 629 years; 61 *Cygni*, 452 years; *σ Coronæ*, 287 years; Castor, 253 years; 70 Ophiuca, 80 years; *ξ Ursæ*, 58 years; *ζ Cancri*, 55 years; and *η Coronæ*, 43 years.

CLUSTERS OF STARS AND NEBULÆ.

16. In the beautiful cluster of stars called the *Pleiades*, six or seven stars are readily discerned by the naked eye; but a telescope, even of moderate power, exhibits within the space they occupy fifty or sixty conspicuous stars. The constellation called *Coma Berenices*, is another group, more diffused, and composed of larger stars.

17. In the constellation *Cancer*, there is a luminous spot or nebula (6.31), called *Praesipe*, or the bee-hive, which a telescope of moderate power resolves entirely into stars. In *Perseus* is another spot, crowded with stars, which become separately visible with a good telescope. There are numerous other luminous spots or nebulæ distributed over the heavens, many of them not visible without the aid of a telescope, which when examined with instruments of the greatest power, are resolved into clusters of stars.

18. Of nebulæ which are not resolvable into clusters of stars, or which have not been thus resolved, one of the most prominent is that near the star ν in *Andromeda*. It is visible to the naked eye, and has often been mistaken for a comet.

CHAPTER XVI.

NAUTICAL ASTRONOMY.

1. Some of the most useful practical applications of astronomy are those which serve to make known to the navigator his latitude and longitude, when at sea. The continual agitation in the motion of a ship does not permit the use of instruments which are adjusted by a plumb line or spirit level. The astronomical instruments used at sea, are the *Hadley*'s *Quadrant, the Sextant, and Circle of Reflection*, which are independent of such adjustments. The quadrant is used for observing the altitude of the sun or other heavenly bodies, and the sextant or circle of reflection for measuring the angular distance of the moon from the sun or a star.

2. The *Mile* used in measuring distances at sea, is the 60th part of a degree. So that a mile just corresponds to a minute.

3. The course on which a ship sails is determined by an instrument called a *Mariner's Compass;* and the rate at which she sails, by an instrument called a *Log*. The latter is a piece of board in the form of a sector of a circle, the circular part of which is loaded with lead, so that when in the water it may keep a vertical position. To the log is attached a line of considerable length, divided into spaces called *Knots*, and wrapped round a light reel. The length of a knot is such, that when the log is thrown into the sea, and the line allowed to run freely off the reel, the number of knots which pass off in a half minute, indicates the number of miles the ship is then sailing in an hour.

4. That portion of the surface of the ocean, which a ship traverses during a few hours, or even during a day, does not differ much from a plane. Supposing it to be a plane, and also that the meridians are parallel to one another, let AB, *Fig.* 46. be the track of the ship, NS and N′S′, meridians passing through A and B, and AC and BD parallels of latitude. Then in the triangle DAB, we have,

$$AD = AB \cos DAB,$$

$$\text{Or, Diff. of Lat.} = \text{Dist.} \times \text{cos. Course.}$$

It is proved by writers on the theory of navigation, that this expression for the difference of latitude is rigidly true, even when the earth is considered as a sphere or spheroid. But in consequence of currents and other causes, the distance and course can never be obtained with great accuracy, and consequently the difference of latitude thus found, must be considered only as an approximation.

5. The triangle ADB, also gives,

$$BD = \text{Dist.} \times \text{sin. Course.}$$

As the meridians are not parallel, but really converge towards the poles, each way from the equator, it is evident that BD is greater than the distance between the meridians on one of the parallels of latitude and less than that on the other, except when A and B are on opposite sides of the equator and equally distant from it. It is in general nearly equal to the distance between the meridians on a parallel of latitude, midway between the parallels passing through A and B. The latitude of this parallel is called the *Middle Latitude*, and is equal to half the sum or half the difference of the latitudes of A and B, according as they are on the same, or on opposite sides of the equator.

6. If AB, *Fig.* 36, be considered as a part of the equator, P its pole, ED a parallel of latitude, and PDA, PEB meridians, passing through any two places, then (11.84) AB, the difference of longitude of the places is equal to ED divided by the cosine of BE. Hence (5), *Fig.* 46.

$$\textit{Diff. of Long. of A and B} = \frac{\text{BD}}{\text{cos Mid. Lat.}} = \frac{\text{Dist.} \times \text{sin. Course}}{\text{cos Mid. Lat.}}.$$

7. The computed differences of longitude and latitude, applied to the longitude and latitude of the place A, give nearly those of the place B, when the distance between the places is not great. The longitude and latitude thus found are called the longitude and latitude by *Dead Reckoning.*

8. As the longitude and latitude found in the preceding manner cannot be depended on, except for a short time, it is necessary that the navigator should be able to determine them by observation. When the weather is favourable the latitude is determined each day at noon, by observation of the sun's altitude about that time. Several altitudes being observed, it is easy to infer the greatest altitude that the sun acquires, which is the meridian altitude. But this altitude is the apparent altitude of the under or upper limb, usually the former, and must be corrected for refraction, parallax and semi-diameter. It also requires another correction. The observation gives the altitude above the visible horizon; and consequently, as the observer is on the deck of the vessel, several feet above the surface of the water, it is too great. A small table accompanies every treatise on navigation, containing a correction, depending on the height of the eye, which is to be subtracted from the observed altitude. This correction is called the *Dip of the Horizon.*

From the correct meridian altitude of the sun, and the sun's declination, given by the Nautical Almanac, the altitude of the point of the equator, that is on the meridian is determined; and thence (4.2), the latitude.

9. Sometimes the sun is hid by clouds, so as to prevent the observation of the meridian altitude, and yet it is visible at other times in the day. In such cases the latitude may be found from two observed altitudes with the interval of time between them. The interval ought if possible to be two or three hours; and one of the altitudes should be as near to noon as circumstances will admit.

10. When the altitudes are taken at different places, as is generally the case at sea, the less one should be reduced to what it would have been, if it had been taken at the same instant, at the place where the other is taken.

This may be done with sufficient accuracy in a very simple manner. Let A, *Fig.* 47, be the place where the less latitude is taken, B, that where the greater is taken, and AS the line in which the horizon is intersected by a vertical circle passing through A and the sun, at the time of observing the less altitude. From the bearing of the sun at that time, and the course the ship is sailing, the angle BAS is known. On AS let fall the perpendicular BD. Then it is evident that the altitudes of the sun, at B and D, at the same time, are the same. But the altitude at D is greater than the altitude at the same time at A, by the number of minutes contained in AD. Consequently AD is the correction of the less altitude. Hence the

$$\textit{Correction} = AD = AB \cos BAS.$$

When the angle BAS is greater than 90°, the cosine is negative, and the correction must be subtracted from the altitude.

11. *Given two altitudes of the sun, with the interval of time between the observations, to find the latitude of the place.*

Let Z, *Fig.* 48, be the zenith of the place at which the greater altitude is taken, P the pole, S the place of the sun at the time of the less altitude, and S′ its place at the time of the greater. As the sun's declination changes but little in the course of a few hours, PS and PS′ may be each considered as equal to the sun's polar distance at the middle of the time between the observations; and consequently the triangle PSS′ may be regarded as isosceles. If PG be perpendicular to SS′, it will bisect it in G. Put,

$H = ES =$ the less altitude reduced,
$H' = FS' =$ the greater altitude,
$D = PS = 90° \pm$ sun's declination,
$A = SPS' =$ interval of time expressed in degrees,
$L = HP =$ latitude, where the greater altitude is taken,
$U = PSZ$, $V = ZSS'$, $W = PSS'$, and $X = SS'$.

Then from the right angled triangle PSG, we have,

$$\sin \tfrac{1}{2} X = \sin \tfrac{1}{2} A \sin D, \text{ and } \cot W = \cos D \tan \tfrac{1}{2} A.$$

From the triangle ZSS′, by an investigation exactly similar to that in article 20th, chap. 9th, we have,

$$\sin \tfrac{1}{2} V = \sqrt{\frac{\cos\left(\frac{H + X + H'}{2}\right) \sin\left(\frac{H + X + H'}{2} - H'\right)}{\cos H \sin X}}$$

$$\text{Then } U = W \mp V$$

The upper sign has place, when the sun passes the meridian on the opposite side of the zenith from the elevated pole, and the under, when it passes on the same side.

Now from the triangle PSZ,

$$\cos ZP = \cos PS \cos ZS + \cos PSZ \sin PS \sin ZS,$$

Or, $\sin L = \cos D \sin H + \cos U \sin D \cos H$

$= \cos D \sin H + (2 \cos^2 \frac{1}{2} U - 1) \sin D \cos H$ (App. 9),

$= 2 \cos^2 \frac{1}{2} U \sin D \cos H - (\sin D \cos H - \cos D \sin H)$,

$= 2 \cos^2 \frac{1}{2} U \sin D \cos H - \sin (D - H)$.

But,

$\sin L = -\cos (90° + L) = 1 - 2 \cos^2 \frac{1}{2} (90 + L)$ (App. 9)

and, $\sin (D - H) = \cos [90° - (D - H)] = 2 \cos^2 \frac{1}{2} [90° - (D - H)] - 1$.

By substituting these values we have,

$\cos^2 \frac{1}{2} (90° + L) = \cos^2 \frac{1}{2} [90° - (D - H)] - \cos^2 \frac{1}{2} U \sin D \cos H$

$$= \cos^2 \tfrac{1}{2} [90° - (D - H)] . \left(1 - \frac{\cos^2 \frac{1}{2} U \sin D \cos H}{\cos^2 \frac{1}{2} [90° - (D - H)]}\right).$$

$$\text{Make } \sin M = \frac{\cos \frac{1}{2} U \sqrt{\sin D \cos H}}{\cos \frac{1}{2} [90° - (D - H)]}$$

Then $\cos^2 \frac{1}{2} (90° + L) = \cos^2 \frac{1}{2} [90 - (D - H)] . (1 - \sin^2 M) = \cos^2 \frac{1}{2} [90° - (D - H)] \cos^2 M$.

Or, $\cos \frac{1}{2} (90° + L) = \cos \frac{1}{2} [90° - (D - H)] \cos. M$.

12. When the latitude is determined, the time may be obtained by an observation of the sun's altitude, a few hours from noon (9.20). Supposing the watch or chronometer, used on board the vessel to have been well regulated and set, previously to leaving port, and that it keeps time accurately, the difference between the time obtained from observation, and that shown by the watch, gives the difference of longitude, in time. But the best time-keeper cannot be entirely depended on, and therefore the longitude, thus obtained, is liable to uncertainty.

13. If the true angular distance between the centres of the moon and sun, or between the centre of the moon and some star near the ecliptic, be obtained from calculations founded on the observed angular distance; and the time when they are at that distance be determined by calculation for the meridian of Greenwich; then the difference between the calculated time, and the time of observation

as reckoned at the meridian of the ship, will give the longitude from Greenwich.

14. The Nautical Almanac contains the distances of the moon from the sun, and from several stars that are best adapted to the purpose. The distances are given to every third hour. It is, therefore, easy to determine by proportion the time when either distance is of a given magnitude.

15. The observed distance must be corrected for the semi-diameter of the moon when the observation is of the moon and a star, and for the semi-diameters of the sun and moon, when it is of those bodies, so as to give the *apparent* distance of the *centres*. To obtain the *true* distance, the apparent distance must be corrected for the effects of refraction and parallax. This requires that the altitudes of the bodies should be known.

16. The altitudes may be taken by two assistant observers, at the same time that the principal one observes the angular distance. If there is but one observer, he can first take several altitudes of the bodies; then several distances; and afterwards several more altitudes, noting the times of all the observations. Thence it is easy to infer, with sufficient accuracy, the altitudes corresponding to the mean of the distances.

17. *Given the apparent distance of the moon and sun, or of the moon and a fixed star, and the altitudes of the bodies, to determine the true distance.*

Let Z, *Fig.* 49, be the zenith, ZH the vertical passing through the moon, and ZO, that passing through the sun, or a star. Then as the moon is more depressed by parallax, than it is elevated by refraction, the apparent place is below the true place. But for the sun or a star as the parallax is very little or insensible, the apparent place is above the true place. Let M be the apparent place, and M′ the true place of the moon; and S the apparent place, and S′ the true place of the sun or star. Put,

$H = HM =$ apparent altitude of the moon,
$H' = HM' =$ true do.
$A = OS =$ apparent altitude of the sun or star,
$A' = OS' =$ true do.

$D = MS =$ apparent distance,
$D' = M'S' =$ true do.

Then in the triangle ZMS, we have, (App. 34),

$$\cos Z = \frac{\cos D - \sin H \sin A}{\cos H \cos A}$$

$$= \frac{\cos D - \cos H \cos A + \cos (H + A)}{\cos H \cos A} \qquad \text{(App. 14)}$$

$$= \frac{\cos D + \cos (H + A)}{\cos H \cos A} - 1.$$

In like manner from the triangle ZM'S' we have,

$$\cos Z = \frac{\cos D' + \cos (H' + A')}{\cos H' \cos A'} - 1.$$

Hence $\dfrac{\cos D + \cos (H + A)}{\cos H \cos A} = \dfrac{\cos D' + \cos (H' + A')}{\cos H' \cos A'}$,

Or, $\cos D' = \dfrac{\cos H' \cos A'}{\cos H \cos A} . [\cos D + \cos (H + A)] - \cos (H' + A')$.

But,

$\cos D + \cos (H + A) = 2 \cos \frac{1}{2} (H + A + D) \cos \frac{1}{2} (H + A - D)$, (App. 22)

$$\cos (H' + A') = 2 \cos^2 \tfrac{1}{2} (H' + A') - 1 \text{ (App. 9)}$$
$$\cos D' = 1 - 2 \sin^2 \tfrac{1}{2} D' \text{ (App. 8)}.$$

Substituting these values, and reducing, we have,

$$\sin^2 \tfrac{1}{2} D' = \cos^2 \tfrac{1}{2} (H' + A') - \frac{\cos \frac{1}{2} (H + A + D) \cos \frac{1}{2} (H + A - D) \cos H' \cos A'}{\cos H \cos A}$$

$$= \cos^2 \tfrac{1}{2} (H' + A') . \left(1 - \frac{\dfrac{\cos \frac{1}{2} (H + A + D) \cos \frac{1}{2} (H + A - D) \cos H' \cos A'}{\cos^2 H \cos A}}{\cos^2 \frac{1}{2} (H' + A')}\right)$$

Make $\sin M =$

$$\frac{\sqrt{\left(\dfrac{\cos \frac{1}{2} (H + A + D) \cos \frac{1}{2} (H + A - D) \cos H' \cos A.}{\cos H \cos A}\right)}}{\cos \frac{1}{2} (H' + A')}$$

Then,

$$\sin^2 \tfrac{1}{2} D' = \cos^2 \tfrac{1}{2} (H' + A') . (1 - \sin^2 M) = \cos^2 \tfrac{1}{2} (H' + A') \cos^2 M,$$

Or, $\sin \frac{1}{2} D' = \cos \frac{1}{2} (H' + A') \cos M$.

CHAPTER XVII.

OF THE CALENDAR.

1. The *Calendar* is a distribution of time into periods of different lengths, as years, months, weeks, and days.

2. It has been shown that the tropical year contains 365 d. 5 h. 48 m. 48 sec. (7.9). But in reckoning time for the common purposes of life, it is most convenient to have the year to contain a certain number of *whole* days. In the calendar established by Julius Cæsar, and thence called the *Julian* calendar, three successive years are made to consist of 365 days each; and the fourth, of 366 days. The year which contains 366 days, is called a *Bissextile* year. It is also frequently called *Leap* year. The others are called *common* years. The added day in a bissextile year is called the *Intercalary* day.

3. According to the Julian calendar, and reckoning from the epoch of the christian era, every year, the number of which is exactly divisible by 4, is a bissextile; and the others are common years.

4. It is evident that the reckoning by the Julian calendar supposes the length of the year to be 365¼ days. A year of this length is called a *Julian Year*. A Julian year, therefore, exceeds the true astronomical year, by 11 m. 12 sec. This difference amounts to 3¼ days in the course of 400 years.

5. At the time of the *Council of Nice*, which was held in the year 325, the vernal equinox fell on the 21st of March, according to the Julian calendar. But by the latter part of the 16th century, in consequence of the excess of the Julian year above the true solar year, it came ten days earlier, that is, on the 11th of March. It was observed that by continuing to reckon according to the Julian calendar, the seasons would fall back, so that in process of time they would correspond to quite different times of the year. This reckoning also led to irregularity in the times of holding certain festivals of the church. The subject,

claiming the attention of Pope Gregory XIII. he, with the assistance of several astronomers, reformed the calendar. To allow for the 10 days, by which the vernal equinox had fallen back from the 21st of March, he ordered that the day following the 4th of October, 1582, should be reckoned the 15th, instead of the 5th. And in order to keep the vernal equinox to the 21st of March, in future, it was concluded that three intercalary days should be omitted every four hundred years. It was also concluded that the omission of the intercalary days should take place in those centurial years, the numbers of which were not divisible by 400. Thus the years 1700, 1800, and 1900, which, according to the Julian calendar, would be bissextiles, would, according to the reformed calendar, be common years.

6. The calendar, thus reformed, is called the *Gregorian calendar.* It is easy to perceive, by a short calculation, that time reckoned by the Gregorian calendar, agrees so nearly with that reckoned by true solar years, that it will require nearly 4000 years to produce the difference of one day.

7. The Gregorian calendar was at once adopted in catholic countries; but in those where the protestant religion prevailed, it did not obtain a place till some time after. In England and her colonies, it was not introduced till the year 1752.* It is now used in all Christian countries, except Russia.

8. The Julian and Gregorian calendars are also desig-

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* At this time there was a difference of 11 days between the Julian and Gregorian calendars, in consequence of the suppression, in the latter, of the intercalary day in 1700. It was therefore enacted by parliament, that 11 days should be left out of the month of September, of the current year, 1752, by calling the day following the 2d of the month, the 14th, instead of the 3d.

Previous to this, years commencing at two different times had been in use, in England. The *historical* year commenced on the 1st of January, as at present. But the *civil* or *legal* year commenced on the 25th of March. Dates in the interval between these times, were frequently expressed by naming both years. Thus in books printed prior to 1752, we often meet with dates expressed as follows: Feb. 2d, 1735–6, or $173\frac{5}{6}$. The same act that introduced the Gregorian calendar, established the 1st of January as the commencement of the civil, as well as of the historical year.

nated by the terms *Old Style* and *New Style*. In consequence of the intercalary days, omitted in the years 1700 and 1800, there is now 12 days difference between them.

9. The year is divided into 12 portions, called calendar months. Each of these contains either 30 or 31 days, except the second month, February, which in a common year contains 28 days, and in a bissextile, 29 days; the intercalary day being added at the last of this month.

10. It was formerly customary to designate the days of the week in the calendar by the first seven letters of the alphabet, always placing them so that A corresponded to the first day of the year, B to the second, C to the third, D to the fourth, E to the fifth, F to the sixth, G to the seventh, A to the eighth, B to the ninth, and so on. According to this arrangement, whatever letter designates any given day of the week in the first part of the year, continues to designate the same throughout the year. The letter designating the first day of the week, or Sunday, is called the *Dominical Letter*.

11. As a common year consists of 365 days, or 52 weeks and 1 day, the last day of each common year must fall on the same day of the week as the first, and the next year must commence one day later in the week. Consequently the day of the week which was the first day of the former year, and was designated by A, is the seventh day of the second year, and is designated by G; that which was the second, and was designated by B, in the former year, is the first, and is designated by A in the second, and so on. It therefore follows, that whatever letter is the dominical letter, in any common year, the letter next preceding it in the alphabet is the dominical letter in the following year; except the former was A, in which case the second is G.

12. In every common year, the first day of March is the 60th day of the year, and consequently corresponded to the letter D. In bissextile years, on account of the intercalation, the 1st of March is the 61st day of the year; but the letter D was still made to correspond to it, and the

letters for the remaining part of the year were arranged accordingly. It therefore follows that, after the 29th of February, any given day of the week was designated by the letter in the alphabet next preceding that by which it was designated in the first two months. Consequently a bissextile had two dominical letters, one of which appertained to January and February, and the other, which was the next preceding letter in the alphabet, appertained to the other ten months.

13. From what has been said, it follows that the dominical letters succeed one another in a retrograde order, that is, in the order G, F, E, D, C, B, A, G, F, &c.; and that each bissextile has two, in the same order.

It is now usual to retain only the dominical letter in the calendar, and to designate the other days of the week by numbers or by their names.

14. The year 1800, which was a common year, commenced on the fourth day of the week, and consequently the dominical letter was the 5th of the alphabet, which is E. From thence, taking into consideration that every four years, in which a bissextile is included, requires five dominical letters in a retrograde order, it is easy to find the dominical letter for any year in the present century. To do this, multiply the number of years above 1800, by 5, and divide the product by 4, neglecting the remainder. Divide the quotient by 7, and subtract the remainder from 5; or from 12, when the remainder is equal to or greater than 5. The last remainder is the number of the dominical letter.

Delambre, in the 38th chapter of his Astronomy, has given the investigation of a formula for finding the dominical letter in any century, according to the Gregorian calendar.

15. There are some periods of time, which, though they are not now much used, it may be proper briefly to notice.

16. The *Solar Cycle* is a period of 28 years, in which, according to the Julian calendar, the days of the week

return to the same days of the month, and in the same order. The first year of the Christian era was the 10th of this cycle. Consequently, if 9 be added to the number of any year, and the sum be divided by 28, the remainder will be the number of the year of the solar cycle. When there is no remainder, the year is the 28th of the cycle.

17. The *Lunar Cycle*, or as it is sometimes called, the *Metonic Cycle*, is a period of 19 years, in which the conjunctions, oppositions, and other aspects of the moon, return on the same days of the year. The synodic revolution of the moon being 29.5305885 days, 235 revolutions are 6939.688 days; which differs only about an hour and a half from 19 Julian years. The number by which the year of the lunar cycle is designated, is frequently called the *Golden Number.*

The first year of the christian era was the 2d of the lunar cycle. Hence to find the year of the cycle, for any given year, add 1 to the number of the year, and divide by 19. The remainder expresses the year of the cycle. If nothing remains, the year is the 19th of the cycle.

18. The *Cycle of the Indiction* is a period of 15 years. This period, which is not astronomical, was introduced at Rome, under the emperors, and had reference to certain judicial acts.

To find the cycle of the indiction for a given year, add 3, and divide by 15. The remainder expresses the year of the cycle.

19. The *Julian Period* is a period of 7980 years, obtained by taking the continued product of the numbers, 28, 19, and 15. After one Julian period the different cycles of the sun, moon, and indiction, return in the same order, so as to be just the same in a given year of the period, as in the same year of the preceding period. The first year of the Christian era was the 4714th of the Julian period. Hence if 4713 be added to the number of a given year, the result will be the year of the Julian period.

20. The *Epact*, as an astronomical term, is the mean age of the moon at the commencement of a year, or in other

words, it is the interval between the commencement of the year and the time of the last mean new moon; and is expressed in days, hours, minutes, and seconds.

21. The *Epact*, as given in the calendar, is nearly the age of the moon at the commencement of the year, expressed in whole days, and was introduced for the purpose of finding the days of mean new and full moon throughout the year, and thence the times of certain festivals. Without entering into any explanation of the reason of the rule, it must suffice here to observe, that the *Epact* for any year during the present century may be found by multiplying the golden number of the year by 11, adding 19 to the product and dividing the sum by 30. The remainder is the *Epact* for the year.

CHAPTER XVIII.

UNIVERSAL GRAVITATION AND SOME OF ITS EFFECTS.

1. It is designed to give in this chapter a general view of some of the effects of the attraction of gravitation, without entering into very minute investigations. The propositions, contained in the first four of the following articles, are demonstrated in treatises on mechanics.

2. If a body, put in motion, be urged towards a fixed point, not in the direction of its motion, by a force continually acting upon it, it will move in a curve; and the straight line drawn from the body to the point, will describe areas proportional to the times.

3. Conversely, if a body move in a curve, in such manner that the straight line drawn from it, to some point, describes areas proportional to the times, the body is urged towards the point by a force continually acting on it.

By Kepler's first and second laws, the planets revolve in curves about the sun, and their radius vectors describe areas proportional to the times. Consequently the planets are urged towards the sun by forces continually acting on them.

4. If a body revolving about a point, be continually urged towards that point by a force which varies inversely as the square of the distance, it will move in an *Ellipse* or some other of the curves, called *Conic Sections*.

5. If a body, continually urged by a force directed to some point, describe an ellipse of which that point is a focus, the force must vary inversely as the square of the distance.

It therefore follows, from Kepler's second law, that each planet is continually urged towards the sun, by a force which varies inversely as the square of the distance from the sun's centre.

6. Since each planet is urged towards the sun by a force varying inversely as the square of the distance, it is reasonable to suppose, instead of a distinct force for each planet, a single force residing in the sun, and varying from planet to planet according to the same law.

7. By taking into view Kepler's third law, for the motions of the planets, it is proved that the sun is the centre of a force, which, acting on the particles of matter in all the planets, and varying in intensity, inversely as the square of the distance from the sun's centre, retains them in their orbits.

8. As the motions of the satellites of Jupiter, Saturn, and Uranus, are conformable to Kepler's third law, it is proved in like manner that each of these planets is the centre of a force which, varying in intensity inversely as the square of the distance from the centre of the planet, extends to the satellites and retains them in their orbits.

9. The earth has but one satellite, and therefore Kepler's third law does not apply to it. But by investigations, founded on the distance which a heavy body falls at the earth's surface in one second of time, compared with the distance which the moon recedes in the same time from a tangent to its orbit, towards the earth, it is proved that

the force of gravity, varying inversely as the square of the distance, extends to the moon and retains it in its orbit.

10. The existence of a similar force, in each of the planets that have no satellites, is inferred from the effects which they are known to produce on one another and on the other planets.

11. The circumstances mentioned in the preceding articles serve to prove that all particles of matter are urged towards one another, with a force which varies inversely as the square of the distance. This force is called the *Force of Gravitation.*

12. A *Projectile Force* is the force by which a body is put in motion.

13. A *Centripetal Force* is the force by which a body revolving about another body is urged towards it.

14. A *Centrifugal Force* is the force by which a body revolving about another body tends to recede from it.

15. *Centripetal* and *Centrifugal* forces are called *Central Forces.*

RELATIVE MASSES OF THE PLANETS.—RELATIVE WEIGHT OF A BODY AT THEIR SURFACES.

16. The relative quantities of matter or masses of the sun, planets, and satellites, may be determined with considerable accuracy, from the effects which they produce in disturbing the motions of each other. For these effects depend on the quantities of matter of the disturbing bodies and on their distances; and the distances are known from the methods of plane astronomy.

17. The masses of those planets which have satellites may be found in a simpler manner and with greater accuracy. If 1 denote the mass of the sun, M the mass of a planet, m the mass of one of its satellites, D the mean distance of the planet from the sun, d the mean distance of the satellite from the planet, and P and p the periodic times of the planet and satellite respectively; then it is proved that

$$\frac{M+m}{1+M}=\frac{d^3}{D^3}\,\frac{P^2}{p^2}.$$

As the mass of the satellite is small compared with that of the planet, and the mass of the planet is small compared with that of the sun, we have,

$$M = \frac{d^3}{D^3} \cdot \frac{P^2}{p^2} \text{ very nearly.}$$

18. The following table exhibits the relative quantities of matter or masses of the sun and planets as given by Laplace in the fifth edition of his *Systeme Du Monde.*

Sun	1
Mercury	$\frac{1}{2025810}$
Venus	$\frac{1}{405871}$
The Earth	$\frac{1}{354936}$
Mars	$\frac{1}{2546320}$
Jupiter	$\frac{1}{1070.5}$
Saturn	$\frac{1}{3512}$
Uranus	$\frac{1}{17918}$

If the mass of the earth be denoted by 1, the mass of the moon, according to the most accurate determination, is $\frac{1}{74 \cdot 95}$.

19. The densities of bodies are proportional to their quantities of matter, divided by their bulks. The following table contains, approximately, the densities of the sun, moon, and planets, the density of the earth being denoted by 1.

Sun	0.252
Mercury	2.585
Venus	1.024
The Earth	1.000
The Moon	0.615
Mars	0.656
Jupiter	0.201
Saturn	0.103
Uranus	0.218

20. Supposing the planets to be exactly spherical and not to revolve on their axes, the weight of the same body at their different surfaces would be proportional to their quantities of matter, divided by the squares of their diameters. But the centrifugal force, at the surface of a planet that revolves on its axis, diminishes the weight of a body, placed on it, particularly near the equator. The diminution thus produced, on any of the planets, is not, however, very considerable. The following table, taken from Vince's Astronomy, exhibits the relative weight, nearly, of a body at the surface of the sun and planets, its weight at the surface of the earth being denoted by 1.

Sun	27.70
Mercury	1.70
Venus	0.98
The Earth	1.00
Mars	0.34
Jupiter	2.33
Saturn	1.02
Uranus	0.93

THE CENTRE OF GRAVITY OF THE SOLAR SYSTEM.

21. As all particles of matter attract each other, the sun must be attracted towards a planet, in like manner as the planet is towards the sun. But as the quantity of matter in the sun is far greater than that in any of the planets, its attraction at a given distance must be proportionably greater.

22. If there were only one planet, the sun and that planet would describe similar ellipses, of which their common centre of gravity would be one of the foci; their distances from that point being always inversely as their quantities of matter. As there are several planets revolving round the sun, the path of the sun's centre must be a more complicated curve. But the quantity of matter in all the planets, taken together, being very small, compared with that in the sun, the extent of the curve described by the sun's centre cannot be very great.

23. It is found by computation, that the distance between

the sun's centre and the centre of gravity of the system, can never be equal to the sun's diameter.

24. It is proved by writers on mechanics that the centre of gravity of a system of bodies is not affected by the mutual actions of these bodies on one another; and that unless there are extraneous actions, the centre of gravity will either remain at rest or move uniformly in a right line.

25. From the minute changes in the situations of some of the fixed stars, called their proper motions (14.21), Dr. Herschel inferred that the centre of gravity, and consequently the whole system of the sun and planets, is in motion towards the constellation Hercules. The observations hitherto made, are not, however, considered sufficient to prove the existence of such a motion.

KEPLER'S LAWS.

26. Kepler's laws, with regard to the motions of the planets, have been thus far considered as rigorously true. It may now be proper to inform the student that the mutual actions of the heavenly bodies on each other cause slight deviations from those laws, as they are stated in the preceding part of the work.

27. If the radius vector and mean distance of a planet be reckoned from the centre of gravity of the system, to the centre of the planet, or when the planet has satellites, to the centre of gravity of the planet and satellites, the first and second laws will hold true, excepting so far as the motion of the planet is affected by the actions of the others.

28. The third law, as applied to any two of the planets, is affected not only by the actions of the other planets, but also by the quantities of matter in the two planets themselves. If p and P be the periodic revolutions of any two of the planets, a and A their mean distances from the centre of gravity of the system, and m and M their quantities

of matter, that of the sun being denoted by 1, then, disregarding the actions of the other planets,

$$p^2 : P^2 :: \frac{a^3}{1 + m} : \frac{A^3}{1 + M}.$$

PROBLEM OF THE THREE BODIES.

29. If we suppose only two bodies to gravitate towards each other, with forces inversely as the squares of their distances, and to revolve about their common centre of gravity, they would move in conic sections, and the radius vectors would describe areas proportional to the times; the centre of gravity either remaining at rest or moving uniformly in a right line. But if there are three bodies, the action of any one on the other two changes the nature of their orbits, so that the determination of their motions becomes a problem of the greatest difficulty, distinguished by the name of *The Problem of the Three Bodies.*

30. The solution of the problem of the three bodies, in its utmost generality, is not within the power of the mathematical sciences as they now exist. Under certain limitations, however, and such as are quite consistent with the condition of the heavenly bodies, it admits of being resolved. These limitations are, that the force which one of these bodies exerts on the other two, is, either from the smallness of that body, or its great distance, very inconsiderable, in respect to the forces which these two exert on one another.

31. The force of the third body is called *a disturbing force,* and its effects in changing the places of the other two bodies are called *the disturbances of the System.*

32. Though the small disturbing forces may be more than one, or though there be a great number of remote disturbing bodies, the computation of their combined effect arises readily from knowing the effect of one; and therefore the problem of three bodies, under the conditions just stated, may be extended to any number.

33. The problem of the three bodies has exercised the

ingenuity of several of the most eminent mathematicians. But Laplace, in the *Mecanique Celeste*, has extended the solution farther than any other person. He has given a very complete investigation of the inequalities, both of the planets and satellites.

INEQUALITIES OF THE MOON.

34. The moon is attracted at the same time by both the earth and sun; it is only, however, the difference between the gravitations of the earth and moon towards the sun that disturbs the motion of the moon about the earth. If the sun were at an infinite distance, they would be attracted equally, and in parallel straight lines; and in that case their relative motions would not be in the least disturbed. But his distance, although very great in respect of that of the moon, yet cannot be supposed infinite; the moon is alternately nearer the sun, and farther from him than the earth, and the straight line which joins her centre and that of the sun, forms with the terrestrial radius vector an angle which is continually varying; thus the sun acts unequally and in different directions on the earth and moon, and from this diversity of action there must result inequalities in her motion which depend on her position in respect of the sun.

35. At the quadratures, the gravity of the moon to the earth is increased in consequence of the sun's action, by a quantity equal to the product of the mass of the sun, by the distance of the moon from the earth, divided by the cube of the earth's distance from the sun; at the syzigies it is diminished by twice this quantity; and the effect in the whole, is a diminution of the moon's gravity, equal to the product of the sun's mass by the moon's mean distance from the earth, divided by twice the cube of the earth's distance from the sun. And the value of the mean diminution is equal to a 368th part of the whole gravity of the moon to the earth.

It is a well known proposition in mechanics, that if AB and AD, *Fig.*

50, represent the quantities and directions of two forces acting on a point or body at A, and the parallelogram ABCD be completed, the diagonal AC will represent the quantity and direction of a single force which would produce the same effect as the two forces. The substitution of a single force as the equivalent of two others, is called the *Composition of Forces.*

On the contrary, if AC represent the quantity and direction of a single force acting on a body at A, and any parallelogram ABCD is described about AC as a diagonal, the adjacent sides AB and AD will represent the quantities and directions of two forces that are just equivalent to the single force. The substitution of two forces, as the equivalent of a single force, is called the *Resolution of Forces.*

Let ACBO, *Fig.* 51, represent the orbit of the moon, which may in this investigation be considered as coinciding with the plane of the ecliptic. Also let S be the sun, E the earth, M the place of the moon in her orbit, and AB perpendicular to SE, the line of the quadratures. Let the line SE represent the force which the sun exerts on the earth at E, or on the moon, when in quadratures, at A and B*. Then, $SM^2 : SE^2 : SE : \frac{SE^3}{SM^2}$ = the force with which the sun acts on the moon at M. In the line MS, produced if necessary, take $MD = \frac{SE^3}{SM^2}$; then MD represents the force which the sun exerts on the moon at M. Let the force MD be resolved into the two MH and MG, one of which, MH, is equal and parallel to ES. Then since the force MH is equal and parallel to ES, it will have no tendency to change the relative motions or positions of the earth and moon. The other force MG, will therefore represent, in quantity and direction, the whole effect of the sun's action in disturbing the moon's motion in her orbit. Let SM be produced to meet the diameter AB in N. Then because the angle ESN is very small, being when greatest only about 7′, the line SN may be considered equal to SE. Hence,

$$MD = \frac{SE^3}{SM^2} = \frac{SN^3}{SM^2} = \frac{(SM + MN)^3}{SM^2}$$
$$= \frac{SM^3 + 3\,SM^2 \times MN + 3\,SM \times MN^2 + MN^3}{SM^2}.$$

But as MN is very small compared with SM, the two terms $3\,SM \times MN^2$, and MN^3 may be omitted. Therefore,

$$MD = \frac{SM^3 + 3\,SM^2 \times MN}{SM^2} = SM + 3\,MN.$$
$$\text{Or, } SD = 3\,MN.$$

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* Strictly speaking, as the quantity of matter in the earth is greater than that in the moon, the forces which the sun exerts on the earth and moon when at equal distances, are not equal. But the effects of those forces, in moving the bodies, are equal, and it is these effects which is the subject under consideration.

As the angle ESM is very small, and SD is also small, the line DG must very nearly coincide with SE, and consequently the point G with the point L. We may therefore consider ML as the force by which the sun disturbs the motion of the moon. Now,

$$EL + LS = ES = HM = DG = SD + LS, \text{ very nearly,}$$
$$\text{or, } EL = SD, \text{ very nearly.}$$

Hence, if MK be perpendicular to SE, we have,

$$EL = 3\ MN = 3\ EK.$$

Let the force ML be resolved into two others, one MQ in the direction of the radius vector, and the other MP in the direction of a tangent to the orbit at M. Then the force MQ increases or diminishes the gravity of the moon to the earth, according as the point Q falls between E and M, or in EM produced. The other force MP increases or diminishes the moon's angular motion about the earth. Since the moon's orbit does not differ much from a circle, the angle QMP may be considered as a right angle. Put $a = SE$, $r = EM$, and $x =$ the angle AEM, Then,

$$EK = EM \cos MEK = r \sin x,$$
$$EL = 3\ EK = 3\ r \sin x,$$

$$PM = LQ = EL \cos x = 3\ r \sin x \cos x = \frac{3r}{2} \sin 2\ x, \qquad \text{(App. 7).}$$

$$\text{Also, } EQ = EL \sin x = 3\ r \sin^2 x,$$
$$MQ = EQ - EM = 3\ r \sin^2 x - r = -r\,(1 - 3 \sin^2 x).$$

Or using the affirmative sign to denote an increase in the moon's gravity to the earth,

$$MQ = +\,r\,(1 - 3 \sin^2 x).$$

Now if $m =$ the mass of the sun, then the force which the sun exerts on the earth may be expressed by $\frac{m}{a^2}$. Hence,

$$ES : PM :: \frac{m}{a^2} : \text{the force PM. Therefore,}$$

$$\text{the force } PM = \frac{m}{a^2} \cdot \frac{PM}{ES} = \frac{m.3r \sin 2x}{2\,a^3} = \frac{3mr}{2\,a^3} \sin 2\ x. \qquad \text{(A).}$$

In like manner,

$$\text{the force } MQ = \frac{m}{a^2} \cdot \frac{MQ}{ES} = \frac{mr}{a^3} \cdot (1 - 3 \sin^2 x). \qquad \text{(B).}$$

When the moon is in quadratures, $x = 0$ or 180°, and consequently,

$$\text{The force } MQ = +\frac{mr}{a^3}.$$

When the moon is in syzigies, $x = 90°$ or 270°, and, therefore,

$$\text{The force } MQ = -\frac{2mr}{a^3}.$$

The force MQ is $= 0$, when $3 \sin^2 x = 1$, or $\sin x = \sqrt{\frac{1}{3}}$; that is, when $x = 35° \ 15' \ 52''$.

The moon's gravity to the earth is therefore increased while she is within about 35° of the quadratures, on either side, and is diminished in all the remaining part of the orbit; and the greatest diminution is double the greatest increase. It follows, therefore, that in the whole the moon's gravity to the earth is diminished by the action of the sun. An easy investigation, with the aid of differential calculus, proves that the mean diminution is $\frac{mr}{2a^3}$; r representing in this case the mean distance of the moon from the earth. The value of this expression is readily found to be equal to the 368th part of the whole gravity of the moon to the earth, nearly.

36. From the diminution of her gravity by a 368th part, the moon describes her orbit at a greater distance from the earth, with a less angular velocity, and in a longer time, than if she were acted on only by the attraction of the earth.

37. The inequality in the moon's motion, called the *Annual Equation*, (10.29), is the effect of the variation in the distance of the earth from the sun.

Since, in the expression $\frac{mr}{2a^3}$, which designates the mean diminution in the moon's gravity to the earth, the quantities m and r are constant, it follows that the mean diminution is inversely proportional to the cube of the earth's distance from the sun. Hence as the earth approaches the perihelion, its distance diminishing, the mean diminution of the moon's gravity to the earth must increase; the moon's distance from the earth must become greater than it otherwise would be; and consequently its angular motion must be slower. The contrary takes place as the earth approaches the aphelion.

38. The *Evection* is produced by an inequality in the sun's disturbing force, depending on the variation in the moon's distance from the earth, and on the position of the moon with respect to the line of the syzigies.

Let R and r denote the distances of the moon from the earth, in apogee and perigee, when the line of the apsides coincides with the line of the syzigies, X and x, the distances at which the moon would be from the earth, in apogee and perigee, if she was not acted on by the sun, and G and g the perigean and apogean gravities in that case. Also put $n = \frac{m}{a^3}$; and supposing the earth's distance from the sun to remain constant, n will be constant. Then (35), $G - 2rn$ and $g - 2Rn$ will be the perigean and apogean gravities of the moon, when the line of the apsides coincides with the line of the syzigies. Hence,

$$X^2 : x^2 :: G : g,$$

$$\text{and } R^2 : r^2 :: G - 2rn : g - 2Rn.$$

Consequently,

$$\frac{X^2}{x^2} = \frac{G}{g},$$

$$\text{and } \frac{R^2}{r^2} = \frac{G - 2rn}{g - 2Rn}.$$

Now as G is greater than g, and $2rn$ less than $2Rn$, it is evident that,

$$\frac{G - 2rn}{g - 2Rn} \text{ is greater than } \frac{G}{g}.$$

$$\text{Hence, } \frac{R^2}{r^2} \text{ is greater than } \frac{X^2}{x^2},$$

It therefore follows, that when the line of the apsides coincides with the line of the syzigies, the ratio of the apogean distance of the moon to the perigean distance, and consequently the eccentricity of the orbit is increased by the action of the sun. In like manner it may be shown that when the line of the apsides coincides with the line of the quadratures, the sun's action diminishes the eccentricity of the orbit. The change in the eccentricity of the orbit produces a change in the equation of the centre; which change is the evection.

39. The *Variation* is produced by a part of the sun's disturbing force, which acts in the direction of a tangent to the moon's orbit.

It has been shown (35.A), that MP, the part of the sun's force which acts in the direction of a tangent to the orbit, is equal to $\frac{3mr}{2a^3} . \sin 2x$. Hence, supposing the earth's distance from the sun and the moon's distance from the earth to remain constant, this force is proportional to sin $2x$; that is, to the sine of twice the distance of the moon from the quadratures. It is therefore greatest in the octants and nothing in the syzigies and quadratures.

Supposing the moon to set out from the quadrature A, the tangential force MP continually accelerates her motion, till she arrives at the sizigy C; the force then changes its direction and retards her motion. Consequently at C the motion is greatest. As the moon advances from C, her motion is continually retarded till she arrives at B, where it is least. It is then accelerated till it becomes greatest at O, and again retarded till it become least at A. Hence, as the motion is greatest in the syzigies and least in the quadratures, and as the degree of retardation is the same as that of acceleration, we may infer that the mean motion* has place when the moon is in the octants.

Now as the moon moves from the quadrature A with a motion less than her mean motion, her mean place will be in advance of her true place, and will become more and more so, till at the octant the true motion is equal to the mean. The difference between the true and mean places is then the greatest. For after that, the true motion being greater than the mean, the true place will approach nearer to the mean, till at the syzigy C, they coincide. It is equally plain, that at the octant between C and B, the moon's true place will be most in advance of the mean place, and that at B, they will again coincide. Corresponding effects take place in the two remaining quadrants.

40. The inequality called the *Acceleration of the Moon* (10.21), by which her velocity appears subject to continual increase, and her period to continual diminution, has been found by Laplace to be a *Secular* Equation, depending on a change in the eccentricity of the earth's orbit, produced by the actions of the planets, and which requires several thousand years to go through its different values.

MOTION OF THE APSIDES OF THE MOON'S ORBIT.

41. The motion of the apsides is produced by the action of the sun, in diminishing the moon's gravity to the earth.

If the moon was only acted on by the earth's attraction, it would describe an ellipse, and its angular motion would be just 180°, from one apsis to the other; or which is the same, from one place where the orbit cuts the radius vector at right angles, to the other. But in consequence of the change produced in the moon's gravity to the earth, by the action of the sun, the moon's path is not an ellipse. When the effect of the sun's action is a diminution of the moon's gravity, she will continually recede from the ellipse that would otherwise be described, her path will be less bent, and she must move through a greater distance before the radius vector intersects the path at right angles. She must therefore move through a greater angular distance than 180°, in going from one apsis to the other, and consequently the apsides will advance. On the contrary, when the gravity is increased by the sun's action, the moon's path will fall within the ellipse which she would otherwise describe, its curvature will be increased, and the distance through which she must move before the radius vector intersects her path at right angles, will be less. The apsides will therefore move backwards. Now it has been shown (35) that the sun's action alternately diminishes and increases the moon's gravity to the earth. The motion of the apsides will therefore be alternately direct and retrograde. But as the diminution has place during a much longer part of the moon's revolution, and is besides greater than the increase, the direct motion will exceed the retrograde. Consequently, in an entire revolution of the moon, the apsides have a progressive motion.

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* The expressions, mean place, true place, mean motion, and true motion, are here to be understood only in relation to the present inequality.

MOTION OF THE MOON'S NODES, AND CHANGE IN THE INCLINATION OF THE ORBIT.

42. The direction, in which the sun's disturbing force acts on the moon, does not, except in some particular cases, coincide with the plane of the moon's orbit; this force therefore produces a tendency in the moon to quit that plane, one of the effects of which is a change in the position of the line of the nodes; and another, is a change in the inclination of the plane of the orbit to that of the ecliptic.

Let OL, *Fig.* 52, be the line passing through the centres of the earth and sun, and IN′ the line of the nodes. These two lines lie in the plane of the ecliptic, which we may consider as designated by the plane of the paper. Let EMHI conceived to be, from EI, above the plane of the paper, be the plane of the moon's orbit, NM a part of the northern half of the orbit, and AB a plane, seen edgewise, perpendicular to the line EL. When the moon is in this latter plane it is in quadrature.

Let ML designate the quantity and direction of the sun's disturbing force when the moon is at M. Now when the line of the nodes coincides with OL the line of the syziges, ML will coincide with the plane of the moon's orbit, and will therefore have no tendency to make the moon deviate from that plane. Also, since EL is equal to three times the distance of the moon from AB (34), when the moon is in the plane AB, that is when she is in quadrature, L will coincide with E, and consequently ML will be in the plane of the orbit, and will have no tendency to make the moon move from it. At all other times, the force ML, not acting in the plane of the orbit, will tend to make the moon quit that plane; or instead of supposing the moon continually passing from one plane to another, we may conceive the plane itself to change its position.

Let LH be drawn perpendicular to IHME the plane of the moon's orbit. Then if MH be joined, and the parallelogram MHLK be completed, the lines MH and MK will represent, in quantities and directions, two forces that are together equivalent to ML. The force MH acting in the plane of the orbit, has no tendency to change the position of that plane. The tendency of the other force MK, acting at right angles to the plane of the orbit, will be to bend the moon's path towards the ecliptic, or from it. When the effect of the force MK is to bend the moon's path towards the ecliptic, the moon will meet the ecliptic sooner than it would otherwise do, and consequently the node will move backward. On the contrary, when the force MK bends the moon's path from the ecliptic, the moon will not meet the ecliptic so soon as it would otherwise do, and therefore the node will move forward. Now it is plain that when the points L and M are on the same side of the line of the nodes, the force MK tends to make the moon's path bend towards the ecliptic; and when they are on opposite sides, it tends to make the path bend from the ecliptic. Hence when the points L and M are on the same side of the line of the nodes, the motion of the nodes is retrograde; and when on opposite sides, it is direct.

When the line of the nodes has the position NN′ the points L and M will be on the same side of it, while the moon is moving from the node N to the next quadrature in EB; and therefore the motion of the nodes is retrograde. When the moon has passed the quadrature, the point L falls on the other side of E, in EO; and therefore while the moon is moving from the quadrature to the next node in EN′, the points L and M will be on opposite sides of the line of the nodes, and the motion of the nodes will be direct. While the moon is moving from the node in EN′ to the quadrature in EA, the motion of the nodes will be again retrograde; and while she is moving from the quadrature in EA, to the node in EI it will be direct. Hence, while the moon is moving from the nodes to the quadratures, the motion of the nodes is retrograde; and while she is moving from the quadratures to the nodes it is direct. It is therefore plain that in this position of the line of the nodes, the retrograde motion has place during a longer portion of the moon's synodic revolution, than the direct motion.

When the line of the nodes has the position *nn′* it is easy to determine, from what has been said, that the motion of the nodes will be direct while the moon is moving from the nodes to the quadratures; but retrograde while she is moving from the quadratures to the nodes; and therefore, that, in the whole synodic revolution of the moon, the retrograde motion has place during a longer time than the direct motion.

It appears then that in each synodic revolution of the moon, the nodes alternately retreat and advance, but that in all cases, except when the line of the nodes nearly coincides with the line of the syzigies, the motion is retrograde during a longer time than it is direct.

Let the plane LIH be perpendicular to IN the line of the nodes. Then the angle LIH is in the inclination of the plane of the moon's orbit to the ecliptic. As LH is perpendicular to the plane of the orbit, the angle IHL is a right angle. Put x = the moon's angular distance from the quadratures, S = IEL, the angle contained between the line of the nodes and the line of the syzigies, I = LIH the inclination of the orbit, and r = EM = radius vector of the moon. Then (35), EL = $3r \sin x$. Hence,

$$LI = EL \sin LEI = 3r \sin x \sin S,$$
$$MK = LH = LI \sin LIH = 3r \sin x \sin S \sin I.$$

Or, (35), using $\frac{m}{a^2}$ to denote the force exerted by the sun on the earth,

$$\text{The force } MK = \frac{m.MK}{a^3} = \frac{3mr}{a^3} \sin x \sin S \sin I.$$

Now during any one revolution of the moon, none of the quantities which enter into the expression for the force MK, varies much, except sin x. And it is easy to perceive, by reference to the figure, that sin x, and consequently the force MK, always acquires its greatest value during the time the motion of the nodes is retrograde.

As in each synodic revolution of the moon, the nodes retreat during a longer time than they advance, and as the force which causes the motion is greatest while they retreat, the retrograde motion must exceed the direct motion, and the result in the whole must be a retrograde movement of the nodes.

When the tendency of the force MK is to bend the moon's path towards the ecliptic, if the moon is then moving from the node to the 90° from it, the inclination of the orbit will be diminished; but if she is moving from the 90° to the node, the inclination will be increased. On the contrary, when the tendency of the disturbing force is to bend the path from the ecliptic, the inclination of the orbit will be increased when the moon is moving in the first 90° from the node, and will be diminished when she is moving from 90° to the node. Hence when NN′ is the line of the nodes, if the moon set out from the quadrature in EA, the inclination of the orbit will be continually diminished till she is 90° past the node N; and will then be increased till she arrives at the quadrature in EB; from thence to the 90° past the node in EN′, the inclination will be again diminished, and will then be increased till she again arrives at the quadrature in EA. The diminution will therefore be greater than the increase. But when nn' is the line of the nodes, if the moon set out from the quadrature in EA, the inclination will only be diminished till she arrives at the 90° from the node in En', and will be increased from thence to the quadrature in EB; it will then be diminished, till she is 90° from the node in En, and will be increased from thence till she returns to the quadrature in EA. The increase will therefore exceed the diminution. Thus in some synodic revolutions of the moon the inclination of the orbit is diminished, and in others it is increased as much. The result is a mean inclination which does not change.

43. Disturbances in the motions of the earth and planets are necessary effects of the actions of these bodies on one another; but it is not designed to take any other notice of them here, than to mention one important fact.

44. Lagrange and Laplace have proved that no terms, except those which alternately increase and diminish, can enter into the expressions for the disturbances of the planets. This proves that the planetary system is stable; that it does not involve any principle of destruction in itself, but is calculated to endure for ever, unless the action of an external power is introduced.

FIGURE OF THE EARTH.

45. It has already been inferred from measurements, (4.8), that the figure of the earth is an oblate spheroid, of which the greater axis, that is, the diameter of the equator, is to the less, the axis of revolution, as 300 to 299.

46. Since the earth revolves on its axis, it is evident that its parts are all under the influence of a centrifugal force, varying with their distances from that axis, and that if the whole were a fluid mass, the columns towards the equator, being composed of parts that, having a greater centrifugal force, tend more to recede from the axis, must extend in length, in order to balance the columns in the direction of the axis. By this means an oblateness or elevation at the equator would be produced, similar, in some degree at least, to that which the earth has been found to possess.

47. A homogeneous fluid of the same mean density with the earth, and revolving on its axis in the same time that the earth does, would be in equilibrium, if it had the figure of an oblate spheroid, of which the axis was to the equatorial diameter as 229 to 230.

48. If the fluid mass, supposed to revolve on its axis, be not homogeneous, but be composed of strata that increase in density towards the centre; the solid of equilibrium will still be an elliptic spheroid, but of less oblateness than if it were homogeneous.

49. Hence, as the ellipticity of the earth is less than $\frac{1}{230}$, being about $\frac{1}{300}$, it is evident, that if the earth is a spheroid of equilibrium, it is denser towards the interior.

50. The greater density of the earth towards the centre has been proved by very accurate observations made on the sides of the mountain Schehallien, in Scotland, by Dr. Maskelyne. From the effect of the mountain in changing the direction of a plumb line suspended near it, and from the known figure and bulk of the mountain determined by a survey, it was found that the mean density of the mountain was to the mean density of the earth nearly as 5 to 9.

51. The inequalities on the surface of the earth, and the unequal distribution of the rocks which compose it, with respect to density, must produce great local irregularities in the direction of the plumb line, and are probably in part the causes of the inequalities observed in the measurement of contiguous arches of the meridian, even when the work has been conducted with the greatest skill and accuracy.

52. From accurate observations of the lengths of pendulums oscillating seconds at places in different latitudes, the relative force of gravity at the places may be determined, and from thence the ellipticity of the earth.

PRECESSION OF THE EQUINOXES AND NUTATION OF THE EARTH'S AXIS.

53. The physical investigation of the precession of the equinoxes is a subject of considerable difficulty. It must suffice here, just to state that the precession is produced by the actions of the sun and moon on those parts of the earth which are on the outside of a sphere, conceived to be described about the earth's axis.

54. The sun's action produces a retrograde movement of the equinoctial points, which is nearly, but not quite uniform. This movement may be separated into two parts; one, a continued mean precession of the equinoxes; the other, an inequality in the precession called the *Solar Nutation* in precession. The inequality in the sun's action

occasions a very small change in the obliquity of the ecliptic, called the *Solar Nu'ation* in the obliquity.

55. The moon's action produces effects similar to those produced by the sun, but greater in degree. One effect is a mean precession of the equinoxes, which, combined with the mean precession produced by the sun, forms the whole *Mean Precession of the Equinoxes.* Another effect is an inequality in the precession called the *Lunar Nutation*, and sometimes, the *Equation of the Equinoxes.*

SECULAR VARIATION OF THE OBLIQUITY OF THE ECLIPTIC.

56. The orbits of the planets not coinciding with the plane of the ecliptic, their actions on the earth tend to make it quit that plane. The effect is, a small variation in the position of the plane of the ecliptic. From these causes, the obliquity of the ecliptic has been, and still continues to be, diminished. The diminution at the present period is about 52″ in a century. In process of time the *same causes* must produce an *increase* in the obliquity.

57. The secular variation of the obliquity of the ecliptic was less in former ages than it is at present. It has now acquired nearly its greatest value, and will begin to decrease about the 23d century of our era. Lagrange has shown that the total diminution in the obliquity, reckoning from that in 1700, must be less than 5° 23′.

DIURNAL ROTATION.

58. It is proved by minute investigation that the actions of the sun and moon, combined with the change in the position of the ecliptic, must produce changes in the duration of a revolution of the earth on its axis, that is, in the length of the day. But the same investigation also proves that those changes are so indefinitely small, that, being periodical, they can never become sensible, even to the nicest observation.

59. When, from the washing of rains, or from other causes, any matter is made to descend from the higher parts of mountains to a position that is nearer the earth's

axis, its velocity will be diminished, and the velocity lost being communicated to the mass must tend to accelerate the diurnal motion. But no changes known to us, in the position of the matter of the earth, can ever produce any sensible alteration in the earth's rotation on its axis.

60. The conclusion drawn from a full examination of the subject is, that the duration of the earth's rotation may be regarded as perfectly unchangeable.

OF THE TIDES.

61. The alternate rise and fall of the surface of the ocean, twice in the course of a lunar day, or of 24 h. 50 m. 48 sec. of mean solar time, is the phenomenon known by the name of the *Tides*. The rise of the water is called the *flowing* of the tide, and the fall is called the *ebbing*.

62. The time from one high water to the next is, at a mean, 12 h. 25 m. 24 sec. The instant of low water is nearly, but not exactly, in the middle of this interval; the tide, in general, taking nine or ten minutes more in ebbing than in flowing.

63. The time of high water is principally regulated by the position of the moon, and in general, in the open sea, is from two to three hours after that body has passed the meridian, either above or below the horizon. But on the shores of the larger continents, and where there are shallows and obstructions to the motion of the water, the interval between the time of the moon's passage of the meridian and the time of high water is very different at different places. The difference is so great, that at many places the time of high water seems to precede the moon's passage.

For any given place, the time of high water is always nearly at the same distance from that of the moon's passage over the meridian.

64. Though the tides seem to be chiefly regulated by the moon, they appear also, in some degree, to be under the influence of the sun. Thus, at the syzigies, when the sun and moon are on the meridian together, supposing other circumstances to be the same, the tides are the

highest; at the quadratures, when the sun and moon are 90° distant, the tides are the least.

65. The tides about the time of the syzigies are called the *Spring Tides;* and those about the time of the quadratures, are called the *Neap Tides.*

66. The highest of the spring tides or the lowest of the neap tides, is not the tide that has place nearest the syzigy or quadrature, but is in general the third, and in some cases the fourth following tide.

At Brest, in France, the tides of the syzigies rise to the height of 19.317 feet, and those of the quadratures only to 9.151 feet; which is not quite half the former quantity. In the Pacific Ocean, the rise in the first case is 5 feet, and in the second, between 2 and 2.5 feet.

67. The height of the tide changes with a change in the moon's distance from the earth. Other circumstances being the same, the tide is highest when the moon is in perigee, and the lowest when she is in apogee.

The tides also depend on the sun's distance from the earth, but in a less degree than on that of the moon. In our winter the spring tides are greater than in the summer, and the neap tides smaller.

68. The tides depend, to some extent, on the positions of the sun and moon with respect to the equator. When the moon is in the northern signs, the tide of the day, in all the northern latitudes, is somewhat greater than that of the night. The contrary has place when the moon is in the southern signs.

69. If the tides be considered relatively to the whole earth, and to the open sea, there is a meridian, about 30° eastward of the moon, where it is always high water, both in the hemisphere where the moon is, and in the opposite one. On the west side of this meridian the tide is flowing, on the east it is ebbing; and on the meridian at right angles to the same it is low water. In consequence of the earth's diurnal rotation, these meridians move westward; but they preserve nearly the same distance from the moon, only approaching a little nearer to her at the syzigies, and going farther off at the quadratures.

The great *wave* which, in this manner, constitutes the tide, is an *undulation* in the waters of the ocean, in which there is very little progressive motion, except when it passes over shallows, or approaches the shores.

70. The facts which have been enumerated clearly indicate that the tides are produced by the actions of the sun and moon; but in a greater degree by that of the moon.

It has been shown (35) that the sun's action increases or diminishes the moon's gravity to the earth, according to her position with respect to the line of the syzigies, or of the quadratures. In like manner, the sun's action increases or diminishes the gravity of a particle of matter at the earth's surface, according to its position with respect to a plane passing through the centre of the earth, at right angles to the line joining the centres of the earth and sun. Within about 35° of this plane on each side, the gravity at the surface is increased; and at the remaining parts, that is for about 55° around the points in which the line of the centres intersects the surface, the gravity is diminished.

Now as the particles of water easily yield to any impression, the surface of the ocean will, in consequence of the change in the gravity of its different parts, assume a figure different from that which it would otherwise have. Around the points in which the line of the centres intersects the surface, the gravity being diminished, the surface will be at a greater distance from the centre; and in the middle parts between these points, the gravity being increased, the surface will be nearer the centre. In consequence of the earth's diurnal rotation, it will successively be different parts of the surface that will thus have the distance from the centre increased and diminished. From what has been said it is easy to perceive that so far as it depends on the sun's action, it is high water at the same time in opposite parts of the earth; and that the consecutive high waters must follow each other at intervals of half a solar day.

It is evident the moon's attraction must produce effects exactly similar to those produced by the sun, and succeeding one another at intervals of half a lunar day.

To show that these effects will be greater in degree, put m and m' for the masses of the sun and moon respectively, and a and r for their distances from the earth, expressed in semidiameters of the earth. Then taking $\frac{m}{a^2}$ to represent the sun's attraction on a particle of matter at the earth's centre, $\frac{m}{(a-1)^2}$ will be the attraction on a particle at the point in the surface nearest the sun, $\frac{m'}{r^2}$ the moon's attraction on a particle at the centre, and $\frac{m'}{(r-1)^2}$, its attraction on a particle in the surface nearest the moon. Hence the difference between the sun's attraction on a particle at the centre and on one at the nearest point in the surface will be,

$$\frac{m}{(a-1)^2} - \frac{m}{a^2} = m.\frac{2\,a-1}{a^2.(a-1)^2} = \frac{2m}{a^3}, \text{ nearly.}$$

Also the difference between the moon's attraction on a particle at the centre and on one at the nearest point in the surface will be,

$$\frac{m'}{(r-1)^2} - \frac{m'}{r^2} = m'.\frac{2\,r-1}{r^2.(r-1)^2} = \frac{2m'}{r^3} \text{ nearly.}$$

Taking the values of m, m, a, and r, the value of the latter expression will be found to be nearly three times that of the former.

A similar result would be obtained for the relative effects of the attractions of the sun and moon on a particle in any other part of the surface.

71. At the time of the syzigies the actions of the sun and moon are combined in producing the tides; but at the quadratures they act in opposition to each other. The result is, much greater tides at the syzigies than at the quadratures. Observations have made known that the former are to the latter, nearly as 2 to 1, Consequently the effect of the moon's action must be to that of the sun, nearly as 3 to 1, as found above.

72. The relative effects of the actions of the sun and moon in producing the tides, must depend on their distances and masses; and as their distances and relative effects are known, their relative masses may from thence be determined.

73. Great extent is necessary, in order that the sea should be sensibly affected by the actions of the sun and moon; for it is only by the inequality of that action, on different parts of the mass of waters, that their equilibrium is disturbed; and this inequaltity cannot sensibly have place, unless a great extent of water is included. Hence in lakes and small inland seas, there are no perceptible tides.

74. The tides which are experienced in narrow seas, and on shores far from the main body of the ocean, are not produced in those seas by the direct actions of the sun and moon, but are waves propagated by the great diurnal undulation. In some bays and narrow seas, the accumulated tide thus becomes very great. At certain places in the Bay of Fundy the tide rises to the height of 60 or 70 feet.

APPENDIX TO PART I.

CONTAINING TRIGONOMETRICAL FORMULÆ.—TWO PROPOSITIONS IN CONIC SECTIONS.—ANALYTICAL FORMULÆ FOR ECLIPSES.

Many of the Trigonometrical formulæ, included in the following collection, are used in the present work. They are introduced here and numbered in order to facilitate the references. Their demonstrations may be seen in any complete treatise on Trigonometry. Nearly all of them are contained and demonstrated in a good work on the subject by Lacroix, which has been translated and published at Cambridge, New England. Those which are not contained in that work, are easily deduced from others that are.

For a single arc or angle a, the radius being $= 1$.

1. $\sin^2 a + \cos^2 a = 1$
2. $\sin a = \tan a \cos a$
3. $\sin a = \dfrac{\tan a}{\sqrt{1 + \tan^2 a}}$
4. $\cos a = \dfrac{1}{\sqrt{1 + \tan^2 a}}$
5. $\tan a = \dfrac{\sin a}{\cos a}$
6. $\cot a = \dfrac{1}{\tan a} = \dfrac{\cos a}{\sin a}$
7. $\sin a = 2 \sin \frac{1}{2} a \cos \frac{1}{2} a$
8. $\cos a = 1 - 2 \sin^2 \frac{1}{2} a$
9. $\cos a = 2 \cos^2 \frac{1}{2} a - 1$
10. $\tan \frac{1}{2} a = \dfrac{\sin a}{1 + \cos a}$
11. $\cot \frac{1}{2} a = \dfrac{\sin a}{1 - \cos a}$
12. $\tan^2 \frac{1}{2} a = \dfrac{1 - \cos a}{1 + \cos a}$

For two arcs a and b, of which a is supposed to be the greater.

13. $\sin (a \pm b) = \sin a \cos b \pm \cos a \sin b$
14. $\cos (a \pm b) = \cos a \cos b \mp \sin a \sin b$
15. $\tan (a \pm b) = \dfrac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$
19. $\sin a \cos b = \frac{1}{2} \sin (a + b) + \frac{1}{2} \sin (a - b)$
17. $\cos a \sin b = \frac{1}{2} \sin (a + b) - \frac{1}{2} \sin (a - b)$
18. $\sin a \sin b = \frac{1}{2} \cos (a - b) - \frac{1}{2} \cos (a + b)$
19. $\cos a \cos b = \frac{1}{2} \cos (a - b) + \frac{1}{2} \cos (a + b)$
20. $\sin a + \sin b = 2 \sin \frac{1}{2} (a + b) \cos \frac{1}{2} (a - b)$
21. $\sin a - \sin b = 2 \cos \frac{1}{2} (a + b) \sin \frac{1}{2} (a - b)$
22. $\cos b + \cos a = 2 \cos \frac{1}{2} (a + b) \cos \frac{1}{2} (a - b)$
23. $\cos b - \cos a = 2 \sin \frac{1}{2} (a + b) \sin \frac{1}{2} (a - b)$
24. $\tan a + \tan b = \dfrac{\sin (a + b)}{\cos a \cos b}$

25. $\tan a - \tan b = \dfrac{\sin (a-b)}{\cos a \cos b}$

26. $\cot b + \cot a = \dfrac{\sin (a+b)}{\sin a \sin b}$

27. $\cot b - \cot a = \dfrac{\sin (a-b)}{\sin a \sin b}$

28. $\tan \frac{1}{2}(a+b) = \dfrac{\sin a + \sin b}{\cos a + \cos b}$

29. $\tan \frac{1}{2}(a-b) = \dfrac{\sin a - \sin b}{\cos a + \cos b}$

30. $\cot \frac{1}{2}(a+b) = \dfrac{\sin a - \sin b}{\cos b - \cos a}$

31. $\cot \frac{1}{2}(a-b) = \dfrac{\sin a + \sin b}{\cos b - \cos a}$

32. $\dfrac{\tan \frac{1}{2}(a+b)}{\tan \frac{1}{2}(a-b)} = \dfrac{\sin a + \sin b}{\sin a - \sin b}$

For a Spherical Triangle, in which A, B, and C, are the angles, and *a*, *b*, and *c*, the opposite sides, as in *Fig.* 53.

33. $\sin A \sin b = \sin B \sin a$

34. $\cos a = \cos A \sin b \sin c + \cos b \cos c$

35. $\cos A = \cos a \sin B \sin C - \cos B \cos C$

36. $\cot a = \dfrac{\cot A \sin B + \cos B \cos c}{\sin c}$

37. $\cot A = \dfrac{\cot a \sin b - \cos C \cos b}{\sin C}$

38. $\sin \frac{1}{2} A = \sqrt{\dfrac{\sin \frac{1}{2}(a+b-c) \sin \frac{1}{2}(a+c-b)}{\sin b \sin c}}$

39. $\tan \frac{1}{2}(b+a) = \tan \frac{1}{2} c \dfrac{\cos \frac{1}{2}(B-A)}{\cos \frac{1}{2}(B+A)}$

40. $\tan \frac{1}{2}(b-a) = \tan \frac{1}{2} c \dfrac{\sin \frac{1}{2}(B-A)}{\sin \frac{1}{2}(B+A)}$

41. $\tan \frac{1}{2}(B+A) = \cot \frac{1}{2} C \dfrac{\cos \frac{1}{2}(b-a)}{\cos \frac{1}{2}(b+a)}$

42. $\tan \frac{1}{2}(B-A) = \cot \frac{1}{2} C \dfrac{\sin \frac{1}{2}(b-a)}{\sin \frac{1}{2}(b+a)}$

43. $\begin{cases} \cot \frac{1}{2} C = \tan \frac{1}{2}(B-A) \dfrac{\sin \frac{1}{2}(b+a)}{\sin \frac{1}{2}(b-a)} \\ \cot \frac{1}{2} C = \tan \frac{1}{2}(B+A) \dfrac{\cos \frac{1}{2}(b+a)}{\cos \frac{1}{2}(b-a)} \end{cases}$

44. $\begin{cases} \tan \frac{1}{2} c = \tan \frac{1}{2}(b-a) \dfrac{\sin \frac{1}{2}(B+A)}{\sin \frac{1}{2}(B-A)} \\ \tan \frac{1}{2} c = \tan \frac{1}{2}(b+a) \dfrac{\cos \frac{1}{2}(B+A)}{\cos \frac{1}{2}(B-A)} \end{cases}$

For a right angled spherical triangle in which C is the right angle, and the opposite side c, the hypothenuse, as in *Fig.* 54.

45. $\cos c = \cos a \cos b$
46. $\cos c = \cot A \cot B$
47. $\sin a = \sin c \sin A$
48. $\tan a = \sin b \tan A$
49. $\tan a = \cos B \tan c$
50. $\cos A = \sin B \cos a$

51. *If* ADBL *Fig.* 55, *be an Ellipse,* AB *the transverse axis,* E *and* F *the foci,* C *the centre, and* D *a point in the curve, then,*

$$\mathrm{ED} = \frac{\mathrm{AC}^2 - \mathrm{EC}^2}{\mathrm{AC} - \mathrm{EC} \cos \mathrm{AED}}.$$

Let DH be perpendicular to AB. Then,

$$\begin{aligned} \mathrm{ED}^2 &= \mathrm{DH}^2 + \mathrm{EH}^2 = \mathrm{DH}^2 + (\mathrm{EC} + \mathrm{CH})^2 \\ &= \mathrm{DH}^2 + \mathrm{EC}^2 + 2\,\mathrm{EC} \times \mathrm{CH} + \mathrm{CH}^2, \\ \text{and } \mathrm{FD}^2 &= \mathrm{DH}^2 + \mathrm{FH}^2 = \mathrm{DH}^2 + (\mathrm{EC} - \mathrm{CH})^2 \\ &= \mathrm{DH}^2 + \mathrm{EC}^2 - 2\mathrm{EC} \times \mathrm{CH} + \mathrm{CH}^2. \end{aligned}$$

Hence by subtraction,

$$\mathrm{ED}^2 - \mathrm{FD}^2 = 4\mathrm{EC} \times \mathrm{CH}.$$

$$\begin{aligned} \text{But } \mathrm{ED}^2 - \mathrm{FD}^2 &= (\mathrm{ED} + \mathrm{FD}) \times (\mathrm{ED} - \mathrm{FB}) \\ &= 2\mathrm{AC} \times (\mathrm{ED} - \mathrm{FD}). \end{aligned}$$

Therefore,

$$2\mathrm{AC} \times (\mathrm{ED} - \mathrm{FD}) = 4\,\mathrm{EC} \times \mathrm{CH},$$

$$\text{Or, } \mathrm{ED} - \mathrm{FD} = \frac{2\mathrm{EC} \times \mathrm{CH}}{\mathrm{AC}}.$$

But $\mathrm{ED} + \mathrm{FD} = 2\mathrm{AC}$.

Hence by addition,

$$2\,\mathrm{ED} = 2\mathrm{AC} + \frac{2\mathrm{EC} \times \mathrm{CH}}{\mathrm{AC}}$$

$$\mathrm{ED} = \mathrm{AC} + \frac{\mathrm{EC} \times \mathrm{CH}}{\mathrm{AC}} \qquad \text{(A)}$$

$$\begin{aligned} \mathrm{ED} \times \mathrm{AC} &= \mathrm{AC}^2 + \mathrm{EC} \times \mathrm{CH} \\ &= \mathrm{AC}^2 + \mathrm{EC} \times (\mathrm{EH} - \mathrm{EC}) \\ &= \mathrm{AC}^2 + \mathrm{EC} \times (\mathrm{ED} \cos \mathrm{AED} - \mathrm{EC}) \\ &= \mathrm{AC}^2 + \mathrm{EC} \times \mathrm{ED} \cos \mathrm{AED} - \mathrm{EC}^2, \\ \mathrm{ED} \times \mathrm{AC} - \mathrm{ED} \times \mathrm{EC} \cos \mathrm{AED} &= \mathrm{AC}^2 - \mathrm{EC}^2 \\ \mathrm{ED} \times (\mathrm{AC} - \mathrm{EC} \cos \mathrm{AED}) &= \mathrm{AC}^2 - \mathrm{EC}^2 \end{aligned}$$

$$\mathrm{ED} = \frac{\mathrm{AC}^2 - \mathrm{EC}^2}{\mathrm{AC} - \mathrm{EC} \cos \mathrm{AED}}.$$

52. *If the circle* AGBM, *Fig.* 55, *be described about* AB, *as a diameter, and* HG *be produced to meet it in G, then,*

$$\mathrm{ED} = \mathrm{AC} - \mathrm{EC} \cos \mathrm{BCG}.$$

From the preceding demonstration, we have (A),

$$\begin{aligned} ED &= AC + \frac{EC \times CH}{AC} \\ &= AC + \frac{EC \times CG \cos ACG}{AC} \\ &= AC + \frac{EC \times AC \cos ACG}{AC} \\ &= AC + EC \cos ACG \\ &= AC - EC \cos BCG. \end{aligned}$$

INVESTIGATION OF FORMULÆ FOR COMPUTING ECLIPSES OF THE SUN, OCCULTATIONS AND TRANSITS.

53. Let O, *Fig.* 60, be the centre of the earth; A, a given place on its surface; S and M, the centres of the sun and moon; X, M′, and Z′, the points in which straight lines from O, through S, M, and A, meet the celestial sphere; EQ, the equator; E, the vernal equinox; P, the north pole of the equator, and PB, PC, and PD, declination circles, passing through X, M′ and Z′. Also, let BY and XZ be each a quadrant. Then, since BY is a quadrant, Y is the pole of ZB; and consequently OY is perpendicular to OX and OZ. And since XZ is a quadrant, OZ is perpendicular to OX. Hence OX, OY and OZ form a system of rectangular axes, having their origin in O.

54. Put $A = EC =$ right ascension of the moon
$A' = EB =$ “ “ sun
$\mu = ED =$ “ “ zenith, or sidereal time
$D = CM' =$ Declination of moon
$D' = BX =$ “ sun
$\varphi =$ Geographic latitude of place,
$\varphi' = DZ =$ declin. of geocent. zenith, or reduced lat. of place
$\pi =$ Equatorial horizontal parallax of moon
$\pi' =$ “ “ “ sun
$d =$ Horizontal semidiameter of moon
$d' =$ “ “ sun
$\delta =$ Augmented semidiameter of moon
$\delta' =$ “ “ sun

$$\left.\begin{array}{l} r = OM \\ r' = OS \\ \rho = OA \end{array}\right\} \begin{array}{l} \text{The distances of the centres of the moon and} \\ \text{sun, and of the place A, from the earth's centre,} \\ \text{the equatorial radius of the earth being} = 1 \end{array}$$

$\Sigma =$ Angle SAM = apparent distance of the centres of the sun and moon.

Also put, $a = \cos M'OX$ $\alpha = \cos Z'OX$
$b = \cos M'OY$ $\beta = \cos Z'OY$
$c = \cos M'OZ$ $\gamma = \cos Z'OZ.$

Then (Analytical Geometry),

$$\left.\begin{array}{c} a^2 + b^2 + c^2 = 1, \qquad \alpha^2 + \beta^2 + \gamma^2 = 1 \\ a\alpha + b\beta + c\gamma = \cos M'OZ' \end{array}\right\} \qquad (1)$$

55. In the spherical triangle XPM′, we have,

$$\cos M'X = \cos XPM' \sin PX \sin PM' + \cos PX \cos PM'$$

$$\text{or, } a = \cos (A - A') \cos D \cos D' + \sin D \sin D'.$$

The right ascensions of Y and Z are A′ + 90° and A′ + 180, respectively, and their declinations are 0° and 90° — D′. If, therefore, instead of A′ and D′, in the expression for a, we substitute successively these right ascensions and declinations, we shall obtain expressions for b and c. We shall thus have,

$$\left.\begin{aligned} a &= \cos (A - A') \cos D \cos D' + \sin D \sin D' \\ b &= \sin (A - A') \cos D \\ c &= -\cos (A - A') \cos D \sin D' + \sin D \cos D' \end{aligned}\right\} \quad . \quad . \quad (2)$$

In like manner we have,

$$\left.\begin{aligned} \alpha &= \cos (\mu - A') \cos \varphi' \cos D' + \sin \varphi' \sin D' \\ \beta &= \sin (\mu - A') \cos \varphi' \\ \gamma &= -\cos (\mu - A') \cos \varphi' \sin D' + \sin \varphi' \cos D' \end{aligned}\right\} \quad . \quad . \quad (3)$$

56. Referring S, M and A, to the rectangular axes OX, OY and OZ,

the co-ordinates of S, are r', 0, 0
" of M, " ar, br, cr
" of A, " $\alpha\rho$, $\beta\rho$, $\gamma\rho$.

Hence, if the equations of the straight line AM, joining A and M, be

$$\left.\begin{aligned} y &= px + E \\ z &= qx + E' \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

we shall have,

$$p = \frac{br - \beta\rho}{ar - \rho\alpha} = \frac{b - \rho\beta \sin \varpi}{a - \rho\alpha \sin \varpi}$$

$$q = \frac{cr - \rho\gamma}{ar - \rho\alpha} = \frac{c - \rho\gamma \sin \varpi}{a - \rho\alpha \sin \varpi}.$$

Also, if the equations of the straight line AS, joining A and S, be,

$$\left.\begin{aligned} y &= p'x + F \\ z &= q'x + F' \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (5)$$

we shall have,

$$p' = \frac{-\rho\beta}{r' - \rho\alpha} = \frac{-\rho\beta \sin \varpi'}{1 - \rho\alpha \sin \varpi'}$$

$$q' = \frac{-\gamma\rho}{r' - \rho\alpha} = \frac{-\rho\gamma \sin \varpi'}{1 - \rho\alpha \sin \varpi'}.$$

And from the equations of the lines AM and AS, we have,

$$\tan \Sigma = \frac{\sqrt{(p - p')^2 + (q - q')^2 + (pq' - p'q)^2}}{1 + pp' + qq'} \quad . \quad . \quad (6)$$

57. This expression for the tangent of the *apparent distance* of the centres is rigidly accurate, but the calculation from it would be very tedious. When, however, the distance is small, as is always the case in eclipses of the sun, it may be reduced without any practical diminution of its accuracy.

Put $\tan\sigma = \sqrt{(p-p')^2 + (q-q')^2}$ . . . . (7)

$$\tan s = \frac{q'.(p-p') - p'.(q-q')}{p'.(p-p') + q'.(q-q')} \quad . \quad . \quad (8)$$

$$\tan f = \sqrt{p'^2 + q'^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Then, $\sec^2 s = 1 + \tan^2 s = \dfrac{(p'^2+q'^2).(p-p')^2+(p'^2+q'^2).(q-q')^2}{[p'.(p-p')+q'.(q-q')]^2}$

$$= \frac{\tan^2\sigma\,\tan^2 f}{[p'.(p-p')+q'.(q-q')]^2}.$$

Hence, $\sec s = \dfrac{\tan\sigma\,\tan f}{p'.(p-p')+q'.(q-q')} = \dfrac{\tan\sigma\,\tan f}{pp'+qq'-\tan^2 f}$

or, $\cos s = \dfrac{pp'+qq'-\tan^2 f}{\tan\sigma\,\tan f}$. Consequently,

$$pp'+qq' = \cos s\,\tan\sigma\,\tan f + \tan^2 f.$$

or, $1+pp'+qq' = 1+\cos s\,\tan\sigma\,\tan f + \tan^2 f.$

Again $\sin s = \dfrac{\tan s}{\sec s} = \dfrac{q'.(p-p')-p'.(q-q')}{\tan\sigma\,\tan f} = \dfrac{pq'-p'q}{\tan\sigma\,\tan f}$

or, $(pq'-p'q)^2 = \sin^2 s\,\tan^2\sigma\,\tan^2 f$

and $(p-p')^2+(q-q')^2+(pq'-p'q)^2 = \tan^2\sigma + \sin^2 s\,\tan^2\sigma\,\tan^2 f.$

Hence $\tan\Sigma = \dfrac{\sqrt{\tan^2\sigma + \sin^2 s\,\tan^2\sigma\,\tan^2 f}}{1+\cos s\,\tan\sigma\,\tan f+\tan^2 f}$

$$= \frac{\tan\sigma\sqrt{1+\sin^2 s\,\tan^2 f}}{1+\cos s\,\tan\sigma\,\tan f+\tan^2 f}$$

or, $\tan\Sigma\,(1+\tan^2 f) + \cos s\,\tan\Sigma\,\tan\sigma\,\tan f = \tan\sigma\sqrt{1+\sin^2 s\,\tan^2 f}$

Consequently $\tan\sigma = \dfrac{\tan\Sigma\,(1+\tan^2 f)}{\sqrt{1+\sin^2 s\,\tan^2 f} - \cos s\,\tan\Sigma\,\tan f}.$

Now $\tan(\sigma \backsim \Sigma) = \dfrac{\tan\sigma \backsim \tan\Sigma}{1+\tan\sigma\,\tan\Sigma}.$

Hence, $\tan(\sigma \backsim \Sigma) < \tan\sigma \backsim \tan\Sigma$

$$< \tan\Sigma.\left(\frac{1+\tan^2 f}{\sqrt{1+\sin^2 s\,\tan^2 f} - \cos s\,\tan\Sigma\,\tan f} \backsim 1\right)$$

$$< \tan\Sigma.\frac{(1+\tan^2 f) \backsim (\sqrt{1+\sin^2 s\,\tan^2 f} - \cos s\,\tan\Sigma\,\tan f)}{\sqrt{1+\sin^2 s\,\tan^2 f} - \cos s\,\tan\Sigma\,\tan f}$$

$$< \tan\Sigma.\frac{(\cos s\,\tan\Sigma\,\tan f + 1 + \tan^2 f) \backsim \sqrt{1+\sin^2 s\,\tan^2 f}}{\sqrt{1+\sin^2 s\,\tan^2 f} - \cos s\,\tan\Sigma\,\tan f}$$

Now, it is evident that the fractional factor will have its greatest value, when $\cos s = 1$, and consequently $\sin s = 0$. Hence,

$$\tan(\sigma \backsim \Sigma) < \tan\Sigma\,\frac{(\tan\Sigma\,\tan f + 1 + \tan^2 f) \backsim 1}{1-\tan\Sigma\,\tan f}$$

$$< \tan \Sigma \tan f \, \frac{\tan \Sigma + \tan f}{1 - \tan \Sigma \tan f}$$

$$< \tan \Sigma \tan (\Sigma + f) \tan f$$

$$< \tan^2 (\Sigma + f) \tan f.$$

But since (*eq.*1) $\beta^2 + \gamma^2 = 1 - \alpha^2$, we have (*eq.*9),

$$\tan f = \sqrt{p'^2 + q'^2} = \frac{\varrho \sin \pi' \sqrt{\beta^2 + \gamma^2}}{1 - \varrho\alpha \sin \pi'} = \frac{\varrho \sin \pi' \sqrt{1 - \alpha^2}}{1 - \varrho\alpha \sin \pi'}$$

Consequently for the greatest value of $\tan f$, we have,

$$\tan f = \frac{\sin \pi'}{1 - \sin \pi'}.$$

Whence it is easily found that f cannot exceed $9''$.

Therefore, $\tan (\sigma \backsim \Sigma) < \tan^2 (\Sigma + 9''). \tan 9''$.

$$\text{or, } \sigma \backsim \Sigma < \tan^2 (\Sigma + 9''). \, 9''.$$

If we take $(\Sigma + 9'') = 1°$, which is greater than it can ever be, at the time of an eclipse of the sun, we find $\tan^2 (\Sigma + 9''). \, 9'' = 0''.0027$. Hence at the time of an eclipse the difference between σ and Σ is less than $0''.003$. We may therefore consider them equal. Consequently we have (eq. 7),

$$\tan \Sigma = \sqrt{(p - p')^2 + (q - q')^2} \qquad (10)$$

58. From (56), we have,

$$p - p' = \frac{b - \varrho\beta \sin \pi}{a - \varrho\alpha \sin \pi} + \frac{\varrho\beta \sin \pi'}{1 - \varrho\alpha \sin \pi'}$$

$$= \frac{b}{a - \varrho\alpha \sin \pi} - \varrho\beta. \left(\frac{\sin \pi}{a - \varrho\alpha \sin \pi} - \frac{\sin \pi'}{1 - \varrho\alpha \sin \pi'} \right)$$

$$= \frac{b}{a - \varrho\alpha \sin \pi} - \varrho\beta. \frac{\sin \pi - a \sin \pi'}{(a - \varrho\alpha \sin \pi).(1 - \varrho\alpha \sin \pi')}.$$

If we put Δ = angle M'OX = geocentric distance of sun and moon's centres, we have,

$$\sin \pi - a \sin \pi = \sin \pi - \cos \Delta \sin \pi'$$

$$= \sin \pi - \sin \pi' + 2 \sin^2 \tfrac{1}{2} \Delta \sin \pi'.$$

But, $\sin (\pi - \pi') = \sin \pi \cos \pi' - \cos \pi \sin \pi'$

$$= \sin \pi - \sin \pi' + 2 \sin^2 \tfrac{1}{2} \pi \sin \pi' - 2 \sin^2 \tfrac{1}{2} \pi' \sin \pi.$$

or, $\sin \pi - \sin \pi' = \sin (\pi - \pi') - 2 \sin^2 \tfrac{1}{2} \pi \sin \pi' + 2 \sin^2 \tfrac{1}{2} \pi' \sin \pi$.

Hence, $\sin \pi - a \sin \pi' = \sin (\pi - \pi') + 2 (\sin^2 \tfrac{1}{2} \Delta - \sin^2 \tfrac{1}{2} \pi) \sin \pi' + 2 \sin^2 \tfrac{1}{2} \pi' \sin \pi$.

At the time of an eclipse, Δ cannot exceed $\Sigma + \pi$, and consequently must be less than 2π. Therefore by taking successively $\Delta = 0$, and $\Delta = 2\pi$, it is easily ascertained that the rejection of the last two terms in the above equation will occasion less error than would result from an error of $0''.004$ in the value of π. Hence we may consider,

$$\sin \pi - a \sin \pi' = \sin (\pi - \pi') \qquad (11)$$

Consequently, $p - p' = \dfrac{b}{a - \varrho\alpha \sin \pi} - \varrho\beta. \dfrac{\sin (\pi - \pi')}{(a - \varrho\alpha \sin \pi).(1 - \varrho\alpha \sin \pi')}$

$$= \frac{b}{a - \rho\alpha \sin \varpi} - \rho\beta . \frac{\sin(\varpi - \varpi')}{a - \rho\alpha \sin \varpi} . \left(1 + \rho\alpha \sin \varpi' + \frac{\rho^2\alpha^2 \sin^2 \varpi'}{1 - \rho\alpha \sin \varpi'}\right)$$

$$= \frac{b - \rho\beta \sin(\varpi - \varpi') . (1 + \rho\alpha \sin \varpi')}{a - \rho\alpha \sin \varpi} - \frac{\rho^3\alpha^2\beta \sin(\varpi - \varpi') \sin^2 \varpi'}{(a - \rho\alpha \sin \varpi) . (1 - \rho\alpha \sin \varpi')} .$$

Rejecting the last term, which is evidently much less than those already rejected, we have,

$$p - p' = \frac{b - \rho\beta \sin(\varpi - \varpi') . (1 + \rho\alpha \sin \varpi')}{a - \rho\alpha \sin \varpi} .$$

In like manner,

$$q - q' = \frac{c - \rho\gamma \sin(\varpi - \varpi') . (1 + \rho\alpha \sin \varpi')}{a - \rho\alpha \sin \varpi} .$$

Hence (eq. 10),

$$\tan^2 \Sigma = \frac{[b - \rho\beta \sin(\varpi - \varpi')(1 + \rho\alpha \sin \varpi')]^2 + [c - \rho\gamma \sin(\varpi - \varpi')(1 + \rho\alpha \sin \varpi']^2}{(a - \rho\alpha \sin \varpi)^2} \qquad (12)$$

59. In the triangle AOM, we have,

$$\text{AM}^2 = \text{OM}^2 - 2\,\text{AO}.\,\text{OM}.\cos \text{AOM} + \text{AO}^2$$

$$\frac{\text{AM}^2}{\text{OM}^2} = 1 - 2\,\text{AO}.\,\frac{1}{\text{OM}}.\cos \text{AOM} + \text{AO}^2.\,\frac{1}{\text{OM}^2}$$

$$\text{or, } \frac{\sin^2 d}{\sin^2 \delta} = 1 - 2\rho \sin \varpi \cos \text{AOM} + \rho^2 \sin^2 \varpi \qquad (13)$$

From equation (10), we have,

$$\begin{aligned}
\tan^2 \Sigma &= (p - p')^2 + (q - q)^2 \\
&= p^2 + q^2 - 2\,(pp' + qq') + p'^2 + q'^2 \\
&= p^2 + q^2 - 2\,(\cos s \tan \sigma \tan f + \tan^2 f) + \tan^2 f \\
&= p^2 + q^2 - (2 \cos s \tan \sigma \tan f + \tan^2 f) \\
&= \frac{(b - \rho\beta \sin \varpi)^2 + (c - \rho\gamma \sin \varpi)^2}{(a - \rho\alpha \sin \varpi)^2} - (2 \cos s \tan \sigma \tan f + \tan^2 f).
\end{aligned}$$

Or, $(a - \rho\alpha \sin \varpi)^2 \tan^2 \Sigma = (b - \rho\beta \sin \varpi)^2 + (c - \rho\gamma \sin \varpi)^2 - (2 \cos s \tan \sigma \tan f + \tan^2 f)\,(a - \rho\alpha \sin \varpi)^2$.

Put $(2 \cos s \tan \sigma \tan f + \tan^2 f) . (a - \rho\alpha \sin \varpi) = g$.

Then, $(a - \rho\alpha \sin \varpi)^2 \tan^2 \Sigma = (b - \rho\beta \sin \varpi)^2 + (c - \rho\gamma \sin \varpi)^2 - g$

Adding $(a - \rho\alpha \sin \varpi)^2$ to each side, we have,

$(a - \rho\alpha \sin \varpi)^2 \sec^2 \Sigma = (a - \rho\alpha \sin \varpi)^2 + (b - \rho\beta \sin \varpi)^2 + (c - \rho\gamma \sin \varpi)^2 - g$.

Developing the terms of the second member of the equation, and substituting for $a^2 + b^2 + c^2$, $\alpha^2 + \beta^2 + \gamma^2$, and $a\alpha + b\beta + c\gamma$, their equals, (eq. 1), we have,

$$\begin{aligned}
(a - \rho\alpha \sin \varpi)^2 \sec^2 \Sigma &= 1 - 2\rho \sin \varpi \cos \text{AOM} + \rho^2 \sin^2 \varpi - g \\
&= \frac{\sin^2 d}{\sin^2 \delta} - g.
\end{aligned}$$

Or, $\dfrac{(a - \varrho\alpha \sin \pi)^2 \sin^2 \delta}{\cos^2 \Sigma} = \sin^2 d - g \sin^2 \delta$.

Rejecting the term $g \sin^2 \delta$, which, as is easily found, will not occasion a greater error than would result from an error of 0″.001 in the moon's semi-diameter, we have,

$$\frac{(a - \varrho\alpha \sin \pi)^2 \sin^2 \delta}{\cos^2 \Sigma} = \sin^2 d$$

$$\text{or, } \sin \delta = \frac{\sin d \cos \Sigma}{a - \varrho\alpha \sin \pi} \quad - \quad - \quad - \quad - \quad (14)$$

60. In the triangle AOS, we have,

$$\frac{AS^2}{OS^2} = 1 - 2AO.\frac{1}{OS} \cos AOX + AO^2.\frac{1}{OS^2}.$$

$$\text{or, } \frac{\sin^2 d'}{\sin^2 \delta'} = 1 - 2\,\varrho\alpha \sin \pi' + \varrho^2 \sin^2 \pi'$$

$$= (1 - \varrho\alpha \sin \pi')^2 + \varrho^2.(1 - \alpha^2) \sin^2 \pi'$$

$$\text{or, } \sin^2 d' = (1 - \varrho\alpha \sin \pi')^2 \sin^2 \delta' + \varrho^2 (1 - \alpha^2) \sin^2 \delta' \sin^2 \pi'.$$

Rejecting the last term, which is extremely small, we obtain,

$$\sin \delta' = \frac{\sin d'}{1 - \varrho\alpha \sin \pi'} \qquad (15)$$

61. Now, at the time of the beginning or end of the eclipse, we have,

$$\Sigma = \delta + \delta', \quad \text{or, } \delta = \Sigma - \delta'.$$

$$\text{Hence, } \sin \delta = \sin \Sigma \cos \delta' - \cos \Sigma \sin \delta'.$$

Or, without sensible error,

$$\sin \delta = \sin \Sigma \cos d' - \cos \Sigma \sin \delta'.$$

Substituting for $\sin \delta$ and $\sin \delta'$, their values (eq. 14 and eq. 15), and dividing by $\cos \Sigma \cos d'$, there results

$$\tan \Sigma = \frac{\tan d'}{1 - \varrho\alpha \sin \pi'} + \frac{\sin d}{(a - \varrho\alpha \sin \pi) \cos d'}.$$

Equating the square of this value of $\tan \Sigma$ with the value of $\tan^2 \Sigma$ (eq. 12), we have,

$$\left(\frac{\tan d'}{1 - \varrho\alpha \sin \pi'} + \frac{\sin d}{(a - \varrho\alpha \sin \pi) \cos d'}\right)^2$$
$$= \left(\frac{b - \varrho\beta \sin (\pi - \pi').\ (1 - \varrho\alpha \sin \pi')}{a - \varrho\alpha \sin \pi}\right)^2$$
$$+ \left(\frac{c - \varrho\gamma \sin (\pi - \pi').\ (1 - \varrho\alpha \sin \pi')}{a - \varrho\alpha \sin \pi}\right)^2$$

Or

$$\left(\frac{(a - \varrho\alpha \sin \pi) \tan d'}{1 - \varrho\alpha \sin \pi'} + \frac{\sin d}{\cos d'}\right)^2 = \left(b - \varrho\beta \sin (\pi - \pi').\ (1 - \varrho\alpha \sin \pi')\right)^2$$
$$+ \left(c - \varrho\gamma \sin (\pi - \pi').\ (1 - \varrho\alpha \sin \pi')\right)^2$$

$$\text{But, } \frac{(a - \varrho\alpha \sin \varpi) \tan d'}{1 - \varrho\alpha \sin \varpi'} = (a - \varrho\alpha \sin \varpi).$$

$$\left(1 + \varrho\alpha \sin \varpi' + \frac{\varrho^2 \alpha^2 \sin^2 \varpi'}{1 - \varrho\alpha \sin \varpi'}\right) \tan d' = a \tan d' - \varrho\alpha$$

$$(\sin \varpi - a \sin \varpi') \tan d' - \frac{\varrho^2 \alpha^2 (\sin \varpi - a \sin \varpi') \tan d' \sin \varpi'}{1 - \varrho\alpha \sin \varpi'}.$$

Rejecting the last term, which is very small, and substituting $\sin (\varpi - \varpi')$ for $(\sin \varpi - a \sin \varpi')$, (11), we have,

$$\frac{(a - \varrho\alpha \sin \varpi) \tan d'}{1 - \varrho\alpha \sin \varpi'} = a \tan d' - \varrho\alpha \sin (\varpi - \varpi') \tan d'.$$

Hence,

$$\left(a \tan d' - \varrho\alpha \sin (\varpi - \varpi') \tan d' + \frac{\sin d}{\cos d'}\right)^2$$
$$= \left(b - \varrho\beta \sin (\varpi - \varpi') . (1 - \varrho\alpha \sin \varpi')\right)^2$$
$$+ \left((c - \varrho\gamma \sin (\varpi - \varpi') . (1 - \varrho\alpha \sin \varpi')\right)^2 \qquad (16)$$

62. As no quantity has been rejected that can produce an error, so great as would result from an error of 0″.004 in the places, parallaxes or semidiameters of the sun or moon, the formula just obtained, may be regarded in practice as rigidly accurate.

It may indeed be further reduced, and still have the requisite practical precision. A little examination will show, that if the term $\varrho\alpha \sin \varpi'$ in the factor $(1 + \varrho\alpha \sin \varpi')$ be rejected, and if $\cos (A - A') \cos (D - D')$, be substituted for a, the resulting error will be within the limit of $\frac{1}{10}$ of a second in arc. We shall thus have,

$$\left(\cos (A - A') \cos (D - D') \tan d' - \varrho\alpha \sin (\varpi - \varpi') \tan d' + \frac{\sin d}{\cos d'}\right)^2$$
$$= \left(b - \varrho\beta \sin (\varpi - \varpi')\right)^2 + \left(c - \varrho\gamma \sin (\varpi - \varpi')\right)^2$$

Or,

$$\left(\frac{\cos (A - A') \cos (D - D') \tan d'}{\sin (\varpi - \varpi')} - \varrho\alpha \tan d' + \frac{\sin d}{\cos d' \sin (\varpi - \varpi')}\right)^2$$
$$= \left(\frac{b}{\sin (\varpi - \varpi')} - \varrho\beta\right)^2 + \left(\frac{c}{\sin (\varpi - \varpi')} - \varrho\gamma\right)^2 \qquad (17)$$

63. In order to observe accurately the time of the beginning of an eclipse it is necessary to know the part of the sun's disc at which the contact take place. This point is usually designated by the angular distance, either from the highest or vertical point of the disc, or from the most northerly point.

Considering S and M as the places of the sun and moon, at the commencement of the eclipse, the angle which the plane ASM makes with the geocentric vertical plane Z′OS, passing through the sun's centre, will evidently express the distance of the point of contact, from the sun's geocentric vertex.

Let OG and OH be drawn, respectively parallel to AS and AM.

Then the angle made by the plane OGH with the vertical plane Z'OS, or which is the same Z'OG, will be equal to that made by ASM with the same plane, and will therefore express the distance of the point of contact from the vertex. Also let SG and SH, each drawn perpendicular to OX, meet OG and OH, in G and H. Then since the equations of AM are (eq. 4),

$$\begin{aligned} y &= px + \mathrm{E} \\ z &= qx + \mathrm{E}' \end{aligned}$$

it follows that the equations of OH, parallel to AM, and passing through the origin, are,

$$\left.\begin{aligned} y &= px \\ z &= qx \end{aligned}\right\} \quad - \quad - \quad - \quad - \qquad (18)$$

In like manner, the equations of OG, are (eq. 5)

$$\left.\begin{aligned} y &= p'x \\ z &= q'x \end{aligned}\right\} \quad - \quad - \quad - \quad - \qquad (19)$$

If now we assume OS = 1, the co-ordinates of S, will be 1, 0, 0. And by taking $x = 1$, in the equations (19 and 18) we find the corresponding co-ordinates of H and G. We thus have,

the co-ordinates	of S,	1,	0,	0	
"	of G,	1,	p',	q'	(20)
"	of H,	1,	p,	q,	

Hence the equation of the plane OGH, passing through the origin O, and the points G and H, is,

$$(pq' - p'q)\,x + (q - q')\,y - (p - p')\,z = 0 \qquad (21)$$

Also the equation of the plane Z'OG, which passes through O, G, and S, is

$$0\,.\,x + q'y - p'z = 0 \quad - \quad - \quad - \quad - \qquad (22)$$

If therefore we put V = the angle made by the plane OGH with the plane Z'OG, we have,

$$\tan \mathrm{V} = \frac{\sqrt{(pq' - p'q)^2\cdot q'^2 + (pq' - p'q)^2\cdot p'^2 + (pq' - p'q)^2}}{(q - q)'\cdot q' + (p - p')\cdot p'.}$$

$$= \frac{(pq' - p'q)\,\sqrt{1 + p'^2 + q'^2}}{p'\cdot(p - p') + q'\cdot(q - q')}.$$

But $pq' - p'q = q'\cdot(p - p') - p'\cdot(q - q')$, and (eq. 9) $\sqrt{1 + p'^2 + q'^2} = \sqrt{1 + \tan^2 f} = \sec f$. Hence,

$$\tan \mathrm{V} = \frac{q'\cdot(p - p') - p'\,(q - q')}{p'\,(p - p') + q'\,(q - q')}\,\sec f.$$

As f cannot exceed $9''$, the factor, $\sec f$, cannot, as is easily proved, affect the value of V, in any case, more than a small fraction of a second. Hence we may consider

$$\tan V = \frac{q'(p-p')-p'(q-q')}{p'(p-p')+q'(q-q')} = \frac{\frac{p-p'}{q-q'}-\frac{p'}{q'}}{1+\frac{p-p'}{q-q'}\cdot\frac{p'}{q'}}$$

$$\text{Put } \tan P = \frac{p-p'}{q-q'} = \frac{b-\rho\beta\sin(\pi-\pi')}{c-\rho\gamma\sin(\pi-\pi')} = \frac{\frac{b}{\sin(\pi-\pi')}-\rho\beta}{\frac{c}{\sin(\pi-\pi')}-\rho\gamma}, \qquad (23)$$

$$\text{and } \tan Q = \frac{p'}{q'} = \frac{\rho\beta}{\rho\gamma} \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

$$\text{Then, } \tan V = \frac{\tan P - \tan Q}{1+\tan P\tan Q} = \tan(P-Q)$$

$$\text{Hence, } V = P - Q \qquad (25)$$

From the equation of the vertical plane Z′OG, we find that the tangent of the angle which it makes with the plane, POX, which is the plane of xz, is equal to $\frac{p'}{q'}$. This angle is therefore (eq. 24), equal to Q. Consequently as the plane POX passes through the north pole and the sun's centre, Q expresses the distance of the vertical point of the sun's disc from the north point. Hence, as $V + Q = P$, it follows that P expresses the distance of the point of contact from the north point of the disc.

64. Put $R = a\tan d' = \frac{\cos(A-A')\cos(D-D')\tan d'}{\sin(\pi-\pi')}$

$$R' = \rho a\tan d' = \left(\rho\cos\varphi'\cos(\mu-A')\cos D' + \rho\sin\varphi'\sin D'\right)\tan d'$$

$$r = \frac{\sin d}{\cos d'\sin(\pi-\pi')}$$

$$k = R - R' + r$$

$$p = \frac{b}{\sin(\pi-\pi')} = \frac{\sin(A-A')\cos D}{\sin(\pi-\pi')}$$

$$q = \frac{c}{\sin(\pi-\pi')} = \frac{\sin D\cos D' - \cos(A-A')\cos D\sin D'}{\sin(\pi-\pi')}$$

$$u = \rho\beta = \rho\cos\varphi'\sin(\mu-A')$$

$$v = \rho\gamma = \rho\sin\varphi'\cos D' - \rho\cos\varphi'\cos(\mu-A')\sin D'.$$

Then, for the time of beginning or end of the eclipse, we have (eq. 17),

$$k^2 = (p-u)^2 + (q-v)^2 \qquad (26)$$

In annular eclipses, if we take $k = R - R' - r$, the formula will correspond to the time when the eclipse begins or ceases to be annular. And in total eclipses, if we take $k = r - (R - R')$, it will correspond to the time when the eclipse begins or ceases to be total.

65. Before proceeding to the solution of the above equation, it will be convenient to have a formula for the nearest apparent approach of the centres of the sun and moon. If (in equation 12) the term $\rho\alpha \sin \pi'$ in the factor $(1 + \rho\alpha \sin \pi')$ be rejected (art. 62), we have,

$$\tan^2 \Sigma = \frac{\left(b - \rho\beta \sin(\pi - \pi')\right)^2 + \left(c - \rho\gamma \sin(\pi - \pi')\right)^2}{(a - \rho\alpha \sin \pi)^2}.$$

$$\text{Whence } \tan \Sigma = \frac{\sin(\pi - \pi')}{a - \rho\alpha \sin \pi} \surd$$

$$\left[\left(\frac{b}{\sin(\pi - \pi')} - \rho\beta\right)^2 + \left(\frac{c}{\sin(\pi - \pi')} - \rho\gamma\right)^2\right]$$

$$= \frac{\sin(\pi - \pi')}{a - \rho\alpha \sin \pi} \sqrt{(p - u)^2 + (q - v)^2} \qquad (27)$$

$$\text{Put } z = \sqrt{(p - u)^2 + (q - v)^2}$$

$$\text{Then } \tan \Sigma = \frac{\sin(\pi - \pi')}{a - \rho\alpha \sin \pi}.\ z.$$

The factor $\frac{\sin(\pi - \pi')}{a - \rho\alpha \sin \pi}$ changes its value so little, during the continuance of an eclipse, that the change in the value of $\tan \Sigma$ may be regarded as depending entirely on the change in the value of z. Hence the least value of $\tan \Sigma$, and consequently the nearest approach of the centres, will have place, when z has its least value.

As the value of z continually decreases while it is approaching its least value, and afterwards continually increases, it is evident that whatever value it has at any time prior to the time of this least value, it must at a subsequent time acquire an equal value. It is also evident that the less the interval is between any two times when the values of z are equal, the nearer will either of these times be, to the time of the least value. Consequently if t be the interval between the times of two equal values of z, the least value must have place when $t = 0$.

66. Let p', q', u' and v' be, respectively, the hourly changes of the values of p, q, u, and v, at any proposed time, supposing the rate of change which each has at that time to continue uniform.

Then assuming the values of p, q, u, and v, and also of p', q', u', and v', to be the values which those quantities have at some time T, so near the time of the least value of z, that the rate of change of each of the quantities p, q, u, and v, may be regarded as uniform, during the interval t, and putting z' for the value of z at the time T, we have,

$$z' = \sqrt{(p - u)^2 + (q - v)^2}$$

Also for the time T + t, we have,

$$z' = \surd\left[\left(p - u + (p' - u')\,t\right)^2 + \left(q - v + (q' - v')\,t\right)^2\right].$$

In order to abridge the expressions, put $x = p - u$, $y = q - v$, $x' = p' - u'$, and $y' = q' - v'$. Then,

$$z' = \sqrt{x^2 + y^2}$$
$$z' = \sqrt{(x + x't)^2 + (y + y't)^2}$$
$$\text{Hence, } (x + x't)^2 + (y - y't)^2 = x^2 + y^2$$
$$\text{or, } 2\,xx't + 2\,yy't + x'^2t^2 + y'^2t^2 = 0$$
$$\text{or, } xx' + yy' + \frac{x'^2 + y'^2}{2}.\; t = 0$$

Consequently, taking $t = 0$, (art. 65), we have at the time of the least value of z, or which is the same, at the time of the *nearest apparent approach of the centres.*

$$xx' + yy' = 0. \qquad - \quad - \quad - \quad - \quad - \qquad (28)$$

Solutions of the equations for the time of the nearest apparent approach of the centres of the sun and moon; and for the times of the beginning and end of the eclipse.

67. As the quantities x' and y' are not constant, but change with the time, the equations (28 and 26) do not admit of direct solutions. They may, however, be readily solved by successive approximations.

Let T be the time of the conjunction of the sun and moon in right ascension, or any other time near to the time of nearest approach of the centres; and let t be the interval between the time T, and the time of nearest approach. Also let the values of x, y, x' and y', be those which they have at the time T. Then, supposing the values of x' and y' to remain constant during the interval t, we obviously have (eq. 28),

$$(x + x't)\,x' + (y + y't)\,y' = 0.$$

Whence, putting $n^2 = x'^2 + y'^2$ we obtain,

$$t = -\frac{xx' + yy'}{n^2} \qquad - \quad - \quad - \quad - \qquad (29)$$

The value of t, added to the time T or subtracted from it, according as t is affirmative or negative, will give the *approximate time of the nearest approach of the centres.*

68. If, now, T be the approximate time of nearest approach, and the values of x, y, x' and y', at the time T, be taken, the true time may, by repeating the calculation, be found very nearly. This second approximation may in general be regarded as giving the true time. If the utmost precision is desired, the calculation may be again repeated, using the values of the quantities, at the time last found.

69. Let, now, T be the true time of the nearest approach of the centres, and t the interval between the time T, and the time of beginning or end of the eclipse. Also let the values of x, y, x', and y', at the time T, be represented by a, b, a', and b', respectively; and let the value of k be the

value which it has at that time. Then supposing the values of x' and y' to continue during the eclipse, equal to a' and b', the value of k to be constant, and putting $m^2 = a^2 + b^2$, and $n^2 = a'^2 + b'^2$, we have, (eq. 26),

$$k^2 = (a \pm a't)^2 + (b \pm b't)^2 = m^2 \pm 2\,(aa' + bb')\,t + n^2t^2$$

But (eq. 28), $aa' + bb' = 0$. Hence,

$$k^2 = m^2 + n^2t^2,$$

$$\text{or, } t = \frac{\sqrt{(k^2 - m^2)}}{n} \quad - \quad - \quad - \quad - \qquad (30)$$

The value of t, subtracted from T, and added to it, will give *approximate times* of the *beginning* and *end* of the eclipse.

70. Let the values of x, y, x', and y', at the approximate time of beginning, be represented by c, d, c', and d', respectively, and let the value of k be that which it has at that time. Also let t and t' be respectively the intervals that would elapse between the approximate and true times of beginning, and the time of nearest approach of the centres, if the values of x' and y' continued to be equal to c' and d'. Then the difference between t and t' will evidently be the interval between the approximate and true times of beginning, very nearly. The error there is in considering it accurately so, depends on the small changes in the values of k, x', and y', during this interval; which, as the interval is always short, may in general be disregarded.

Taking the values of the quantities as above assumed, and putting $\mathrm{M}^2 = c^2 + d^2$, and $n^2 = c'^2 + d'^2$, we have,

$$\mathrm{M}^2 = (a - c't)^2 + (b - d't)^2 = m^2 - 2\,(ac' + bd')t + n^2t^2$$

$$\text{and } k^2 = (a - c't')^2 + (b - d't')^2 = m^2 - 2\,(ac' + bd')\,t' + n^2\,t'^2.$$

$$\text{Hence } t = \sqrt{\left(\frac{\mathrm{M}^2 - m^2}{n^2} + \frac{(ac' + bd')^2}{n^4}\right)} + \frac{ac' + bd'}{n^2},$$

$$t' = \sqrt{\left(\frac{k^2 - m_2}{n^2} + \frac{(ac' + bd')^2}{n^4}\right)} + \frac{ac' + bd'}{n^2},$$

$$\text{and } t \backsim t' = \sqrt{\left(\frac{\mathrm{M}^2 - m^2}{n^2} + \frac{(ac' + bd')^2}{n^4}\right)} \backsim \sqrt{\left(\frac{k^2 - m^2}{n^2} + \frac{(ac' + bd')^2}{n^4}\right)}$$

Since (eq. 28), $aa' + bb' = 0$, the quantity $\frac{(ac' + bd')^2}{n^4}$, only differs from 0, in consequence of the changes in the values of x' and y', during the interval t. It will always be very small in comparison with $\frac{\mathrm{M}^2 - m^2}{n^2}$. Hence, as $\frac{\mathrm{M}^2 - m^2}{n^2}$, and $\frac{k^2 - m^2}{n^2}$, are always nearly equal, the difference between the above radical quantities will not be sensibly affected by the rejection of the term $\frac{(ac' + bd')^2}{n^4}$ from each. We therefore have,

$$t \backsim t' = \frac{\sqrt{\mathrm{M}^2 - m^2}}{n} \backsim \frac{\sqrt{k^2 - m^2}}{n} \quad - \quad - \qquad (31)$$

The difference between t and t', being added to the approximate time of beginning when M^2 is greater than k^2, but subtracted when M^2 is less than k^2, will give the *true time of beginning* very nearly. When greater precision is desired, the calculation may be repeated, taking the time last found for the approximate time.

The *true time of the end* of the eclipse is found in a similar manner.

71. It is not necessary that the values of x' and y', or of $(p' - u')$ and $(q' - v')$, the quantities which they represent, should be obtained with great precision. We may therefore, in finding p' and q', make use of approximate values of p and q. Now, as the sines of small arcs are very nearly proportional to the arcs themselves, and as $\cos(A - A')$ at the time of an eclipse of the sun, differs but little from a unit, we have for near approximate values of p and q, (art. 64),

$$p = \frac{A - A'}{\varpi - \varpi'} \cos D$$

$$q = \frac{D - D'}{\varpi - \varpi'}.$$

Hence, disregarding the variations, in $\cos D$, and $(\varpi - \varpi')$, we have,

$$p' = \frac{\text{hourly change of } (A - A')}{\varpi - \varpi'} \cos D \quad - \quad - \qquad (32)$$

$$q' = \frac{\text{hourly change of } (D - D')}{\varpi - \varpi'} \quad - \quad - \quad - \qquad (33)$$

If the value of u', increased or decreased uniformly, the difference between the values of u, found for two different times, separated by an interval of an hour, would be the value of u' at the middle of the hour. But the increase or decrease is not uniform. If, however, we find the value of u' from the values of u, taken at an interval of only a few minutes, instead of an hour, the error arising from this cause may be disregarded.

Let T be any given time for which the value of u', is required, and let t be some small interval of time, for instance, the $\frac{1}{20}$ of an hour. Then the difference between the values of u, at the times $T - t$, and $T + t$, will be $\frac{1}{10}$ of the value of u', at the time T.

As the hourly increase of the arc $(\mu - A')$ is always 15°, very nearly, its increase in the time t will be 45'. Hence, taking for $(\mu - A')$, the value which it has at the time T, the value of u at the time $T - t$, is $\varrho \cos \varphi' \sin(\mu - A' - 45')$, and at the time $T + t$, it is $\varrho \cos \varphi' \sin(\mu - A' + 45')$. Therefore,

$$\begin{aligned} \frac{u'}{10} &= \varrho \cos \varphi' \Big(\sin(\mu - A' + 45') - \sin(\mu - A' - 45')\Big) \\ &= 2\varrho \cos \varphi' \cos(\mu - A') \sin 45', \text{ (art. 21)} \\ &= 2\varrho \cos \varphi' \cos(\mu - A') . \frac{2700}{206264.8}. \end{aligned}$$

Or, $u' = \varrho \cos \varphi' \cos (\mu - A'). \frac{54000}{206264.8}$. - - - (34)

In like manner, considering D′ as constant, we find,

$$v' = \varrho \cos \varphi' \sin (\mu - A) \sin D'. \frac{54000}{206264.8}. \quad - \quad - \quad (35)$$

OF THE QUANTITY OF THE ECLIPSE.

72. Let w = the digits eclipsed. Then taking for Σ, the value which it has at the time of nearest apparent approach of the centres, we have,

$$\Sigma = \gamma + \delta - 2\gamma. \frac{w}{12} = \gamma + \delta - \gamma. \frac{w}{6}.$$

Hence, $w = \frac{6\,(\gamma + \delta - \Sigma)}{\gamma}$.

Or, taking the sines of γ, δ and Σ, instead of the arcs themselves,

$$w = \frac{6\,(\sin \gamma + \sin \delta - \sin \Sigma)}{\sin \gamma} \quad - \quad - \quad - \quad (36)$$

Substituting a, for $\cos (A - A') \cos (D - D')$, (art. 62), we have from (art. 64),

$$R - R' = \left(\frac{a}{\sin (\varpi - \varpi')} - \varrho\alpha\right) \tan d' = \frac{a - \varrho\alpha \sin (\varpi - \varpi')}{\sin (\varpi - \varpi')} \tan d'$$

$$= \text{(art. 61)}, \frac{a - \varrho\alpha \sin \varpi}{\sin (\varpi - \varpi')}. \frac{\tan d'}{1 - \varrho\alpha \sin \varpi'}$$

$$= \frac{a - \varrho\alpha \sin \varpi}{\cos d' \sin (\varpi - \varpi')}. \frac{\sin d'}{1 - \varrho\alpha \sin \varpi'}$$

$$= \text{(eq. 15)}, \frac{a - \varrho\alpha \sin \varpi}{\cos d' \sin (\varpi - \varpi')} \sin \gamma.$$

Therefore, $\sin \gamma = (R - R') \cos d'. \frac{\sin (\varpi - \varpi')}{a - \varrho\alpha \sin \varpi}$.

Also, $\sin \delta = \text{(eq. 14)}, \frac{\sin d \cos \Sigma}{a - \varrho\alpha \sin \varpi}$

$$= \text{(art. 64)}, r \cos d' \cos \Sigma. \frac{\sin (\varpi - \varpi')}{a - \varrho\alpha \sin \varpi}.$$

And since, (art. 69), m is the value of $\sqrt{x^2 + y^2}$, or of $\sqrt{(p - u)^2 + (q - v)^2}$, at the time of nearest apparent approach of the centres, we have (eq. 27),

$$\tan \Sigma = m. \frac{\sin (\varpi - \varpi')}{a - \varrho\alpha \sin \varpi}.$$

$$\text{or, } \sin \Sigma = m \cos \Sigma. \frac{\sin (\varpi - \varpi')}{a - \varrho\alpha \sin \varpi}.$$

If these values of sin δ', sin δ and sin Σ, be substituted in the expression for w, (eq. 36), rejecting the factor $\frac{\sin(\pi - \pi')}{a - \rho\alpha \sin \pi}$, which will be common to each term of the numerator and denominator, we have,

$$w = \frac{6\left(R - R' + r \cos \Sigma - m \dfrac{\cos \Sigma}{\cos d'}\right)}{R - R'}.$$

Or, rejecting the factors cos Σ and $\frac{\cos \Sigma}{\cos d'}$, the effect of which on the value of w cannot in any case amount to $\frac{1}{1000}$ of a digit, and putting k, for $R - R' + r$, (art. 64), we have,

$$w = \frac{6\,(k - m)}{R - R'} \qquad (37)$$

Formulæ in which the Longitude and Latitude of the sun and moon are employed, instead of the Right Ascension and Declination.

73. If we now regard EQ, *Fig.* 60, as representing the ecliptic, PB, PC, and PD, will be circles of latitude.

Put $L = EC =$ longitude of the moon
$L' = EB =$ " " sun
$L'' = ED =$ " " geocent. zenith
$\lambda = CM' =$ latitude of the moon
$\lambda' = BX =$ " " sun
$\lambda'' = DZ' =$ " " geocent. zenith
$\varepsilon =$ obliquity of the ecliptic.

Then in like manner as in (art. 55), we have,

$$\begin{aligned}
a &= \cos(L - L') \cos \lambda \cos \lambda' + \sin \lambda \sin \lambda' \\
b &= \sin(L - L') \cos \lambda \\
c &= -\cos(L - L') \cos \lambda \sin \lambda' + \sin \lambda \cos \lambda' \\
\alpha &= \cos(L'' - L') \cos \lambda'' \cos \lambda' + \sin \lambda'' \sin \lambda' \\
\beta &= \sin(L'' - L') \cos \lambda'' \\
\gamma &= -\cos(L'' - L') \cos \lambda'' \sin \lambda' + \sin \lambda'' \cos \lambda'
\end{aligned}$$

The arc λ' it always extremely small, and the quantities a, α, β, and γ, are not required with great precision. Hence, as cos $(L - L')$ at the time of an eclipse, differs but little from a unit, we have, without sensible error in the result,

$$\left.\begin{aligned}
a &= \cos(L - L') \cos \lambda \\
b &= \sin(L - L') \cos \lambda \\
c &= \sin(\lambda - \lambda')
\end{aligned}\right\} \quad - \quad - \quad - \quad (38)$$

$$\left.\begin{aligned}
\alpha &= \cos(L'' - L') \cos \lambda'' \\
\beta &= \sin(L'' - L') \cos \lambda'' \\
\gamma &= \sin \lambda''
\end{aligned}\right\} \quad - \quad - \quad - \quad (39)$$

74. By introducing the right ascension and declination of the zenith, into the expressions for α, β, and γ, instead of the longitude and latitude, we obtain other formulæ, which, though consisting of several terms, are nevertheless more convenient in their practical application to the calculation of an eclipse.

Let EQ, *Fig.* 61, be the equator, EC, the ecliptic, P and p, their poles, Z, the zenith of a place, PA, a circle of declination through Z, and pB, a circle of latitude through the same point. Also let pPFD, ZF, and ZE, be arcs of great circles. Since the arc pD passes through the poles of EQ and EC, it is perpendicular to these circles, and the arcs ED and EF are quadrants.

In the triangle PpZ, we have,

$$\cos pZ = \cos pPZ \sin PZ \sin Pp + \cos PZ \cos Pp$$

$$\text{or, } \sin \lambda'' = -\sin \mu \cos \varphi' \sin \iota + \sin \varphi' \cos \iota.$$

$$\text{Hence, } \gamma = \sin \varphi' \cos \iota - \cos \varphi' \sin \iota \sin \mu \quad - \quad . \quad (40)$$

In the right angled triangles EAZ and EBZ,

$$\cos BE \cos BZ = \cos EZ = \cos AE \cos AZ$$

$$\text{or, } \cos L'' \cos \lambda'' = \cos \mu \cos \varphi'$$

And in the triangles FBZ and FPZ, the former of which is right angled,

$$\cos BF \cos BZ = \cos FZ = \cos FPZ \sin PF \sin PZ + \cos PF \cos PZ$$

$$\text{or, } \sin L'' \cos \lambda'' = \sin \mu \cos \iota \cos \varphi' + \sin \iota \sin \varphi'.$$

Substituting in the expression for β, these values of $\cos L'' \cos \lambda''$ and $\sin L'' \cos \lambda''$, we have,

$$\begin{aligned}\beta &= \sin (L'' - L') \cos \gamma'' \\ &= \sin L'' \cos \lambda'' \cos L' - \cos L'' \cos \lambda'' \sin L' \\ &= \cos \varphi' \cos \iota \sin \mu \cos L' + \sin \varphi' \sin \iota \cos L' - \cos \varphi' \cos \mu \sin L' \\ &= \sin \varphi' \sin \iota \cos L' + \cos \varphi' \sin \mu \cos L' - \cos \varphi' \cos \mu \sin L' - 2 \cos \varphi' \sin^2 \tfrac{1}{2} \iota \sin \mu \cos L'\end{aligned}$$

$$\text{or, } \beta = \sin \varphi' \sin \iota \cos L' + \cos \varphi' \sin (\mu - L') - 2 \cos \varphi' \sin^2 \tfrac{1}{2} \iota \sin \mu \cos L' \quad - - - - - - - - - - \quad (41)$$

In like manner we find,

$$\alpha = \sin \varphi' \sin \iota \sin L' + \cos \varphi' \cos (\mu - L') - 2 \cos \varphi' \sin^2 \tfrac{1}{2} \iota \sin \mu \sin L' \quad - - - - - - - - - \quad (42)$$

75. Substituting the preceding values of a, b, &c. in the expressions for R, R′, p, q, &c. (art. 64), and determining the values of p', q', &c. as in (art. 71), we obtain,

$$R = \frac{\cos (L - L') \cos \lambda \tan d'}{\sin (\pi - \pi')}$$

$$R' = (\varrho \sin \varphi' \sin \iota \sin L' + \varrho \cos \varphi' \cos (\mu - L') - 2 \varrho \cos \varphi' \sin^2 \tfrac{1}{2} \iota \sin \mu \sin L') \tan d'$$

$$r = \frac{\sin d}{\cos d' \sin (\pi - \pi')}$$

$$p = \frac{\sin (L - L') \cos \lambda}{\sin (\pi - \pi')}$$

$$q = \frac{\sin (\lambda - \lambda')}{\sin (\pi - \pi')}$$

$$u = \varrho \sin \varphi' \sin \iota \cos L' + \varrho \cos \varphi' \sin (\mu - L') - 2 \varrho \cos \varphi' \sin^2 \tfrac{1}{2} \iota \sin \mu \cos L'$$

$$v = \varrho \sin \varphi' \cos \iota - \varrho \cos \varphi' \sin \iota \sin \mu$$

$$p' = \frac{\text{hourly change of } (L - L')}{\pi - \pi'} \cos \lambda$$

$$q' = \frac{\text{hourly change of } \lambda}{\pi - \pi'}$$

$$u' = \varrho \cos \varphi' \cos (\mu - L') \tfrac{54000}{206265} - 2 \varrho \cos \varphi' \sin^2 \tfrac{1}{2} \iota \cos \mu \cos L' \tfrac{54148}{206265}$$

$$v' = - \varrho \cos \varphi' \sin \iota \cos \mu \tfrac{54148}{201265}.$$

It may be observed, that the last term of the value of u is equal to the product of the last term of v, multiplied by $\tan \frac{1}{2} \iota \cos L'$; and the last term of the value of u' is equal to the product of the value of v' multiplied by the same factor.

When the places of the sun and moon are obtained from the Nautical Almanac or other similar ephemeris, the calculation of an eclipse may be performed with nearly equal facility, whether the places are denoted by right ascensions and declinations, or by longitudes and latitudes. But when the data are obtained from tables of the sun and moon, it is more convenient to use the formulæ in which longitudes and latitudes are employed.

OF THE GENERAL ECLIPSE.

76. From the formulæ that have been investigated for a given place, we may easily deduce others, that will serve to determine various circumstances with regard to the eclipse for the earth in general.

Substituting $\varrho\beta$ and $\varrho\gamma$, in the place of u and v, in the equation for the beginning and end of an eclipse (eq. 26), we have,

$$k^2 = (p - \varrho\beta)^2 + (q - \varrho\gamma)^2$$

$$\text{or, } \frac{k^2}{\varrho^2} = \left(\frac{p}{\varrho} - \beta\right)^2 + \left(\frac{q}{\varrho} - \gamma\right)^2 \qquad - \quad - \quad (43)$$

As it is not important to ascertain the circumstances of the general eclipse with great precision, it is usual in calculating them to consider $\varrho = 1$, and to omit the small term $\varrho\alpha \tan d'$, in the value of k. We thus have,

$$k^2 = (p - \beta)^2 + (q - \gamma)^2 \qquad - \quad - \quad - \quad (44)$$

When by formulæ, deduced from this equation, the approximate longitude and latitude of a place where any particular phenomenon of the eclipse will occur, have been determined, the values of ϱ, and $\varrho\alpha$ tan d', may be found. Hence, if a more exact determination is required, it may be obtained by repeating the calculation, with the true value of k, and with $\frac{k}{\varrho}$, $\frac{p}{\varrho}$ and $\frac{q}{\varrho}$, instead of k, p and q.

77. Considering m, n, g and h, as *affirmative* quantities, put,

$$\begin{aligned} p &= m \sin M & p' &= n \sin N \\ q &= m \cos M & q' &= n \cos N \\ \beta &= g \sin Q & \gamma &= h \sin S \\ \gamma &= g \cos Q & \alpha &= h \cos S \end{aligned}$$

As p' is always affirmative, the arc N cannot exceed 180°. The other arcs M, Q and S, may be taken less than 180°, or not exceeding 180°, provided they are considered *negative*, when their sines are negative. Restricting thus the extent of the arcs, when either of the quantities p, β, or γ is negative, the corresponding arc will be negative.* As α is always affirmative when the sun is not below the horizon, cos S will be affirmative. Consequently S cannot exceed 90°.

78. Multiply the first of the equations (8) by sin D′, and the third by cos D′, and add the products. Also multiply the first by cos D′, and the third by sin D′, and subtract the second product from the first. We thus obtain,

$$\left.\begin{aligned} \sin \varphi' &= \alpha \sin D' + \gamma \cos D' \\ \cos (\mu - A') \cos \varphi' &= \alpha \cos D' - \gamma \sin D' \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (45)$$

Substituting in these, the values of α and γ (art. 77), we have,

$$\sin \varphi' = h (\cos S \sin D' + \sin S \cos D') = h \sin (S + D') \quad (46)$$

$$\cos (\mu - A') \cos \varphi' = h (\cos S \cos D' - \sin S \sin D') = h \cos (S + D') \quad (47)$$

We have also, (eq. 9),

$$\sin (\mu - A') \cos \varphi' = \beta = g \sin Q \qquad (48)$$

Dividing equations (48) and (46), each by (47), we have,

$$\tan (\mu - A') = \frac{g \sin Q}{h \cos (S + D')} \qquad (49)$$

$$\tan \varphi' = \cos (\mu - A') \tan (S + D').$$

But (art. 77), $g \cos Q = \gamma = h \sin S$.

Consequently, $\frac{g}{h} = \frac{\sin S}{\cos Q}$.

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* The sines and tangents of negative arcs have *contrary* signs to those of affirmative arcs of the same numerical value. But the signs of cosines of negative and affirmative arcs of the same numerical value are the same.

Also (4. 12. A), $\tan\varphi = \frac{1}{1-e^2}\tan\varphi'$.

$$\text{Hence, } \tan(\mu - A') = \frac{\sin S}{\cos Q}\cdot\frac{\sin Q}{\cos(S+D')} = \frac{\sin S\tan Q}{\cos(S+D')}. \qquad (50)$$

$$\tan\varphi = \frac{1}{1-e^2}\cos(\mu - A')\tan(S+D') \qquad (51)$$

The value of $(\mu - A')$ found for a given phenomenon of the general eclipse, at a given time at any first meridian, being subtracted from its value at the same time, at the first meridian, will give the longitude of the place which has the phenomenon at that time. The longitude will be *west* or *east*, according as it is *affirmative* or *negative*.

To find the equation for the beginning and end of the general eclipse.

79. The general eclipse must evidently begin and end when the sun is in the horizons of the places where it begins and ends. But when the sun is in the horizon, the angle Z′OZ, *Fig.* 60, is 90°; and consequently $\alpha = 0$. Hence, in this case, the second of the equations (1), becomes,

$$1 = \beta^2 + \gamma^2 = g^2\sin^2 Q + g^2\cos^2 Q = g^2$$

Consequently, $g = 1$, $\beta = \sin Q$, and $\gamma = \cos Q$.

Substituting these values in (eq. 44), we have,

$$\begin{aligned} k^2 &= (p - \sin Q)^2 + (q - \cos Q)^2 \\ &= 1 + p^2 + q^2 - 2(p\sin Q + q\cos Q) \\ &= 1 + p^2 + q^2 - 2m(\sin M\sin Q + \cos M\cos Q) \\ &= 1 + p^2 + q^2 - 2m\cos(Q - M). \end{aligned}$$

Hence, as (art. 77), $p^2 + q^2 = m^2$, we have,

$$\cos(Q - M) = \frac{1 + m^2 - k^2}{2m} \quad . \quad . \quad . \quad . \quad (52)$$

Now, as cos (Q — M) cannot exceed a unit, it is evident that when the least value of the second member of this equation exceeds a unit, there cannot be an eclipse on any part of the earth. It is also evident that the general eclipse must begin or end when this member becomes equal to a unit. Hence, for the beginning or end of the general eclipse, we have

$$1 = \frac{1 + m^2 - k^2}{2m}$$

$$k^2 = m^2 - 2m + 1$$

$$\text{or, } \pm k = m - 1.$$

$$\text{Hence, } \quad (1 \pm k)^2 = m^2 = p^2 + q^2 \quad . \quad . \quad . \quad (53)$$

With the upper sign, this equation corresponds to the times of beginning and end of the *general eclipse*. With the under one it corresponds to the latest time at which the eclipse can end at sunrise, and the earliest, at which it can begin at sunset.

When the least value of $(p^2 + q^2)$ is greater than $(1 + k)^2$, there can not be an eclipse. When it is between $(1 + k)^2$ and $(1 - k)^2$, the phenomenon of beginning and ending in the horizon continues throughout the duration of the eclipse.

For a *central eclipse*, $k = 0$. Hence the equation corresponding to the times when an eclipse begins or ceases to be central, is,

$$1 = p^2 + q^2 \quad - \quad - \quad - \quad - \quad (54)$$

When the least value of $(p^2 + q^2)$ exceeds a unit, the eclipse cannot be central.

80. Using p, q, p', q' and $(1 + k)$, instead of $p - u$, $q - v$, $p' - u'$, $q' - v'$, and k, the times of the middle and of the beginning and end of the general eclipse, may be found in the same manner as the times of nearest approach of the centres, and beginning and end of an eclipse for a particular place. As p', q', and k, vary but little during the continuance of an eclipse, the approximate times may be regarded as the true times, and consequently the operations need not be repeated. Taking 1 instead of $(1 + k)$, the times of the beginning and end of the central eclipse may be found in like manner.

To find the places at which the general or central eclipse begins or ends.

81. Since $\cos(Q - M) = 1$, we have $Q - M = 0$, or $Q = M$. Also since $\alpha = 0$, we have (art. 77), $\cos S = 0$, and $S = \pm 90^\circ$. Hence,

$$\sin S = \pm 1,$$
$$\cos(S + D') = \cos(\pm 90^\circ + D') = \mp \sin D'$$
$$\text{and } \tan(S + D') = \tan(\pm 90^\circ + D') = -\cot D'.$$

Also, as $g = 1$, we have (art. 77),

$$h \sin S = \gamma = \cos Q,$$
$$h = \frac{\cos Q}{\sin S} = \pm \cos Q,$$

and consequently, $h \cos(S + D') = -\cos Q \sin D'$. (55)

Hence as $Q = M$, we have, (eq. 49),

$$\tan(\mu - A') = \frac{\sin M}{-\cos M \sin D'} = \frac{m \sin M}{-m \cos M \sin D'}$$

$$\text{or, } \tan(\mu - A') = \frac{p}{-q \cos D'} = -\frac{\tan M}{\sin D'} \quad - \quad - \quad - \quad (56)$$

Also, (eq. 51), $\tan \phi = -\frac{1}{1 - e^2} \cos(\mu - A') \cot D'$, (57)

To find for a given time the places that will at that time have the beginning or end of the eclipse in the horizon.

82. The values of p and q at the given time being known, the values

of M and m become known, and thence (eq. 52), the value of $(Q - M)$. Whence Q becomes known. Equation (52) may, however, be changed into another which in some cases is better adapted to the practical determination of $(Q - M)$. Thus subtracting each member from a unit, we have,

$$1 - \cos(Q - M) = 1 - \frac{1 + m^2 - k^2}{2m} = \frac{k^2 - (1 \backsim m)^2}{2m}$$

$$= \frac{\Big(k + (1 \backsim m)\Big) . \Big(k - (1 \backsim m)\Big)}{2m}.$$

But, (art. 8), $1 - \cos(Q - M) = 2 \sin^2 \tfrac{1}{2}(Q - M)$.

$$\text{Hence, } \sin \tfrac{1}{2}(Q - M) = \sqrt{\frac{\Big(k + (1 \backsim m)\Big) \Big(k - (1 \backsim m)\Big)}{4m}} \qquad (58)$$

Since, as is evident from (eq. 48), β must be negative when the sun is in the eastern horizon, and affirmative when it is in the western, it follows that Q must be *negative* or *affirmative*, according as the sun is *rising* or *setting*.

As, (eq. 55), $k \cos(S + D') = - \cos Q \sin D'$, we have (eq. 49).

$$\tan(\mu - A' = \frac{\sin Q}{- \cos Q \sin D'} = - \frac{\tan Q}{\sin D'} \qquad (59)$$

The latitude may be found from (eq. 57).

As an eclipse cannot be visible to those places which remain below the horizon during its continuance, if a series of places be determined that will have the end at sunrise, and another that will have the beginning at sunset, it is evident that a curve line passing through the places in the first series will form the western limit of visibility of the eclipse; and another passing through the places in the second series will form the eastern limit. When the phenomena of beginning and end in the horizon continue throughout the duration of the eclipse, these two lines will meet, and thus form a limit, either to the north or south, as well as to the east and west.

To find a northern or southern limit of visibility of the eclipse.

83. If we combine the equation for the time of the nearest approach of the centres (eq. 28), with that for the time of beginning and end (eq. 26), considering the values of p and q, the same in each, the resulting equation must necessarily appertain to a place that will have the beginning, nearest approach of the centres, and end, at the same instant of time; and that will therefore have only a simple contact of the limbs. A curve line passing through a series of such places will form a northern or southern limit of visibility.

Taking instead of x, y, x', and y', their values (art. 66), we have for the time of the nearest approach of the centres, (eq. 28),

$$(p-u).\ (p'-u')+(q-v).\ (q'-v')=0,$$

$$\text{or,}\quad q-v=-(p-u).\ \frac{p'-u'}{q'-v'}.$$

$$\text{Put,}\qquad \tan N'=\frac{p'-u'}{q'-v'}. \qquad - \qquad - \qquad (60)$$

$$\text{Then,}\quad q-v=-(p-u)\tan N' \qquad - \qquad - \qquad (61)$$

This value of $(q-v)$, substituted in the equation for the beginning and end (eq. 26), gives,

$$\begin{aligned} k^2 &= (p-u)^2+(p-u)^2\tan^2 N' \\ &= (p-u)^2\,(1+\tan^2 N'). \end{aligned}$$

$$\text{Hence,}\quad p-u=\pm k\cos N'$$

$$\text{Also (eq. 61),}\quad q-v=-(p-u)\tan N'=\mp k\sin N'$$

$$\text{Consequently,}\quad \begin{aligned} u &= p\mp k\cos N' \\ v &= q\pm k\sin N' \end{aligned}$$

Or, since (art. 64), $u=\varrho\beta$, and $v=\varrho\gamma$, we have,

$$\left.\begin{aligned} \beta &= \frac{p\mp k\cos N'}{\varrho} \\ \gamma &= \frac{q\pm k\sin N'}{\varrho} \\ \text{and (1),}\quad \alpha &= \sqrt{1-(\beta^2+\gamma^2)} \end{aligned}\right\} \qquad - \qquad - \qquad (62)$$

As u' and v' are contained in the expression for tan N', and as these depend on the situation of the place required, the value of N' can not be found till this has been obtained. But N' never differs greatly from N (art. 77). Therefore, taking $\varrho=1$, we have for an approximate determination of the required place,

$$\left.\begin{aligned} \tan N &= \frac{p'}{q'} \\ \beta &= p\mp k\cos N \\ \gamma &= q\pm k\sin N \\ \alpha &= \sqrt{1-(\beta^2+\gamma^2} \end{aligned}\right\} \qquad - \qquad - \qquad (63)$$

When with these values of α, β, and γ, the approximate situation of the required place has been found by (art. 78), we may obtain approximate values of u' and v', and thence of N'. Then with the values of α, β, and γ, as expressed in (eqs. 62), we may, by repeating the calculation, obtain a result that will be very nearly accurate.

84. This repetition of the calculation is, however, in general, unnecessary. For although the place first found may sometimes be considerably distant from that obtained by repeating the calculation, it will nevertheless be near the limiting line of visibility; being situated easterly or,

westerly from the second place, within the limiting line, but so near it as to have only a very small eclipse.

Supposing a small eclipse to occur at the approximate place, let t be the interval between the time for which the calculation is made and the time of nearest approach of the centres at this place; considering t affirmative, when the time of nearest approach is the later of the two, but negative, when it is the earlier. Then, as we may, without material error, consider $(p - u)$ and $(q - v)$ the same as $(p - \beta)$ and $(q - \gamma)$, we have for the values of $(p - u)$ and $(q - v)$, at the time of nearest approach,

$$p - u = \pm k \cos \mathrm{N} + (p' - u')\, t$$
$$q - v = \mp k \sin \mathrm{N} + (q' - v')\, t$$

These values substituted in the equation for the time of nearest approach of the centres (eq. 61), gives,

$$\begin{aligned}\mp k \sin \mathrm{N} + (q' - v')\, t &= \Big(\mp k \cos \mathrm{N} - (p' - u')\, t\Big) \tan \mathrm{N}' \\ &= \mp k \cos \mathrm{N} \tan \mathrm{N}' - t\,(p' - u') \tan \mathrm{N}' \\ &= \mp k \cos \mathrm{N} \tan \mathrm{N}' - t\,(q' - v') \tan^2 \mathrm{N}'.\end{aligned}$$

Hence, $$(q' - v')\, t = \mp \frac{k \cos \mathrm{N} \tan \mathrm{N}' - k \sin \mathrm{N}}{1 + \tan^2 \mathrm{N}'}.$$

Adding $\mp k \sin \mathrm{N}$ to each member, we have,

$$\begin{aligned}\mp k \sin \mathrm{N} + (q' - v')\, t &= \mp k \left(\sin \mathrm{N} + \frac{\cos \mathrm{N} \tan \mathrm{N}' - \sin \mathrm{N}}{1 + \tan^2 \mathrm{N}'}\right) \\ &= \mp k \sin \mathrm{N}' \cos (\mathrm{N} - \mathrm{N}').\end{aligned}$$

But taking a, b, and m, as in (art. 69), we have,

$$\begin{aligned} m^2 &= a^2 + b^2 \\ &= \Big(\mp k \sin \mathrm{N} + (q' - v')\, t\Big)^2 + \frac{\Big(\mp k \sin \mathrm{N} + (q' - v')\, t\Big)^2}{\tan^2 \mathrm{N}'} \\ &= \frac{\Big(\mp k \sin \mathrm{N} + (q' - v')\, t\Big)^2}{\sin^2 \mathrm{N}'} \\ &= \frac{k^2 \sin^2 \mathrm{N}' \cos^2 (\mathrm{N} - \mathrm{N}')}{\sin^2 \mathrm{N}'} = k^2 \cos^2 (\mathrm{N} - \mathrm{N}').\end{aligned}$$

Hence $m = k \cos (\mathrm{N} - \mathrm{N}')$

As $(\mathrm{N} - \mathrm{N}')$ is always a small arc, its cosine must be nearly equal to a unit, and consequently m must be nearly equal to k. It follows, therefore (eq. 37), that at the place found by the approximate calculation, there can only be a very small eclipse.

To find for a given time the place that will have the greatest phase of the eclipse in the horizon.

85. Since (art. 79), $\beta = \sin Q$, and $\gamma = \cos Q$, we have (art. 64), $u = \rho\beta = \rho \sin Q$, and $v = \rho\gamma = \rho \cos Q$. Substituting these values in the equation for the nearest approach of the centres (61), we have,

$$q - \rho \cos Q = -(p - \rho \sin Q) \tan N',$$

$$q \cos N' - \rho \cos Q \cos N' = -p \sin N' + \rho \sin Q \sin N'$$

$$\cos Q \cos N' + \sin Q \sin N' = \frac{1}{\rho}(p \sin N' + q \cos N')$$

$$\text{or, } \cos(Q - N') = \frac{1}{\rho}(p \sin N' + q \cos N').$$

Taking N, instead of N′, and $\rho = 1$, we have for an approximate determination of the required place,

$$\cos(Q - N) = p \sin N + q \cos N) \qquad (64)$$

$$\text{or, } \sin \tfrac{1}{2}(Q - N) = \sqrt{\frac{1 - (p \sin N + q \cos N)}{2}} \qquad (65)$$

The value of Q being found, the values of $(\mu - A')$ and ϕ may be found from equations 59 and 57.

To find the place that will have the sun centrally eclipsed at a given time.

86. For a central eclipse $\Sigma = 0$. Consequently from equation (27) we have,

$$o = (p - \rho\beta)^2 + (q - \rho\gamma)^2.$$

But when the sum of the squares of two quantities is $= 0$, each quantity must be $= 0$. Hence,

$$p - \rho\beta = 0 \qquad\qquad q - \rho\gamma = 0$$

$$\beta = \frac{p}{\rho} \qquad\qquad \gamma = \frac{q}{\rho}$$

$$\text{or, } \frac{\beta}{\gamma} = \frac{p}{q}.$$

Hence, (art. 77), $\tan Q = \tan M$, and $Q = M$.

Also,

$$\alpha = \sqrt{\left(1 - (\beta^2 + \gamma^2)\right)} = \frac{1}{\rho}\sqrt{\left(\rho - (p^2 + q^2)\right)} = \frac{1}{\rho}\sqrt{(\rho^2 - m^2)}$$

$$\text{Consequently,} \qquad \tan S = \frac{\gamma}{\alpha} = \frac{q}{\sqrt{(\rho^2 - m^2)}} \qquad (66)$$

Or, for an approximate determination,

$$\tan S = \frac{q}{\sqrt{(1 - m^2)}} \qquad (67)$$

As Q is equal to M, we have (eqs. 50 and 51),

$$\tan(\mu - A') = \frac{\sin S}{\cos M} \cdot \frac{\sin M}{\cos(S + D')} = \frac{\sin S \tan M}{\cos(S + D')}.$$

$$\tan \varphi = \frac{1}{1 - e^2} \cos(\mu - A') \tan(S + D')$$

The arc $(\mu - A')$ must be numerically greater or less than 90°, according as $(S + D')$ is greater or less than 90°, and it must have the same sign as the arc M.

87. Considering p' and q' as constant throughout the continuance of the eclipse, the determination of a series of places that will have a given phenomenon, may be facilitated by calculating for equal times before and after the middle of the general eclipse.

From the equations (art. 77) we have,

$$mn \sin M \cos N = pq'$$
$$mn \cos M \sin N = qp'$$
$$mn \sin M \sin N = pp'$$
$$mn \cos M \cos N = qq'.$$

Hence,

$$mn \sin(M - N) = pq' - qp'$$
$$mn \cos(M - N) = pp' + qq'$$

Let T be the time of middle of the general eclipse, and t an interval of time, not exceeding half the duration of the eclipse. Also let a and b be the values of p and q, respectively, at the time T. Then at the time $T \mp t$, we shall have,

$$pq' - qp' = aq' - bp' \mp t.(p'q' - p'q') = aq' - bp',$$
$$pp' + qq' = ap' + bq' \mp t.(p'^2 + q'^2) = \mp tn^2.$$

But, since, $\quad ap' + bq' = 0,$

$$b = -\frac{ap'}{q'}, \text{ or, } bp' = -\frac{ap'^2}{q'},$$

$$\text{and, } aq' - bp' = aq' + \frac{ap'^2}{q'} = \frac{a(p'^2 + q'^2)}{q'} = \frac{an^2}{q'}.$$

Consequently at the time $T \mp t$,

$$pq' - qp' = \frac{an^2}{q'}$$
$$pp' + qq' = \mp tn^2,$$
$$\text{or, } \quad mn \sin(M - N) = \frac{an^2}{q'}$$
$$mn \cos(M - N) = \mp tn^2.$$

Therefore,

$$\tan(M - N) = \frac{a}{\mp tq'}.$$

Thus we may have by the same trigonometrical operation, the values of (M — E), and consequently as N is constant, the values of M, at the times $T - t$ and $T + t$. Also, as $m = \surd(p^2 + q^2)$, it is easily seen that its values at these two times will be the same, and consequently (art. 79), the values of cos (Q — M) will likewise be the same.

For the values of (Q — N), article 85, at the times $T - t$ and $T + t$, we have,

$$\begin{aligned} \cos (Q - N) &= p \sin N + q \cos N \\ &= a \sin N + b \cos N \mp t\,(p' \sin N + q' \cos N) \\ &= \mp t\,(p' \sin N + q' \cos N) \\ &= \mp t\,(n \sin^2 N + n \cos^2 N) \end{aligned}$$

$$\text{or, } \cos (Q - N) = \mp tn.$$

88. By using the equations (39) instead of the equations (3), we may readily obtain formulæ, depending on the longitudes and latitudes of the sun and moon, for determining the longitude and latitude of the zenith of the place that will have a given phenomenon of the eclipse at a given time. With the longitude and latitude of the zenith, its right ascension and declination may be found (6.20). The latter is the geocentric latitude of the place, and the former subtracted from the right ascension of the zenith at the same time at the first meridian, gives the longitude.

OCCULTATIONS AND TRANSITS.

89. Considering the quantities which have been used to refer to the sun, as referring to a fixed star or planet, (equation 17) evidently applies to the beginning or end of an occultation of the body, by the moon. For a fixed star, as its semidiameter and parallax are insensible, the formulæ becomes,

$$\left(\frac{\sin d}{\sin \varpi}\right)^2 = \left(\frac{b}{\sin \varpi} - \varrho\beta\right)^2 + \left(\frac{c}{\sin \varpi} - \varrho\beta\right)^2$$

$$\text{or, } (0.2725)^2 = \left(\frac{b}{\sin \varpi} - \varrho\beta\right)^2 + \left(\frac{c}{\sin \varpi} - \varrho\gamma\right)^2.$$

90. If the quantities which have been used to refer to the moon, be considered as referring to Mercury or Venus, the times of beginning and end of a transit of either of these bodies may be calculated from equation 16, or the first equation in (art. 62); observing that the last term of the first member must be taken affirmative for an external contact, but *negative* for an internal contact.

AN

ELEMENTARY TREATISE

ON

ASTRONOMY.

PART II.

Catalogue of the Tables with observations respecting some of them.

TABLE I.

Latitudes and Longitudes from the Meridian of Greenwich, of some cities and other conspicuous places.

The Latitudes and Longitudes of several of the places in the United States, are given according to the determinations of R. T. Payne, the editor of the Astronomical part of the *American Almanac*, a valuable work, published annually in Boston.

TABLE II.

Mean Astronomical Refractions.

TABLE III.

Mean New Moons, &c. in January. The time of mean new moon in January of each year has been diminished by 15 hours, which has been added to the equations in Table VI. Thus, 4 h. 20 m. has been added to the first equations; 10 h. 10 m. to the second; 10 minutes to the third; and 20 minutes to the fourth. By this means the equations for finding the approximate time of new or full moon, are all made additive.

TABLES IV., V., and VI.

These tables are used with the preceding one, in finding the approximate time of new or full moon.

TABLE VII.

Sun's Mean Longitude, the longitude of the perigee, and Arguments for finding some of the small equations of the sun's place. They are all given for mean noon at the meridian of Greenwich, on the first of January for common years, and on the second of January for bissextiles. The sun's longitudes and the longitudes of his perigee have each been diminished by 2°. As each is diminished by the same quantity, the mean anomaly, which is obtained by subtracting the longitude of the perigee from the sun's longitude, and which is the argument for the equation of the centre, is not affected. The Argument I., is for the equation depending on the action of the moon; Argument II., is for that depending on the action of Jupiter; Argument III., is for that depending on the action of Venus; and Argument N, is for the Nutation, or equation of the equinoxes.

Of the 2° which has been subtracted from the sun's mean longitudes, 1° 59′ 30″ is added to the equation of the centre, and 10″ to each of the small equations due to the actions of the Moon, Jupiter, and Venus.

TABLE VIII.

Motions of the Sun and Perigee and change in the Arguments, for Months.

TABLES IX. and X.

Sun's Hourly Motion and Semidiameter.

TABLES XI. and XII.

Sun's Motions for the Days, Hours, Minutes, and Seconds.

TABLE XIII.

Equation of the Sun's Centre.

TABLE XIV.

Small equations of Sun's Longitude.

TABLE XV.

Mean Obliquity of the ecliptic for the beginning of each year contained in the table.

TABLE XVI.

Nutation in Longitude, Right Ascension and Obliquity of the Ecliptic.

TABLE XVII.

Equation of Time, to convert Apparent into Mean time.

TABLE XVIII.

Earth's Radius Vector.

TABLE XIX.

Mean Right Ascensions and Declinations of some of the fixed stars, for the beginning of 1820, with their annual variations.

TABLE XX.

Epochs of the Moon's Mean Longitude, and of the Arguments for finding the Equations which are necessary in determining the True Longitude and Latitude of the Moon. They are all given for mean noon at the meridian of Greenwich, on the first of January for common years, and on the second of January for bissextiles. The Argument for the Evection is diminished by 29′, the Anomaly by 1° 59′, the Argument for the Variation by 8° 59′, the Mean Longitude by 9° 44′; and the Supplement of the Node is increased by 7′. This is done to balance the quantities which are applied to the different equations to render them affirmative.

TABLES XXI. to XLII., inclusive.

These tables, together with Table XX, are for finding the Moon's True Longitude, Latitude, and Equatorial Parallax.

TABLE XLIII.

Reductions of the Moon's Parallax and of the Latitude of a place, according to the ellipticity $\frac{1}{300}$. The mean value of the reduction of the parallax is given, which is sufficiently accurate for calculations in which the moon's place is obtained from the tables in this work.

TABLES XLIV. and XLV.

Moon's Semidiameter and the augmentation of the semidiameter depending on the altitude.

TABLES XLVI. to LIV., inclusive.

Moon's Hourly Motions in Longitude and Latitude.

TABLE LV.

Contains 5 pages of the Nautical Almanac, taken from the month of May, of that for 1836.

TABLE LVI.

Second differences. This table is useful for finding from the Nautical

Almanac, the Moon's longitude or latitude for any time between noon and midnight.

TABLES LVII. and LVIII.

Third and Fourth differences. These tables are used with the preceding one, when great accuracy is required.

TABLE LIX.

Mercury's Epochs.

TABLES LX., LXI., and LXII.

Mercury's motion for months, days, hours, minutes, and seconds.

TABLES LXIII. and LXIV.

Equation of Mercury's Centre and small equations.

TABLE LXV.

Mercury's Radius Vector.

TABLE LXVI.

Reduction of Mercury's orbit longitude to the ecliptic longitude.

TABLE LXVII.

Heliocentric Longitude and Latitude, &c., of Venus at the times of transit over sun's disc in 1874 and 1882. This table is placed in the postion it occupies, in consequence of the convenient space for it on the page.

TABLES LXVIII. and LXIX.

Mercury's Heliocentric Latitude, Cosine of Latitude, and Aberration in Longitude.

TABLE LXX.

Logistical Logarithms. This table is convenient in working proportions, when the terms are minutes and seconds, or degrees and minutes, or hours and minutes.

TABLE LXXI.

Reductions of Moon's Parallax and of the geographic latitude of a place to the geocentric, according to the compression $\frac{1}{300}$. This table may be used when the reductions are required more accurately than as given in table XLIII.

TABLES LXXII. to LXXVI., inclusive.

These are tables used in calculating eclipses and occultations.

TABLE LXXVII.

Logarithmic Sines, Cosines, and Tangents, and Logarithm of tan ½ . cos L, or logarithm C.

TABLE LXXVIII.

Logarithms from 0 to 1, to three decimal figures, with the corresponding natural numbers.

TABLE LXXIX.

Squares of numbers from 0.01 to 9.00, to two decimal figures.

TABLE LXXX.

Squares of numbers from 0.01 to 6.50, complete.

TABLES LXXXI. to LXXXIV., inclusive.

These are tables calculated by Gaus, for finding the Aberration and Nutation of a star in Right Ascension and Declination.

SCHOLIUM. The tables of the sun were abridged from Delambre's Solar Tables. Commencing with the year 1836, in the table of epochs, the mean longitude of the sun and the longitude of the perigee have been corrected according to the determinations of Bessel, as given in No. 133 of the *Astronomische Nachrichten*. The numbers in the preceding part of the table have not been altered, as a change in these would have required a change in a number of the examples.

The tables of the moon have been abridged from Burckhardt's Lunar Tables, and the tables of Mercury, from Lindenau's tables of this planet.

In each set of these tables, some small equations and also the tenths of seconds have been omitted, and consequently the quantities obtained from them will be liable to small errors. None of these errors will, however, exceed a few seconds.

The heliocentric longitude, latitude, &c. of Venus, at the times of the next two transits, have been calculated from Lindenau's tables of this planet.

OBSERVATIONS AND RULES, RELATIVE TO QUANTITIES WITH DIFFERENT SIGNS.

It is frequently convenient, in computations, to designate certain quantities by the *affirmative* sign +, prefixed; and others by the *negative* sign —. Those which have the affirmative sign prefixed, are called *positive* or *affirmative* quantities; and those with the negative sign, prefixed, are called *negative* quantities.

When a quantity is affirmative, the sign is frequently omitted; but when it is negative, the sign must always be used.

To add quantities, having regard to their signs. When all the quantities have different signs, add them as in common arithmetic, and prefix that sign to the sum. When the quantities have different signs, add the affirmative quantities into one sum, and the negative into another. Then take the difference of these two sums and prefix the sign of the greater. These rules will be illustrated by the following examples.

Add	2'	11"	Add	—3'	51"	Add	—7'	14"
	7	2		—4	10		+8	2
	3	4		—1	15		+3	17
Sum	12	17	Sum	—9	16	Sum	+4	5
Add	+3'	15"	Add	—17'	10"	Add	+3'	1"
	—8	12		— 4	3		—1	15
	—5	1		+12	4		+4	18
	+2	17		+18	59		—6	4
Sum	—7	41	Sum	+ 9	50	Sum	0	0

To Subtract quantities having regard to their signs. Suppose the sign of the quantity which is to be subtracted, to be changed; that is, if it is affirmative, suppose it to be negative; and if it is negative suppose it to be affirmative. Then proceed as in the above rule for adding quantities. Thus,

From	5'	10"	From	4'	11"	From	+2'	5"
Sub.	3	21	Sub.	7	27	Sub.	—1	11
Rem.	1	49	Rem.	—3	16	Rem.	+3	16
From	—8'	29"	From	—2'	18"	From	—4'	17"
Sub.	—3	2	Sub.	—7	11	Sub.	+6	21
Rem.	—5	27	Rem.	+4	53	Rem.	—10	38

To find the Logarithmic Sine, Cosine, Tangent, or Cotangent of an arc, with its proper Sign, from Tables that extend only to each minute of the quadrant.

When the given arc does not exceed 180°. With the given arc, or when it exceeds 90°, with its supplement to 180°, take out from the table the required Sine or Tangent, &c. When there are *seconds*, take out the quantity corresponding to the given degrees and minutes; also take the difference between this quantity and the next following one, in the table. Then 60″ : the odd seconds of the given arc : : the difference : a fourth term. This fourth term, *added* to the quantity taken out, when it is *increasing*, but *subtracted* when it is *decreasing*, will give the required quantity.

When the given arc exceeds 180°. Subtract 180° from it, and proceed as before. When the arc exceeds 270°, it is more convenient, and amounts to the same, to subtract it from 360°.

To determine the Sign of the quantity. Call the arc from 0° to 90°, the *first* quadrant; from 90° to 180°, the *second* quadrant; from 180° to 270°, the *third* quadrant; and from 270° to 360°, the *fourth* quadrant. Then,

The Sine of the arc is *affirmative* for the *first* and *second* quadrants; and *negative* for the *third* and *fourth.*

The Cosine, is *affirmative* for the *first* and *fourth* quadrants; and *negative* for the *second* and *third.*

The Tangent and *Cotangent* are *affirmative* for the *first* and *third* quadrants; and *negative* for the *second* and *fourth.*

By attending to the preceding rules, the student will easily find the Sine, Cosine, &c. of an arc, in either quadrant, with its appropriate sign, as exemplified in the following table:

Arc			Sine	Cosine	Tangent	Cotang.
37°	18′	21″	+9.78252	+9.90060	+ 9.88193	+10.11807
114	35	10	+9.95872	—9.61916	—10.33956	— 9.66044
247	12	36	—9.96470	—9.58811	+10.37659	+ 9.62341
314	17	50	—9.85475	+9.84409	—10.01065	— 9.98935

Note. The signs are seldom placed before affirmative logarithms; but they must not be omitted before negative ones.

The Logarithmic Sine, Cosine, Tangent, or Cotangent of an arc being given, to find the arc.

When the given quantity can be found in the table, under or over its name, take out the corresponding arc. When the given quantity is not found exactly in the table, and the arc is required to seconds, take out the degrees and minutes corresponding to the next less quantity, when that quantity is increasing; but to the next greater when it is decreasing. Take the difference between the quantity corresponding to the degrees taken out, and the next following one in the table; also take the difference between the same quantity and the given one. Then, the first difference : the second : : 60″ : the number of seconds which is to be annexed to the degrees and minutes. Then,

For a Sine. When it is *affirmative,* the required arc will be, either the arc found in the table, or its supplement to 180°. When the sine is *negative*, the required arc will be, either the arc found in the table, increased by 180°, or its supplement to 360°.

For a Cosine. When it is *affirmative*, the required arc will be, either the arc found in the table, or its supplement to 360°. When the cosine is *negative*, the required arc will be, either the supplement of the arc found in the table, to 180°, or that arc increased by 180°.

For a Tangent or Cotangent. When it is *affirmative*, the required arc will be, either the arc found in the table, or that arc, increased by 180°. When the tangent or cotangent is *negative*, the required arc will be, either the supplement of the arc found in the table, to 180°, or its supplement to 360°.

These rules are exemplified by the quantities in the following table :

Sine	+ 9.78252	arc	37°	18′	21″	or	142°	41′	39″
Sine	— 9.85475	arc	225	42	10	or	314	17	50
Cosine	+ 9.90060	arc	37	18	18	or	322	41	42
Cosine	— 9.61916	arc	114	35	11	or	245	24	49
Tangent	+ 9.88193	arc	37	18	21	or	217	18	21
Tangent	—10.33956	arc	114	35	11	or	294	35	11
Cotangent	+ 9.62341	arc	67	12	36	or	247	12	36
Cotangent	— 9.98935	arc	134	17	51	or	314	17	51

Note. Tables which extend only to five decimals, will give the arc, for a tangent or cotangent, true to the nearest second, for a few degrees, near to 0°, 90°, 180°, or 270° : for a sine, near to 0° or 180°; and for a cosine, near to 90° or 270°. In other cases they cannot be depended on to give the seconds accurately. They are, however, sufficient for many calculations; particularly, when the nature of the problem does not make it necessary that the required arc or angle should be determined with great accuracy.

As almost every mathematical student is furnished with a set of such tables, and as an example worked by them will serve as well to illustrate a rule, as if worked by those which are more extensive, they will be used, when necessary, in working the examples and questions in the following problems.

Observations, relative to the Signs of the Logarithms of natural Numbers.

When the logarithm of a natural number is used in calculation, its sign is affirmative or negative, according to that of the number.

When the natural number is a decimal, in order to avoid a difficulty with respect to the sign, the arithmetical complement of the index is used. Thus, when there is no cypher between the decimal point and first significant figure, the index is 9; when there is one cypher, the index is 8; when there are two cyphers, the index is 7; and so on. Thus,

The logarithm of .27 is 9.43136
of .027 is 8.43136
of — .027 is — 8.43136
of .0027 is 7.43136
of — .0027 is — 7.43136

When, in order to get the product or quotient of quantities several logarithms, or logarithms and the arithmetical complements of logarithms, are added together, if they are all affirmative, or if there is an *even* number of *negative* ones, the resulting logarithm will be affirmative; but if there is an *odd* number of *negative* ones, the resulting logarithm will be negative.

When the resulting logarithm of a calculation is the logarithm of a natural number, the number will be affirmative or negative, according to the sign of the logarithm.

When in any of the calculations on the following problems, the resulting logarithm is the logarithm of a natural number, if the index is 9, or near to 9, as 8, 7, &c. the number will be a decimal. When the index is 9, there must be no cypher between the decimal point and first significant figure. When the index is 8, there must be one cypher; when the index is 7, there must be two cyphers; and so on.

PROBLEMS FOR MAKING VARIOUS ASTRONOMICAL CALCULATIONS.

PROBLEM I.*

To work, by logistical logarithms, a proportion, the terms of which are minutes and seconds of a degree, or of time, or hours and minutes.

With the minutes at the top and seconds at the side, or if a term consists of hours and minutes, with the hours at the top and minutes at the side, take from table LXX, the logistical logarithms of the three given terms, and proceed in the usual manner of working a proportion by logarithms. The quantity, in the table, corresponding to the resulting logarithm, will be the fourth term.

Note 1. The logistical logarithm of 60′ is 0.

2. The student will easily perceive that proportions that are worked by logistical logarithms, may also be worked by the common rule in arithmetic.

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* Perhaps, in strict language, this and a few of the following problems are not properly called astronomical. They are, however, for performing subsidiary operations in astronomical calculations.

Exam. 1. When the moon's hourly motion is 31′ 57″, what is its motion in 39 m. 22 sec.? *Ans.* 20′ 58″.

As 60 m. - - - - - -	0
: 39 m. 22 sec. - - - -	1830
: : 31′ 57″ - - - - -	2737
: 20′ 58″ - - - - -	4567

2. If the moon's declination change 2° 29′ in 12 hours, what will be the change in 8 h. 21 m.? *Ans.* 1° 44′.

As 12 h. - - - - - -	6990
: 8 h. 21 m. - - - -	8565
: : 2° 29′ - - - - -	13831
	22396
: 1° 44′ - - - - .	15406

3. When the sun's hourly motion is 2′ 31″, what is its motion in 17 m. 18 sec.? *Ans.* 0′ 44″.

4. When the sun's declination changes 22′ 14″ in 24 hours, what is its change in 19 h. 25 m.? *Ans.* 17′ 59″.

PROBLEM II.

From a table in which quantities are given, for each Sign and Degree of the circle, to find the quantity corresponding to Signs, Degrees, Minutes, and Seconds.

Take out, from the table, the quantity corresponding to the given signs and degrees; also take the difference between this quantity and the next following one. Then 60′ : odd minutes and seconds : : this difference : a fourth term. This fourth term added to the quantity taken out, when the quantities in the table are increasing; but subtracted, when they are decreasing, will give the required quantity.

Note 1. When the quantities change but little from degree to degree, the required quantity may frequently be estimated, without the trouble of making a proportion.

Note 2. The given quantity with which a quantity is taken from a table, is called the *Argument.*

Note 3. In many tables, the argument is given in parts of the circle, supposed to be divided into a 100, a 1000, or 10,000, &c. parts. The method of taking quantities from such tables is the same as is given in the above rule; except that when the argument changes by 10, the first term of the proportion must be 10, and the second, the odd units; when the argument changes by 100, the first term must be 100, and the second, the odd parts between hundreds; and so on.

Exam. 1. Given the argument 1^s 9° 31′ 26″, to find the corresponding quantity in table XXXII. *Ans.* 11° 13′ 57″.

1^s 9° gives 11° 11′ 15″.

The difference between 11° 11′ 15″ and the next following quantity in the table is 5′ 9″.

As 60′ : 31′ 26″ :: 5′ 9″ : 2′ 42″.*
To 11° 11′ 15″
Add 2 42

11 13 57

2. Given the argument 10^s 13° 16′ 54″, to find the corresponding quantity in table XXXV. *Ans.* 93° 32′ 37″.

10^s 13° gives 93° 33′ 40″.

The difference between 93° 33′ 40″ and the next following quantity in the table, is 3′ 43″.

As 60′ : 16′ 54″ : : 3′ 43″ : 1′ 3″
From 93° 33′ 40″
Take 1 3

93 32 37

3. Given the argument 4^s 11° 57′ 10″, to find the corresponding quantity in table XIII. *Ans.* 3° 24′ 12″.

4. Given the argument 3721, to find the corresponding quantity in table XXV. *Ans.* 4′ 52″.

PROBLEM III.

To convert Degrees, Minutes, and Seconds of the Equator into Time.

Multiply the quantity by 4, and call the product of the seconds, thirds; of the minutes, seconds; and of the degrees, minutes.

Exam. 1. Convert 72° 17′ 42″ into time.

72° 17′ 42″
4

4 h. 49 m. 10 sec. 48‴. = 4 h. 49 m. 11 sec. nearly.

2. Convert 117° 12′ 30″ into time. *Ans.* 7 h. 48 m. 50 sec.

3. Convert 21° 52′ 27″ into time. *Ans.* 1 h. 27 m. 30 sec.

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* The student can work the proportion, either by common arithmetic, or by logistical logarithms, as he may prefer.

PROBLEM IV.

To convert Time into Degrees, Minutes, and Seconds.

Reduce the time to minutes, or minutes and seconds; divide by 4, and call the quotient of the minutes, degrees; of the seconds, minutes; and multiply the remainder by 15, for the seconds.

Exam. 1. Convert 5 h. 41 m, 10 sec. into degrees, &c.

h.	m.	sec.
5	41	10
60		
4)341	10	
85°	17′	30″

2. Convert 7 h. 48 m. 50 sec. into degrees, &c. *Ans.* 117° 12′ 30″.
3. Convert 11 h. 17 m. 21 sec. into degrees, &c. *Ans.* 169° 20′ 15″.

PROBLEM V.

The Longitude of two Places, and the Time at one of them being given, to find the corresponding Time at the other.

Express the given time astronomically. Thus, when it is in the morning, add 12 hours, and diminish the number of the day by a unit. When the given time is in the afternoon, it is already in astronomical time.

Find the difference of longitude of the two places, by subtracting the less longitude from the greater, when they are both of the same name, that is, both east, or both west; but by adding the two longitudes together, when they are of different names. When one of the places is Greenwich, the longitude of the other is the difference of longitude.

Then, if the place, at which the time is required, is to the *east* of the other place, *add* the difference of longitude, in time, to the given time; but if it is to the *west*, *subtract* the difference of longitude from the given time. The sum or remainder is the required time.

Note. The longitudes of the places mentioned in the following examples, are given in table I.

Exam. 1. When it is August 8th, 2 h. 12 m. 17 sec. A. M. at Greenwich, what is the time as reckoned at Philadelphia?

	d.	h.	m.	sec.
Time at Greenwich, August, -	7	14	12	17
Diff. of Long., - - -		5	0	40
Time at Philadelphia, - -	7	9	11	37 P. M.

2. When it is April 11th, 3 h. 15 m. 20 sec. P. M. at New York, what is the corresponding time at Greenwich?

	d.	h.	m.	sec.
Time at New York, April,	11	3	15	20
Differ. of Long., - -		4	56	4
Time at Greenwich,	11	8	11	24 P. M.

3. When it is Sept. 10th, 3 h. 20 m. 35 sec. P. M. at Paris, what is the time as reckoned at New Haven?

	h.	m.	sec.
Longitude of Paris, - -	0	9	21E.
do. of New Haven,	4	51	51W.
Diff. of Long., - - -	5	1	12

	d.	h.	m.	sec.
Time at Paris, September,	10	3	20	35
Diff. of Long., - . -		5	1	12
Time at New Haven, -	9	22	19	23

Or Sept. 10th, 10 h. 19 m. 23 sec. A. M.

4. When it is January 15th, 9 h. 12 m. 10 sec. P. M. at Washington, what is the corresponding time at Berlin? *Ans.* Sept. 16, 3 h. 13 m. 52 sec. A. M.

5. When it is Oct. 5th, 7 h. 8 m. A. M. at Quebec, what is the time at Richmond? *Ans.* Oct. 5th. 6 h. 43 m. 18 sec. A. M.

6. When it is noon of the 10th of June at Greenwich, what is the time at Philadelphia? *Ans.* June 10th, 6 h. 59 m. 20 sec. A. M.

PROBLEM VI.

The Apparent *Time being given, to find the corresponding* Mean *Time; or the* Mean *Time being given, to find the* Apparent.

When the given time is not for the meridian of Greenwich, reduce it to that meridian by the last problem. Then, from the tables, take out the sun's Mean Longitude corresponding to this time. Thus, from table VII, take the longitude corresponding to the given year; and from tables VIII, XI, and XII, take the motions in longitude for the months, days, &c. The sum, rejecting 12 signs, when it exceeds that quantity, will be the sun's Mean Longitude, as given by the tables.

With the sun's Mean Longitude, thus found, take the Equation of Time from table XVII. Then, when Apparent Time is given, apply the equation with the Sign it has in the table; but when Mean Time is given, apply it with a contrary Sign; the result will be the Mean or Apparent Time required.

Note 1. In taking the sun's longitude from the tables, it is not necessary to regard the seconds in the given time.

Note 2. The Sun's Mean Longitude, found from the tables in this work, is always two degrees less than its true value; but this difference is allowed for, in arranging the numbers in table XVII.

Note 3. The Equation of Time is given in the Nautical Almanac for each day in the year, at noon, on the meridian of Greenwich, and can easily be found for any intermediate time by proportion.

Exam. 1. On the 15th of August, 1821, when it is 8 h. 15 m. 12 sec. A. M. mean time at Philadelphia, what is the apparent time at the same place?

	d.	h.	m.	sec.
Time at Philadelphia, August 1821,	14	20	15	12
Diff. of Long., - - - -		5	0	40
Time at Greenwich, - - -	15	1	15	52

	M. Long.			
1821 - - - - - -	9ˢ	8°	48′	19″
August - - - - - -	6	28	57	26
15 d. - - - - - -		13	47	57
1 h. - - - - - -			2	28
16 m. - - - - - - -				39
Mean Long. - - - - -	4	21	36	49

The equation of time in table XVII, corresponding to 4ˢ 21° 36′ 49″ is + 4 m. 13 sec.

	d.	h.	m.	sec.
Mean Time at Philadelphia, August 1821,	14	20	15	12
Equation of time, sign changed - .		—	4	13
Apparent time - - - - -	14	20	10	59

Or, August 1821, 15 d. 8 h. 10 m. 59 sec. A. M.

2. On the 18th of October, 1821, when it is 3 h. 21 m. 17 sec. P. M. apparent time at Philadelphia, what is the mean time at Greenwich?

	d.	h.	m.	sec.
Time at Philadelphia, October 1821,	18	3	21	17
Diff. of Long. - - - - -		5	0	40
Time at Greenwich - - -	18	8	21	57

	M. Long.			
1821	9ˢ	8°	48′	19″
October	8	29	4	54
18 d.		16	45	22
8 h.			19	43
22 m.				54
M. Long.	6	24	59	12

Equat. of time — 14 m. 48 sec.

	d.	h.	m.	sec.
Appar. Time at Greenwich, Oct. 1821,	18	8	22	3
Equation of time - - - -			—14	48
Mean Time at Greenwich - -	18	8	7	15

3. On the 15th of May, 1821, when it is 7 h. 12 m. P. M. mean time at Greenwich, what is the apparent time at Boston?

Ans. 2 h. 31 m. 40 sec. P. M.

4. On the 17th of September, 1821, when it is 10 h. 25 m. 32 sec. A. M. apparent time at New York, what is the mean time at Greenwich?

Ans. 3 h. 16 m. 1 sec. P. M.

PROBLEM VII.

To find the Sun's Longitude, Semidiameter, and Hourly Motion, and the apparent Obliquity of the Ecliptic, for a given time, from the Tables.

For the Longitude.

When the given time is not for the meridian of Greenwich, reduce it to that meridian by Prob. V; and if it is apparent time reduce it to mean time by the last problem.

With the mean time at Greenwich, take from tables VII, VIII, XI, and XII, the quantities corresponding to the year, month, day, hour, minute, and second, and find their sums.* The sum in the column of mean longitudes will be the tabular mean longitude of the sun; the sum in the column of perigee, will be the tabular longitude of the perigee; and the sums in the columns I, II, III, and N, will be the arguments for the small equations of the sun's longitude, and for the equation of the equinoxes, which forms one of them.

Substract the longitude of the perigee from the sun's mean longitude, borrowing 12 signs when necessary; the remainder is the sun's Mean Anomaly. With the mean anomaly take the equation of the sun's centre from table XIII; and with the arguments I, II, and III, take the corresponding equations from table XIV. The equation of the centre and the three other equations, added to the mean longitude, gives the sun's true longitude, reckoned from the mean equinox.

With the argument N, take the equation of the equinoxes, or which is the same thing, the nutation in longitude, from table XVI, and apply it, according to its sign, to the true longitude already found, and the result will be the true longitude, from the apparent equinox.

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* In adding quantities that are expressed in signs, degrees, &c., reject 12 or 24 signs, when the sum exceeds either of these quantities. In adding any arguments, expressed in 100, or 1000, &c. parts of the circle, when they are expressed by two figures, reject the hundreds from the sum; when by three figures, the thousands; and when by four figures, the ten thousands.

For the Semidiameter and Hourly Motion.

With the sun's Mean Anomaly, take the Hourly Motion and Semidiameter, from tables IX and X.

For the Apparent Obliquity of the Ecliptic.

To the Mean Obliquity, taken from table XV, apply, according to its sign, the Nutation in Obliquity, taken from table XVI, with the argument N, and the result will be the Apparent Obliquity.

Note. In the Nautical Almanac the sun's Longitude and Semidiameter are given for each day in the year at mean noon; and the Apparent Obliquity of the Ecliptic is given for every tenth day in the year. Either of these quantities may easily be found for any intermediate times by proportion. The sun's Hourly Motion may be found for any time by taking a 24th part of the difference between the longitudes at the preceding and following noons.

Exam. 1. Required the sun's Longitude, Hourly Motion, and Semidiameter, and the Apparent Obliquity of the Ecliptic, on the 18th of October, 1821, at 3 h. 20 m. 24 sec. P. M. mean time at Philadelphia.

	d.	h.	m.	sec.
Mean time at Philadelphia, Oct. 1821,	18	3	20	24
Diff. of Long. - - - - -		5	0	40
Mean time at Greenwich - -	18	8	21	4

	M. Long.	Long. Perigee.	I	II	III	N
1821	9s 8° 48′ 19″	9s 7° 50′ 43″	920	782	260	036
Octob.	8 29 4 54	46	250	684	468	40
18 d.	16 45 22	3	574	43	29	2
8 h.	19 43	0	11	0	0	0
21 m.	52					
4 sec.	0	9 7 51 32	755	509	757	78
		6 24 59 10				
	6 24 59 10					
Eq. Sun's cent.	8 35	9 17 7 38	Mean Anomaly.			
I.	4	Sun's hourly Motion - -			2′	29″
II.	10	Sun's Semidiameter .			16	6
III.	6					
	6 25 8 5	M. Obliq. Ecliptic, 1821,		23°	27′	46″
Nutation.	+ 9	Nutation - - -			+	8
Sun's true long.	6 25 8 14	Appar. Obliquity -		23	27	54

2. Required the sun's longitude, hourly motion, and semidiameter, and the obliquity of the ecliptic, on the 19th of August, 1821, at 7 h. 4 m. 51 sec. A. M. apparent time at Philadelphia. *Ans.* Sun's longitude 4s 26° 6′ 28″; hourly motion 2′ 25″, semidiameter 15′ 51″; obliquity of the ecliptic 23° 27′ 55″.

3. Required the sun's longitude, hourly motion, and semidiameter, and the obliquity of the ecliptic, on the 21st of February, 1824, at 9 h. 6 m. 23 sec. P. M. mean time at Philadelphia. *Ans.* Sun's longitude 11s 2° 27′ 17″; hourly motion 2′ 31″; semidiameter 16′ 12″; obliquity of the ecliptic 23° 27′ 47″.

PROBLEM VIII.

The Obliquity of the Ecliptic and the Sun's longitude being given, to find the Right Ascension and Declination.

For the Right Ascension.

To the Cosine* of the Obliquity, add the Tangent of the Longitude, and reject 10 from the index; the resulting logarithm will be the Tangent of the Right Ascension, which must always be taken in the same quadrant as the Longitude.

For the Declination.

To the Sine of the Obliquity, add the Sine of the Longitude, and reject 10 from the Index; the resulting logarithm will be the Sine of the Declination, which must always be taken out less than 90°; and it will be north or south, according as the sign is affirmative or negative.

Note. The Sun's right ascension, and declination are given, in the Nautical Almanac, for each day in the year. See table LV.

Exam. 1. Given the obliquity of the ecliptic 23° 27′ 40″, and the sun's longitude 125° 31′ 25″, to find the right ascension and declination.

cos. Obliquity	23°	27′	40″	9.96253
tan. Long.	125	31	25	— 10.14635
tan. Right Ascen.	127	53	30	— 10.10888
sin. Obliquity	23°	27′	40″	9.60002
sin. Long.	125	31	25	9.91055
sin. Decl.	18	54	23 N.	9.51057

2. The obliquity of the ecliptic being 23° 27′ 40″, what is the sun's right ascension and declination, when his longitude is 35° 19′ 30″?

Ans. Right ascension 33° 1′ 43″, and declination 13° 18′ 32″ N.

3. Given the obliquity of the ecliptic 23° 27′ 50″, and the sun's longitude 313° 36′ 12″; what is the right ascension and declination?

Ans. Right ascension 316° 4′ 30″, and declination 16° 45′ 29″ S.

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* By the terms Sine, Cosine, &c. are here meant the logarithmic Sine, Cosine, &c. The same is to be understood when the terms are used in the rules for working any of the following problems.

PROBLEM IX.

Given the Obliquity of the Ecliptic and the Sun's Right Ascension, to find the Longitude and Declination.

For the Longitude.

To the arithmetical complement of the Cosine of the Obliquity, add the Tangent of the Right Ascension; and the resulting logarithm will be the Tangent of the Longitude, which must be taken in the same quadrant as the Right Ascension.

For the Declination.

To the Tangent of the Obliquity, add the Sine of the Right Ascension, and reject 10, from the index; the resulting logarithm will be the Tangent of the Declination, which will be north or south, according as the sign is affirmative or negative.

Exam. 1. Given the obliquity of the ecliptic 23° 27′ 50″, and the sun's right ascension 215° 12′ 27″; what is the longitude and declination?

cos. Obliquity	23°	27′	50″	Ar. Co.	0.03748
tan. Right Asc.	215	12	27		9.84857
tan. Long.	217	34	5		9.88605
tan. Obliquity	23°	27′	50″		9.63755
sin. Right Asc.	215	12	27		— 9.76083
tan. Declin.	14	3	0 S.		9.39838

2. When the obliquity of the ecliptic is 23° 27′ 50″, and the sun's right ascension 53° 31′ 20″, what is the longitude and declination?

Ans. Longitude 55° 51′ 16″, and declination 19° 14′ 24″ N.

3. Given the obliquity of the ecliptic 23° 27′ 40″, and the sun's right ascension 187° 15′ 21″; required the longitude and declination.

Ans. Longitude 187° 54′ 6″, and declination 3° 8′ 15″ S.

PROBLEM X.

The Obliquity of the Ecliptic and the Sun's Longitude being given, to find the angle of Position.

To the Tangent of the Obliquity, add the Cosine of the Longitude, and reject 10, from the index; the resulting logarithm will be the Tangent of the angle of Position, which must always be taken less than 90°.

The northern part of the circle of latitude will be to the *West* or *East* of the northern part of the circle of declination, according as the sign of the tangent of the Angle of Position is *affirmative* or *negative*.

Exam. 1. Given the obliquity of the ecliptic 23° 27′ 50″, and the sun's longitude 112° 19′ 17″, to find the angle of Position.

tan. obliquity	23°	27′	50″	9.63755
cos. Long.	112	19	17	—9.57956
tan. Angle of Posit.	9	21	41	—9.21711

The northern part of the circle of latitude lies to the east of the circle of declination.

2. Given the obliquity of the ecliptic 23° 27′ 50″, and the sun's longitude 77° 47′ 30″; what is the angle of position? *Ans.* 5° 14′ 40″; and the northern part of the circle of latitude lies to the west of the circle of declination.

3. When the obliquity of the ecliptic is 23° 27′ 50″, and the sun's longitude 225° 41′ 12″, what is the angle of position? *Ans.* 16° 15′ 7″; and the northern part of the circle of latitude lies to the east of the circle of declination.

PROBLEM XI.

To find, from the Tables, the Moon's Longitude, Latitude, Equatorial Parallax, Semidiameter, and Hourly Motions, in Longitude and Latitude, for a given time.

When the given time is not for the meridian of Greenwich, reduce it to that meridian; and when it is apparent time reduce it to mean time.

With the mean time at Greenwich, take out, from tables XX, XXI, XXII, XXIII, and XXIV, the arguments, numbered 1, 2, 3, &c., to 20, and find their sums, rejecting the ten thousands, in the first nine, and the thousands in the others. The resulting quantities will be the arguments for the first twenty equations of Longitude.

With the same time, and from the same tables, take out the remaining arguments and quantities, entitled Evection, Anomaly, Variation, Longitude, Supplement of the Node, II, V, VI, VII, VIII, IX, and X; and add quantities in the column for the Supplement of the Node.

For the Longitude.

With the first twenty arguments of longitude, take, from tables XXV to XXX, the corresponding equations, and place their sum in the column of Evection. Then, the sum of the quantities in this column will be the corrected argument of Evection.

With the corrected argument of Evection, take the Evection from table XXXI, and add it to the sum of the preceding equations. Place the resulting sum in the column of Anomaly. Then, the sum of the quantities in this column will be the corrected Anomaly.

With the corrected Anomaly, take the Equation of the Centre from table XXXII, and add it to the sum of all the preceding equations. Place the resulting sum in the column of variation. Then, the sum of the quantities in this column will be the corrected argument of variation.

With the corrected argument of Variation, take the variation from table XXXIII, and add it to the sum of all the preceding equations; the result will be the sum of the first twenty-three equations of the Longitude. Place this sum in the column of Longitude. Then, the sum of the quantities in this column will be the Orbit Longitude of the Moon, reckoned from the mean equinox.

Add the Orbit Longitude to the Supplement of the Node. The result will be the argument of the Reduction. It will also be the first argument of Latitude.

With the argument of Reduction, take the Reduction from table XXXIV, and add it to the Orbit Longitude. Also, with the 19th argument, which is the same as argument N, for the Sun's Longitude, take the Nutation in Longitude, from table XVIII, and apply it, according to its sign, to the last sum. The result will be the Moon's true Longitude from the Apparent equinox.

For the Latitude.

Place the sum of the first twenty-three equations of Longitude, taken to the nearest minute, in the column of Arg. II. Then the sum of the quantities in this column will be Arg. II of Latitude, corrected. The Moon's true Longitude is the 3d argument of Latitude. The 20th argument of Longitude is the 4th argument of Latitude. Convert the degrees and minutes, in the sum of the first twenty-three equations of Longitude, into thousandth parts of the circle, by taking from table XXXVIII, the number corresponding to them. Place this number in the columns V, VI, VII, VIII, and IX; but not in column X. Then the sums of the quantities in columns, V, VI, VII, VIII, IX, and X, rejecting the thousands, will be the 5th, 6th, 7th, 8th, 9th, and 10th arguments of Latitude.

With the sum of the Supplement of the Node, and the Moon's Orbit Longitude, which is Arg. 1 of Latitude, take the Moon's distance from the North Pole of the Ecliptic, from table XXXV; and with the remaining nine arguments, take the corresponding equations from tables XXXVI, XXXVII, and XXXIX. The sum of these ten quantities will be the Moon's true distance from the North Pole of the Ecliptic. The difference between this distance and 90°, will be the Moon's true latitude; which will be *North* or *South* according as the distance is *less* or *greater* than 90°.

For the Equatorial Parallax.

With the corrected arguments, Evection, Anomaly, and Variation, take the corresponding quantities from tables XL, XLI, and XLII. Their sum will be the Equatorial Parallax.

For the Semidiameter.

With the Equatorial Parallel, take the Moon's Semidiameter from table XLIV.

For the Hourly Motion in Longitude.

With the arguments, 2, 3, 4, and 5, of Longitude, rejecting the two right hand figures in each, take the corresponding equations from table XLVI. Also with the correct argument of Evection, take the equation from table XLVII.

With the sum of the preceding equations at top, and the correct anomaly at the side, take the equation from table XLVIII. Also with the correct anomaly take the equation from table XLIX.

With the sum of all the preceding equations at the top, and the correct argument of Variation at the side, take the equation from table L. With the correct argument of Variation, take the equation from table LI. And with the argument of Reduction, take the equation from table LII. These three equations added to the sum of all the preceding ones, will give the Moon's Hourly Motion in Longitude.

For the Hourly Motion in Latitude.

With the 1st and 2d arguments of Latitude, take the corresponding quantities from tables LIII and LIV, and find their sum, attending to the signs. Then 32′ 56″ : the moon's true hourly motion in Longitude : : this sum : the moon's true hourly motion in Latitude. When the sign is affirmative, the moon is tending north; and when it is negative, she is tending south.

Exam. 1. Required the moon's longitude, latitude, equatorial parallax, semidiameter, and hourly motions in longitude and latitude, on the 6th of August, 1821, at 8 h. 46 m. 33 sec. A. M. mean time at Philadelphia.

	d.	h.	m.	sec.
Mean time at Philadelphia, August,	5	20	46	33
Diff. of Long., - - - -		5	0	40
Mean time at Greenwich, August,	6	1	47	13

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1821	0027	8365	5389	1368	6970	7714	6319	7024	7800	620	917	842	142	979	067	923	331	134	036	036
August	5804	7776	518	838	1134	8874	2233	2742	5148	873	599	938	245	22	471	759	917	503	31	97
6 d.	137	3249	5201	1435	1678	1860	289	1952	121	351	156	352	171	496	153	183	210	130	1	2
1 h.	1	27	43	12	14	16	2	16	1	3	1	3	1	4	1	2	2	1		
47 m. 13 sec.	1	21	34	9	11	12	2	13	1	2	1	2	1	3						
	5970	9438	1185	3662	9807	8476	8845	1747	3071	849	674	137	560	504	692	867	460	768	68	135

	Evection.	Anomaly.	Variation.	Longitude.	Supp. of Node.	II.	V.	VI.	VII.	VIII.	IX.	X.
1821 .	1s 19° 43′ 47″	8s 9° 54′ 17″	10s 22° 4′ 2″	8s 2° 7′ 41″	0s 13° 3′ 29″	0s 27° 41′	706	711	074	079	637	596
August	7 29 5 59	8 9 46 42	2 4 26 20	9 3 23 46	0 11 13 35	6 24 15	210	371	987	147	597	126
6 d.	1 26 34 57	2 5 19 30	2 0 57 13	2 5 52 55	15 53	1 25 46	170	197	141	169	179	27
1 h.	28 17	32 40	30 29	32 56	8	28	1	2	1	1	1	0
47 m.	22 10	25 35	23 53	25 48	6	22	0	0	0	0	0	
13 sec.	6	7	7	7	0	0	0	0	0	0	0	
Sum of Equa.	36 14	1 47 41	6 1 31	6 32 2		6 32	18	18	18	18	18	
	11 16 51 30	6 27 46 32	3 .4 23 35	7 18 55 15	0 24 33 11	9 25 4	105	299	221	414	432	749
			Reduction	- - 3 15	7 18 55 15							
				7 18 58 30	8 13 28 26	Arg. 1 of Latitude.						
			Nutat. in long.	+ 8								
			Moon's true Longitude	7 18 58 38								

Arg.	Equ. ☽'s Long.		
1	0°	18′	51″
2		1	39
3			20
4		5	42
5		1	36
6		2	39
7		1	39
8			17
9		1	4
10			16
11			14
12			6
13			27
14			20
15			19
16			10
17			8
18			3
19			7
20			17
	0	36	14
Ev.	1	11	27
Sum.	1	47	41
An.	4	13	50
Sum.	6	1	31
Var.		30	31
Sum.	6	32	2

Arg.	Eqs. ☽'s Lat.		
I	94°	43′	57″
II		16	57
☽'s long.			2
20, long.			4
V			45
VI			7
VII			3
VIII			20
IX			18
X			43
	95	3	16
	90	0	0
Moon's Lat.	5	3	16 S.

Arg.	☽'s Eq. Par.	
Ev.	1′	27″
An.	53	0
Var.		4
Moon's Eq. Par.	54	31

Moon's Semidiameter 14′ 51″

Moon's Hourly motion in longitude.

Arguments.	Equations.	
2 of long.	0′	5″
3 do.		2
4 do.		1
5 do.		3
Evection	1	19
	1	30
An. & Sum. Eqs.		6
Anomaly	27	58
	29	34
Var. & Sum. Eqs.		9
Variation		3
Reduction		17
Hourly Motion in Long.	30	3

Moon's Hourly Motion in Latitude.

Arg.	Equations	
I	−0′	51″
II	+	2
Sum.	−0	49

As 32′ 56″ : 30′ 3″ : : — 0′ 49″ : — 0′ 45″ = Moon's hourly motion in latitude tending south.

2. Required the moon's longitude, latitude, equatorial parallax, semidiameter, and hourly motions in longitude and latitude, on the 27th of April, 1821, at 9 h. 43 m. 30 sec. P. M. mean time at Baltimore. *Ans.* Long. 11ˢ 13° 32′ 13″; lat. 6′ 58″ N. equat. par. 60′ 0″; semidiam. 16′ 21″; hor. mot. in long. 36′ 11″; and hor. mot. in lat. 3′ 14″, tending north.

3. What will be the moon's longitude, latitude, equatorial parallax, semidiameter, and hourly motions in longitude and latitude, on the 19th of August, 1822, at 5 h. 56 m. 14 sec. P. M. mean time at Philadelphia? *Ans.* Long. 6ˢ 3° 7′ 12″; lat. 3° 51′ 35″ S.; equat, par. 56′ 19″; semidiam. 15′ 21″; hor. mot. in long. 32′ 7″; and hor. mot. in lat. 2′ 1″, tending south.

PROBLEM XII.

To find the Moon's Longitude, Latitude, Hourly Motions, Equatorial Parallax, and Semidiameter, for a given Time, from the Nautical Almamac.

Reduce the given time to Apparent time at Greenwich. Then,

For the Longitude.

Take from the Nautical Almanac, the two longitudes, for the noon and midnight, or midnight and noon, next preceding the time at Greenwich, and also the two immediately following these, and set them in succession, one under another. Then, having regard to the signs, subtract each longitude from the next following one, and the three remainders will be the *first differences.* Call the middle one A. Subtract each first difference from the following, for the *second differences.* Take the half sum of the second differences, and call it B.

Call the excess of the given time at Greenwich, above the time of the second longitude, T. Then 12 h : T : : A : *fourth term,* which must have the same sign as A.

With the time T at the side, take from table LVI the quantities corresponding to the minutes, tens of seconds, and seconds of B, at the top, the sum of these, with a *contrary* sign to that of B, will be the *correction of second differences.*

The sum of the second longitude, the fourth term, and the correction of second differences, having regard to the signs, will be the required longitude.

For the Hourly Motion in Longitude.

To the logistical logarithm of $\frac{1}{12}$ of T, add the logistical logarithm of B, and find the quantity corresponding to the sum. Call this quantity E, and prefix to it the same sign as that of B.

Or E may be found without logarithms; thus, 12 h. : T : : B : E.

Divide the sum of A, ½ B with is sign changed, and E, by 12, and the quotient will be the required hourly motion in longitude.

For the Latitude.

Prefix to *north* latitudes the *affirmative* sign, but to *south* latitudes the *negative* sign, and then proceed in the same manner as for the longitude. The resulting latitude will be *north* or *south*, according as its sign is *affirmative* or *negative*.

Note. The Moon's Declination may be found in the same manner.

For the Hourly Motion in Latitude.

With T, and the values that A and B have, in finding the latitude, find the hourly motion in latitude, in the same manner as directed for finding the hourly motion in longitude. When the resulting hourly motion in latitude is *affirmative*, the moon is tending *north*; and when it is *negative*, she is tending *south*.

For the Semidiameter and Equatorial Parallax.

The moon's semidiameter and equatorial, horizontal parallax, may be taken from the Nautical Almanac with sufficient accuracy by simply proportioning for the odd time between noon and midnight or midnight and noon.

Exam. 1. Required the moon's longitude, latitude, equatorial parallax, semidiameter, and hourly motions in longitude and latitude from the Nautical Almanac, table LV, on the 4th of May, 1836, at 4 h. 30 m. 8 sec. P. M. mean time at Philadelphia.

		d.	h.	m.	sec.
Mean time at Philadelphia, May - -		4	4	30	8
Diff. of Longitude - - - - -	+		5	0	40
Mean time at Greenwich, May, - -		4	9	30	48

For the Longitude and Hourly Motion in Longitude.

	Longitudes			1st diff.			2d diff.	
3d midn.	260°	32′	39.3″	7	23	46.9	—2′	34.4″
4th noon	267	56	26.2	7	21	12.5	—3	15.1
" midn.	275	17	38.7	7	17	57.4		
5th noon	282	35	36.1					
				7	21	12.5	—2	54.7
					A		B	

T A

h. h. m. sec.

12 : 9 30 48 : : 7° 21′ 12″.5 : 5° 49′ 46″.8, fourth term.

Second longitude	267°	56′	26.2″
Fourth term	5	49	46.8
Cor. for 2d diff.	+		14.4
Moon's true longitude	273	46	27.4

	m.	sec.		
$\frac{1}{12}$ T	47	34	L. L.	1008
B	— 2′	54″.7	L. L.	— 13141
E	— 2	18.5		— 14149

A	7°	21′	12.5″
½ B, sign changed	+	1	27.3
E	—	2	18.5
	12)7	20	21.3
Hor. mot. in long.		36	41.8

For the Latitude and Hourly Motion in Latitude.

	Latitudes	1st diff.	2d diff.
3d midn.	— 2° 41′ 22.2″		
		— 33 52.6	
4th noon	— 3 15 14.8		+ 3 26.3
		— 30 26.3	
" midn.	— 3 45 41.1		+ 3 54.9
		— 26 31.4	
5th noon	— 4 12 12.5		
		— 30 26.3 A	3 40.6 B

	T			A		
h.	h. m. sec.					
12 :	9 30 48	: :	— 30′ 26.3″	:	— 24′ 7.″8, fourth term.	

Second latitude	—3°	15′	14.8″
Fourth term	—	24	7.8
Cor. for 2d diff.	—		18.0
Moon's true latitude	3	39	40.6 S.

	m.	sec.		
$\frac{1}{12}$ T	47	33	L. L.	1008
B	3	40.6	L. L.	12127
E	2	54.9		13135

A	— 30′	26.3″
½ B, sign changed	— 1	50.3
E	+ 2	54.9
	12)— 29	21.7
Moon's hor. mot. in lat.	— 2	26.8, tending south.
Moon's semidiameter	16′	24.2″
" equatorial parallax	60	11.7

2. Required the moon's longitude, latitude, equatorial parallax, semidiameter, and hourly motions in longitude and latitude, on the 6th of May, 1836, at 1 h. 41 m. 40 sec. P. M. mean time at Greenwich.

Ans. Long. 297° 59′ 57.1″; lat. 4° 54′ 18.6″ S; equat. par. 59′ 15.0″; semidiam. 16′ 8.7″; hor. mot. in long. 35′ 34.9″; hor. mot. in lat. 1′ 13.7″, tending south.

Note 1. When the moon's longitude and latitude are required with great precison, the third and fourth differences should be noticed. To do this, take from the ephemeris, the three longitudes or latitudes, preceding the given time, and the three following it, and find the first, second, third, and fourth differences, as directed in the rule, for the first and second differences. Call the middle first difference, A, the half sum of the two middle second differences, B, the middle third difference, C, and the half sum of the fourth differences, D. Then taking T, equal the excess of the given time above the time of the third longitude or latitude, find the fourth term and the correction for second differences, as directed in the rule.

With the time T, and middle third difference, C, take from table LVII, the correction for third differences, which, when T is less than 6 hours, must have the same sign as C, but a *contrary* sign, when T is more than 6 hours.

With the time T, and half sum of fourth differences, D, take from table LVIII, the correction for fourth differences, which must always have the same sign as D.

The sum of the third longitude or latitude, the fourth term, and the corrections for second, third, and fourth differences, having regard to the signs of all the quantities, will be the longitude or latitude required.

Note 2. When great precision is required in the moon's parallax and semidiameter, the corrections for second differences should be applied in the same manner as for the longitude or latitude.

Taking the time, as in the first example, let the moon's longitude be required, as corrected for third and fourth differences.

	Longitudes	1st diff.	2d diff.	3d diff.	4th diff.
3d noon	253° 7′ 3.7″	7° 25′ 35.6″			
" midn.	260 32 39.3	7 23 46.9	—1 48.7		
4th noon	267 56 26.2	7 21 12.5	—2 34.4	—45.7″	+ 5.0″
" midn.	275 17 38.7	7 17 57.4	—3 15.1	—40.7	+ 6.2
5th noon	282 35 36.1	7 14 7.8	—3 49.6	—34.5	
" midn.	289 49 43.9				
		7 21 12.5	—2 54.7	—40.7	+ 5.6
		A	B	C	D

Third longitude - - - - - -	267°	56′	26.2″
Fourth term - - - - - -	5	49	46.81
Cor. for 2d diff. - - - - - -	+		14.31
" 3d " - - - - - -	+		0.31
" 4th " - - - - - -	+		0.06
Moon's true longitude - - - -	273	46	27.7

PROBLEM XIII.

The Moon's Equatorial Parallax, and the Latitude of a Place being given, to find the Reduced Parallax and Latitude.

With the Latitude of the place, take the Reductions from table XLIII,[*] and subtract them from the Parallax and Latitude.

Exam. 1. Given the equatorial parallax 54′ 31″, and the latitude of Philadelphia 39° 57′ N. to find the reduced parallax and latitude.

Equatorial parallax	54′	31″
Reduction		5
Reduced parallax	54	26
Latitude of Philadelphia	39°	57′ N.
Reduction		11
Reduced Lat. of Philadelphia	39	46 N.

2. Given the equatorial parallax 60′ 0″, and the latitude of Boston 42° 23′ N. to find the reduced parallax and latitude. *Ans.* Reduced par. 59′ 55″, and reduced lat. 42° 12′ N.

3. Given the equatorial parallax 57′ 21″, and the latitude of Charleston 32° 50′ N. to find the reduced parallax and latitude. *Ans.* Reduced par. 57′ 18″, and reduced lat. 32° 40′ N.

Note. When the reductions of parallax and latitude are required with precision, they must be taken from table LXXI.

PROBLEM XIV.

To find the Mean Right Ascension and Declination of a Star for a given Time, from the tables.

Take the difference between the time for which the table is constructed and the given time, and multiply the annual variation, by the number of years in this difference; the product will be the variation for the years. Reduce the odd time to days.[*] Then 365 days : number of days : : annual variation : proportional part. This proportional part, added to the variation for the years, will be the whole variation, which applied to the quantity given in the table, with its proper sign, when the given time is after the time for which the table is constructed, but with a *contrary* sign when it is before, will give the required quantity.

Exam. Required the mean right ascension and declination of *Regulus*, on the 15th of June, 1821.

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[*] This, when the given time is after the time for which the table is constructed, may be done very simply by taking from table V, the number of days corresponding to the month, and adding to it the odd days.

Mean right ascen. begin. of 1820, table XIX,	149°	41′	39″
Var. for 1 yr. 166d.	+	1	10
Mean right ascen. required	149	42	49
Mean declin. begin. of 1820,	12°	50′	36″ N
Var. for 1 yr. 166 d.	—		25

2. Required the mean right ascension and declination of β *Tauri*, on the 6th of November, 1822. *Ans.* Mean right ascen. 78° 46′ 29″ and mean declin. 28° 26′ 53″ N.

PROBLEM XV.

To find the Aberration of a Star, in right Ascension and Declination, for a given Day.

Find the mean right ascension and declination of the star, for the given time, by the last problem. Also find the sun's true longitude for noon of the given day by prob. VII, or take it from the Nautical Almanac.

Designate the sun's longitude by $\ominus$, the right ascension of the star by A, and the declination by D.

With the argument $\ominus$, take the quantity x from table LXXXI; add it to $\ominus$, attending to the sign and from the sum subtract A. Then,

For the Aberration in Right Ascension.

With argument $\ominus$, take from the table LXXXI, the log. a, with its proper sign, and to it add the Cosine of $(\ominus + x - A)$, and the arithmetical complement of the Cosine of D, rejecting the tens in the index of the sum. The natural number, corresponding to the resulting logarithm, will be the aberration in right ascension, to be applied to the *mean* right ascension.

For the Aberration in Declination.

Add together the log. a, the Sine of $(\ominus + x - A)$, and the Sine of D, and reject the tens in the index of the sum. Take the natural number, corresponding to the sum, and call it m. With the arguments $\ominus + D$ and $\ominus - D$, or when the declination is south, with these arguments, each increased by VI signs, take the corresponding quantities from table LXXXII. The sum of these quantities, and m, giving attention to the signs, will be the aberration in declination, to be applied to the *mean* declination.

Exam. 1. What are the aberrations in right ascension and declination, of *Regulus*, on the 15th of June, 1821, the sun's longitude on that day, being 2^s 23° 58′ ?

$A = 149° 42' 49'' =$ mean right ascen. of Regulus.
$D = 12 \quad 50 \quad 11$ N = mean declin. do.
$\odot = 2^s \; 23° \; 58' =$ sun's longitude.

$\odot$ - - - - -	2^s 23° 58′
x, from table LXXXI,	+ 0 30
$\odot + x$ - - -	2 24 28
A - - - - -	4 29 43
$\odot + x - A$ - - -	9 24 45 = 294° 45′

log. a, from tab. LXXXI,	- - - -	— 1.3061
cos. $(\odot + x - A)$ - - -	294° 45′	9.6219
cos D - - - - - -	12 50 Ar. Co.	0.0110
Aber. in right ascen.	—8.69″ - - -	— 0.9390
log. a - - - - - - - - -		— 1.3061
sin $(\odot + x - A)$ -	294° 45′ - - -	— 9.9581
sin D - - - -	12 50 N. - -	9.3466
m - - - -	+ 4.08″ - - -	0.6108
Arg. $(\odot + D) = 3^s$ 6° 48′, gives	- - -	+ 0.48″
Arg. $(\odot - D) = 2$ 11 8, gives	- - -	— 1.30
m - - - - - - - - -		+ 4.08
Aber. in declination - - - - - - -		+ 3.26

2, Required the aberrations in right ascension and declination, of *Antares*, on the 11th of March, 1821, the sun's longitude being 11^s 20° 38′. *Ans.* Aber. in right ascen. + 5.43″, in declin. — 0.74″

3. On the 6th of November, 1822, the sun's longitude will be 7^s 13° 32′; what will be the aberrations in right ascension and declination of *β Tauri?* *Ans.* Aber. in right ascension + 18.54″, and in declination + 0.15″.

PROBLEM XVI.

To find the Nutations of a Star in Right Ascension and Declination, for a given Time.

Find the Supplement of the Moon's Node, from tables XX, XXI and XXII, and subtract it from 12^s 0° 7′; the remainder will be the Mean Longitude of the Moon's Ascending Node.

Designate the right ascension of the body by A, the declination by D, and the mean longitude of the moon's node by N.

With the argument N, take the quantity B, from table LXXXIII; add to it N, attedding to the sign, and from the sum, subtract A.

For the Nutation in Right Ascension.

With the argument N, take from table LXXXIII, the log. *b*, with its proper sign, and to it add the Cosine of (N + B — A), and the tangent of D, marking it negative, when the declination is south, and reject the tens in the index of the sum. Apply the natural number corresponding to the sum, to a quantity, taken from table LXXXIV, with the argument N, and the result will be the nutation in right ascension, to be applied to the mean right ascension.

For the Nutation in Declination.

To the log. *b*, add the sine of (N + B — A), or when the declination is south, the sine of (N + B — A + VI^s) rejecting the tens in the index, and the natural number corresponding to the sum will be the nutation in declination, to be applied to the *mean* declination.

Exam. 1. Required the nutations, in right ascension and declination, of *Rigel*, on the 19th of July, 1825.

By prob. XIV, A = 76° 32′ 8″ and D = 8° 24′ 36″ S.

	Supp. of Node		
1825	3^s	0°	25′
July		9	35
19 d			57
	3	10	57
	12	0	7
N	8	19	10
B, from tab. LXXXIII,	—	3	35
N + B	8	15	35
A	2	16	33
N + B — A	5	29	2 = 179° 2′

log. *b*, from tab. LXXXIII		— 0.8623
cos. (N + B — A)	179° 2′	— 9.9999
tan. D	8 25	— 9.1702
nat. numb.	—1″.08	— 0.0324
From tab. LXXXIV.	+ 16.25	
Nut. in right ascen.	+ 15.17	
log. *b*,		—0.8623
sin. (N + B — A + VI^s)	359° 2′	—8.2271
Nut. in declin.	+ 0″.12	+ 9.0894

2. Required the nutations, in right ascension and declination, of *Antares*, on the 11th of March, 1821. *Ans.* Nut. in right ascen. + 3″.70, and in declin. + 9″.23.

3. What will be the nutations, in right ascension and declination of β *Tauri*, on the 6th of November, 1822. *Ans.* Nut. in right ascen. + 14″.61, and in declin + 7″.3.

PROBLEM XVII.

The Obliquity of the ecliptic and the Right Ascension and Declination of a Body being given, to find the Longitude and Latitude.

Designate the obliquity of the ecliptic by E. To the tangent of the declination marked negative when the declination is south, add the arithmetical complement of the sine of the right ascension; the result will be the tangent of an arc, which call B. The arc B must be taken according to the sign, but always less than 180°.

For the Longitude.

Add together the cosine of the difference between B and E, the tangent of the right ascension, and the arithmetical complement of the cosine of B, rejecting ten from the index; the result will be the tangent of the longitude, which must be taken according to the sign, observing also that the longitude and right ascension are always, either both between 90° and 270°, or reckoning in the order of the signs, both between 270° and 90.°

For the Latitude.

To the tangent of the difference between B and E, which must be marked negative, not only when the difference is greater than 90°, but also when E is greater than B, add the sine of the longitude, rejecting ten from the index; the result will be the tangent of the latitude, which must always be taken less than 90°, and will be *north* or *south*, according as the sign is affirmative or negative.

Note. When the mean obliquity of the ecliptic and the mean right ascension and declination are used, the results will be the mean longitude and latitude. But when the apparent obliquity of the ecliptic, found by prob. VII, and the apparent right ascension and declination, found by applying to the mean right ascension and declination, the aberrations and nutations, obtained by problems XV and XVI, are given, the results will be the apparent longitude and latitude.

Exam. 1. On the 10th of April, 1821, the mean right ascension of *Arcturus* was 211° 52′ 37″, the mean declination 20° 7′ 4″ N., and the mean obliquity of the ecliptic 23° 27′ 46″. What were its longitude and latitude?

tan. declin.	20° 7′ 4″ N.	9.56384
sin. right asc.	211 52 37 Ar. Co.	—0.27729
tan. B	145 15 13	—9.84113
E	23 27 46	
cos. (B ∽ E)	121 47 27	—9.72166
tan. right asc.	211 52 37	9.79371
cos. B	145 15 13 Ar. Co.	—0.08529
tan. long.	201 44 16	9.60066
tan. (B ∽ E)	121° 47′ 27″	—10.20774
sin. long.	201 44 16	— 9.56863
tan. lat.	30 51 37 N.	9.77637

2. Given the obliquity of the ecliptic 23° 27′ 47″, the right ascension of *Rigel*, 76° 28′ 21″, and the declination 8° 25′ 2″ S., on the 1st of January, 1820, to find the longitude and latitude. *Ans.* Long. 74°. 18′ 51″, and lat. 31° 8′ 45″ S.

3. On the first of January, 1821, the right ascension of *Procyon* was 112° 28′ 49″, the declination 5° 40′ 35″ N., and the obliquity of the ecliptic 23° 27′ 46″. What were its longitude and latitude? *Ans.* 113° 18′ 55″, and 15° 59′ 0″ S.

PROBLEM XVIII.

The Obliquity of the Ecliptic, and the Longitude and Latitude of a Body, being given, to find the Right Ascension and Declination.

Designate the obliquity of the ecliptic by E. To the tangent of the latitude, marked negative when the latitude is south, add the arithmetical complement of the sine of the longitude; the result will be the tangent of an arc, which call B. The arc B must be taken, according to the sign, but always less than 180°.

For the Right Ascension.

Add together the cosine of the sum of B and E, the tangent of the longitude, and the arithmetical complement of the cosine of B, rejecting ten from the index; the result will be the tangent of the right ascension, which must be taken according to the sign, observing also that the right ascension and longitude are always, either both between 90° and 270°, or reckoning in the order of the signs, both between 270° and 90°.

For the Declination.

To the tangent of the sum of B and E, add the sine of the right ascension, rejecting ten from the index; the result will be the tangent of the declination, which must always be taken less than 90°, and will be *north* or *south*, according as the sign is affirmative or negative.

Note. The quantities found will be mean or apparent, according as the given ones are mean or apparent.

Exam. 1. Given the obliquity of the ecliptic 23° 27′ 46″, the longitude of *Arcturus* 201° 44′ 16″, and the latitude 30° 51′ 37″ N., to find the right ascension and declination.

tan. lat.	30°	51′	37″ N.	9.77637
sin. long.	201	44	16 Ar. Co.	—0.43137
tan B	121	43	28	—10.20774
E	23	27	46	
cos. (B + E)	145	15	14	—9.91471
tan. long.	201	44	16	9.60066
cos. B	121	47	28 Ar. Co.	—0.27834
tan. right asc.	211	52	36	9.79371
tan. (B + E)	145°	15′	14″	—9.84113
sin. right asc.	211	52	36	—9.72271
tan. declin.	20	7	5 N.	9.56384

2. Given the obliquity of the ecliptic 23° 27′ 47″, the longitude of *Rigel* 74° 18′ 51″, and the latitude 31° 8′ 45″ S., to find the right ascension and declination. *Ans.* Right ascen. 76° 28′ 21″, and declin. 8° 25′ 1″ S.

3. When the obliquity of the ecliptic was 23° 27′ 46″, the longitude of *Procyon* 113° 18′ 55″, and the latitude 15° 59′ 0″, S., what were the right ascension and declination? *Ans.* Right ascen. 112° 28′ 48″, and declin. 5° 40′ 35″ N.

PROBLEM XIX.

The Obliquity of the Ecliptic, and the Longitude and Declination of a body being given, to find the Angle of Position.

Add together the cosine of the longitude, the sine of the obliquity, and the arithmetical complement of the cosine of the declination, taking them all affirmative, and reject ten from the index; the result will be the sine of the angle of position; which, in all cases where the problem is used in calculating an occultation of a planet or star, by the moon, must be taken less than 90°.

When the longitude is less than 90° or more than 270°, the northern part of the circle of latitude lies to the *west* of the circle of declination; but when the longitude is between 90° and 270°, it lies to the *east*.

Exam. 1. Given the obliquity of the ecliptic 23° 27′ 46″, the longitude of *Arcturus* 201° 44′ 16″, and the declination 20° 7′ 5″ N., to find the angle of position.

cos. long.	201°	44′	16″		9.96797
sin. obliq.	23	27	46		9.60005
cos. declin.	20	7	5	Ar. Co.	0.02734
sin. ang. posit.	23	11	46		9.59536

The circle of latitude lies to the east of the declination.

2. Given the obliquity of the ecliptic 23° 27′ 47″, the longitude of *Rigel* 74° 18′ 51″, and the declination 8° 25′ 1″ S.; required the angle of position. *Ans.* 6° 14′ 50″.

3. When the obliquity of the ecliptic was 23° 27′ 46″, the longitude of *Procyon* 113° 18′ 55″, and the declination 5° 40′ 35″ N., what was the angle of position? *Ans.* 9° 6′ 43″.

PROBLEM XX.

To find the Heliocentric Longitude and Latitude, and the Radius Vector of Mercury, for a given time.

Reduce the given time to mean time at Greenwich. Take from table LIX, the mean longitude of Mercury, the longitudes of the aphelion and node, and the arguments II and III, corresponding to the given year. Under the three former, place the motions for the months, days, hours, minutes and seconds of the given time, taken from tables LX to LXII; and under each of the latter, place the number D, in table LX, corresponding to the given month, and also the number expressing the day of the month, diminished by a unit. Add together the quantities in each column, rejecting 12 signs when either sum in the first three columns admits the rejection, but setting down the whole amount in each of the last two columns. Subtract the resulting longitude of the aphelion from the mean longitude of the planet, and the remainder will be the mean anomaly.

With the mean anomaly as the argument, take the equation of the centre from table LXIII, and applying it according to its sign to the mean longitude, add to the result the equations II and III, taken from table LXIV, with their respective arguments. The sum will be the *orbit longitude* of Mercury.

From the orbit longitude subtract the longitude of the node, and the remainder will be the argument for the latitude; it will also be the argument for the reduction to the ecliptic. With this argument take the reduction from table LXVI, and apply it according to its sign to the orbit longitude. The result will be the heliocentric ecliptic longitude, reckoned from the mean equinox. With the argument N, found from the solar tables for the given time, take the nutation in longitude from table XVI, and apply it according to its sign to the longitude from the mean equinox, and it will give the longitude from the true equinox.

With the argument of latitude take from table LXVIII, the latitude, and also its secular variation. Multiply the secular variation by the number of years, the given time is subsequent to 1800, and divide the product by 100. The result added to the latitude taken from the table, will give the correct heliocentric latitude.

With the mean anomaly as the argument, take the radius vector from table LXV.

Exam. 1. Required the heliocentric longitude and latitude, and the radius vector of Mercury, on the 13th of June, 1836, at 20 h. 12 m. 30 sec. mean time at Greenwich.

	M. Long.				Aphelion.				Node.				II.	III.
1836.	9s	8°	52′	11″	8s	14°	53′	43″	1s	16°	22′	26″	377	1925
June	8	22	2	29				23				18	152	152
13th.	1	19	6	31				2				1	12	12
20 h.		3	24	37										
12 m.			2	3	8	14	54	8	1	16	22	45	541	2089
30 sec.				5	7	23	27	56	8	0	23	32		
	7	23	27	56	11	8	33	48	6	14	0	47		
Eq. Cent.	+	6	55	27	M. Anom.				Arg. Lat.					
									N					
	8	0	23	23			1836.		842					
Eq. II.			+	4			June		22					
Eq. III.			+	5			13 d.		2					
Orbit Long.	8	0	23	32					866					
Reduct.		—	6	16										
	8	0	17	16										
Nut.			—	13										
Hel. Long.	8	0	17	3										

Hel. Lat. from table	1°	41′	29.6″ S.
Sec. Variation		+	1.5
True Hel. Lat.	1	41	31 S.

Radius Vector 0.46287.

2. Required the heliocentric longitude and latitude, and the radius vector of Mercury, on the 17th of November, 1837, at 11 h. 29 m. 20 sec. mean time at Philadelphia. *Ans.* Long. 221° 47′ 1″; Lat. 0° 33′ 55″ N. Rad. Vect. 0.44780.

PROBLEM XXI.

The heliocentric longitude and latitude, and the radius vector of Mercury at a given time being given, to find its geocentric longitude and latitude and its horizontal parallax and semidiameter at that time.

For the Geocentric Longitude.

To the sun's longitude at the given time, found by prob. VII, add

180° 0′ 20″, and the sum will be the earth's longitude at that time.* Find the earth's radius vector, by adding to the radius vector taken from table XVIII, with the sun's mean anomaly as the argument, the perturbations taken from the small table on the same page, with the arguments I, II, and III.

Subtract the longitude of the earth from the heliocentric longitude of the planet; the remainder, if less than 180°, will be the angle of commutation, to be marked *west;* but if the remainder is greater than 180°, its supplement to 360° will be the angle of commutation, to be marked *east.* Take half the angle of commutation, and subtracting it from 90°, call the remainder A.

Add together the cosine of the planet's heliocentric latitude, the logarithm of its radius vector, and the arithmetical complement of the logarithm of the earth's radius vector, rejecting the tens from the index of the sum, and the result will be the tangent of an arc B.

To the tangent of the difference between the arc B and 45°, add the tangent of the arc A, rejecting ten from the index of the sum, and the result will be the tangent of an arc C. Subtracting C from A, the remainder will be the angle of elongation, of the same name as the angle of commutation.

If the angle of elongation is *east, add* it to the sun's longitude increased by 20″; but if it is *west, subtract* it from the sun's longitude thus increased; and the sum or remainder will be the true geocentric longitude.

Add together the arcs A and C, and the sum will be the annual parallax. With the elongation, annual parallax, and geocentric latitude as arguments, find the aberration in longitude from table LXIX, and applying it to the true longitude, the result will be the apparent longitude.

For the Geocentric Latitude.

Add together the tangent of the heliocentric latitude, the sine of the elongation, and the arithmetical complement of the sine of the commutation, rejecting ten from the index of the sum, and the result will be the tangent of the true geocentric latitude, which will be of the same name as the heliocentric latitude.

With the angle of elongation increased by 270°, and the annual parallax and geocentric longitude, each increased by 90°, as arguments, take from table LXIX, parts I, II, and III, respectively, of the aberration in longitude, and add them together, having regard to their signs. Multiply the sum by the multiplier, taken from a small table at the bottom of page 74, and the product will be the first three parts of the aberration in latitude. Take from the other small table on the same page, part IV of the aberration in latitude, and add it to the former three, attending to the signs, and the sum will be the aberration in latitude. Considering the true geocentric latitude as affirmative or negative, according as it is north or south, add to it the aberration in latitude, and the result will be the apparent latitude.

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* The sun's longitude found from the tables is the apparent longitude as affected by aberration, and is therefore 20″ less than the true longitude.

For the Horizontal Parallax and Semidiameter.

Add together the constant logarithm 0.93337, the sine of the true geocentric latitude, the arithmetical complement of the sine of the heliocentric latitude, and the arithmetical complement of Mercury's radius vector, rejecting ten from the index of the sum, and the result will be the logarithm of the horizontal parallax, in seconds.

To the constant logarithm 9.57584, add the logarithm of the horizontal parallax, and the sum, rejecting ten from the index, will be the logarithm of the semidiameter, in seconds.

Note 1. The true geocentric longitude and latitude and the horizontal parallax of the planet Venus, may be found in the same manner. The constant logarithm 9.98302 added to the logarithm of the horizontal parallax, and ten rejected from the index of the sum, will give the logarithm of the semidiameter.

2. The geocentric longitude and latitude of a superior planet may also be found in the same manner, except that in finding the arc B, one ten must be retained in the index of the tangent, and the sum of C and A must be taken for the elongation, instead of their difference.

3. At the time of conjunction, the angles of commutation and elongation are each nothing, and consequently the geocentric latitude cannot be found by the rule. In this case, add the cosine of the heliocentric latitude to the logarithm of the planet's radius vector, rejecting ten from the index of the sum, and the result will be the logarithm of the curtate distance of the planet. Take the difference between the curtate distance and the earth's radius vector, or the sum of the two, according as the conjunction is inferior or superior, and add together the arithmetical complement of the logarithm of this difference or sum, the logarithm of the planet's radius vector and the sine of the heliocentric latitude, rejecting ten from the index of the sum, and the result will be the tangent of the true geocentric latitude.

Exam. 1. Required the geocentric longitude and latitude, the horizontal parallax, and the semidiameter of Mercury at the time given in the first example of the last problem.

For the Geocentric Longitude.

Sun's longitude found by prob. VII.	83°	13′	14″
Add - - - - - - - - -	180	0	20
Earth's longitude - - - - - -	263	13	34

Sun's anom. 5^s 12° 32′ 2″, gives, tab. XVIII,	1.01592
Arg. I, 999, gives	8
" II, 926, "	4
" III, 924, "	2
Earth's radius vector	1.01606

Heliocentric long. Mercury	240°	17′	3″
Longitude of Earth	263	13	34
	337	3	29
	360	0	0
Commutation	22	56	31 E
	11	28	15
	90	0	0
A =	78	31	45

Mercury's hel. lat.	1° 41′ 31″ S		cos.	9.99981
" rad. vect.	0.46287		log.	9.66546
Earth's " "	1.01606	Ar. Co.	"	9.99309
B	24° 28′ 58″		tan.	9.65836
45° — B	20° 31′ 2″		tan.	9.57313
A	78 31 45		tan.	10.69267
C	61 31 52		tan.	10.26580
Elongation	16 59 53 E			

Sun's long. + 20″	83°	13′	34″
Elongation	16	59	53 E
Mercury's true longitude	100	13	27

Elongation 17° 0′ gives, tab. LXIX, part I,	— 19″
Ann. par. 140 14 " " " II,	+ 25
Geoc. Long. " " " III,	— 6
Aber. in long.	0

Hence the apparent geocentric longitude is in this case the same as the true.

For the Geocentric Latitude.

Hel. lat.	1° 14′ 31″ S	tan.	8.47038
Elongation	16 59 53	sin.	9.46589
Commutation	22 56 31	Ar. Co. "	0.40916
True geoc. lat.	1 16 9 S	tan.	8.34543

Elong.	+ 270° = 287° 0′	Part I,	— 6″
Ann. par.	+ 90 = 230 4	" II,	+ 21
Geoc. long.	+ 90 = 190 13	" III,	+ 3
			+ 18
Multiplier - - - - - - - - - -			0.02
			+ 0.36″
Arg. of lat. 194° 1′, gives part IV,			+ 3.0
Aber. in lat. - - - - - - - -			+ 3″

True geoc. lat. - - - - -	— 1° 16′ 9″
Aber. in lat. - - - - -	+ 0 0 3
Appar. geoc. lat. - - - - -	1 16 6 S.

		C. log.	0.93337
True geoc. lat.	1° 16′ 9″	sin.	8.34535
Hel. lat.	1 41 31	Ar. Co. "	1.52980
Rad. vect.	0.46287	Ar. Co. log.	0.33454
Hor. parallax	13.9″		1.14306
		C. log.	9.57584
Semidiam.	5.2″		0.71890

2. Required the apparent geocentric longitude and latitude, the horizontal parallax and the semidiameter of Mercury at the time given in the second example of last problem. *Ans.* App. geoc. long. 230° 59′ 53″; app. geoc. lat. 0° 10′ 42″ N.; hor. par. 6.0″; and semidiam. 2.3″.

PROBLEM XXII.

The Right Ascension of a Fixed Star on any given day being given, to find the time of its passage over the meridian of the given place.

For mean noon of the given day, reduced to Greenwich time, find the sun's tabular mean longitude and the argument N, from tables VII, VIII, XI, and XII. To the tabular mean longitude, increased by 2°, apply according to its sign, the nutation in right ascension, taken from table XVI, with the argument N. The result converted into time, will be the sidereal time of mean noon, at the given place.*

Subtract the sidereal time of mean noon from the apparent right ascension of the star, increasing the latter by 24 hours when necessary. Then, as 24 h. : the remainder : : 23 h. 56 m. 4 sec. : the time of the star's passage over the meridian, expressed in mean solar time.

Note. The sidereal time of mean noon at at any place may be readily found from the Nautical Almanac, as follows :—As 1 h. : the difference of longitude between the given place and Greenwich, expressed in time : : 9.8565 seconds : a fourth term. To the sidereal time of mean noon at Greenwich as given in the Nautical Almanac for the given day, *add* this fourth term when the longitude of the given place is *west*; but when the longitude of the place is *east*, *subtract* the fourth term from the sidereal time of mean noon at Greenwich. When the sidereal time of mean noon at the given place is thus found, and subtracted from the apparent right ascension of the star, the remainder may be easily and accurately converted into mean solar time by the table of time equivalents, given at the end of the Nautical Almanac.

EXAM. 1. The apparent right ascension of *Regulus* on the 2d of May, 1836, was 9 h. 59 m. 38 sec. Required the time of its passage over the meridian of Philadelphia on that day.

	h.	m.	sec.
Mean noon Philadelphia, - - - - - -	0	0	0
Diff. of long. - - - - - - - -	5	0	40
Greenwich time - - - - - -	5	0	40

	Sun's M. Long.				N.
1336	9ˢ	9°	10′	6″	842
May	3	28	16	40	18
2 d.			59	8	
5 h.			12	19	
40 sec.				2	
	1	8	38	15	860
		2	0	0	
	1	10	38	15	
Nut. R. A.			—	13	
	1	10	38	2	

h. m. sec.

1 10 38 2 = 40° 38′ 2″ = 2 42 32 = sid. time of mean noon at Phila.

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* The sidereal time of mean noon, and the right ascension of the meridian or mid-heaven or zenith, at mean noon, are all equivalent expressions.

	h.	m.	sec.
Apparent right ascen. of Regulus - - -	9	59	38
Sidereal time of mean noon at Phila. - -	2	42	32
Remainder - - - - - - - -	7	17	6

h. h. m. sec. h. m. sec. h. m. sec.
24 : 7 17 6 :: 23 56 4 : 7 15 54 = mean time of passage over the meridian.

2. Required the time at which *Arcturus* passed the meridian of New York, on the 10th of May, 1836, its apparent right ascension at that time being 14 h. 8 m. 13 sec. *Ans.* 10 h. 52 m. 22 sec. mean time.

PROBLEM XXIII.

To reduce the time of the Moon's Passage over the Meridian of Greenwich, as given in the Nautical Almanac, to the time of its passage over the meridian of any other place.

Take the difference between the times of the moon's passage at Greenwich on the given day and the next following or next preceding day, according as the place has *west* or *east* longitude. Then, as 24 h. : the longitude of the place in time :: this difference : a fourth term. This fourth term is the reduction, and being *added* to the time of the moon's passage over the meridian of Greenwich on the given day, when the place is in *west* longitude, but *subtracted* when it is in *east* longitude, will give the required time of passage, in mean time, of the given place.

Note. The times of the passages of the larger planets over the meridian of Greenwich are given in the Nautical Almanac for each day, and of the four small planets for every fourth day. The time of passage of any of them over a given meridian may be found in the same manner as the moon's passage, except that for either of the small planets, four days must be used instead of one day, and when the motion of a planet is retrograde, that is when the time of passage on a given day is *earlier* than on the preceding day, the reduction must be applied in a *contrary* manner to the direction in the rule, by *subtracting* for a place with *west* longitude, and *adding* for a place with *east* longitude.

Exam. 1. Required the time of the moon's passage over the meridian of Philadelphia, on the 19th of May, 1836.

	h.	m.
Passage at Greenwich on 20th, tab. LV - -	3	58 0
" " 19th, " - -	3	7.5
Difference - - - - - - - - -	0	50.5

h. h. m. m. m.
24 : 5 1 :: 50.5 : 10.6

	h.	m.
Passage at Greenwich on 19th - - - - -	3	7.5
Reduction, add - - - - - - -	0	10.6
Passage at Phila. - - - - - - -	3	18.1

2. Required the time of the moon's passage over the meridian of Washington, on the 4th of May, 1836. *Ans.* 15 h. 55.3 m.

PROBLEM XXIV.

The Latitude of a place and the Sun's Declination at apparent noon being given, to find the time of his rising and setting.

To the tangent of the latitude of the place, add the tangent of the sun's declination, rejecting ten from the index ; the result will be the sine of the *ascensional difference*, which must be taken less than 90°, and reduced to time.

The ascensional difference, added to 6 hours, when the latitude and declination are both of the same name, that is both north or both south, but subtracted from 6 hours, when they are of different names, will give the *semidiurnal arc.*

The semidiurnal arc expresses the apparent time of sunset, and subtracted from 12 hours, gives the apparent time of sunrise. The apparent times may, if required, be changed into mean times, by applying the equation of time.

Note 1. In the above rule, no notice is taken of the change in the sun's declination between noon and the time of his being in the horizon, nor of the effects of refraction and parallax in changing the time of his rising or setting. When this time is required with precision, the declination should be found for the approximate time of rising or setting as obtained by the above rule ; the calculation may then be performed by the formula in art. 24, chap. ix.

2. The rising or setting of a planet or star may be found by calculating the semidiurnal arc by the above rule, and subtracting it from the time of the body's passage over the meridian for the rising, and adding it for the setting.

Exam. 1. Required the time of the sun's rising and setting at Philadelphia, on the 25th of January, 1821, the declination at noon of that day being 18° 52′ S.

Lat. - - -	39° 57′ N. - - -		tan.	9.92304
Decl. - -	18 52 S. - -		"	9.53368
Asc. diff. - -	16 38 - - -		sin.	9.45672
	4			
	1 h. 6 m. 32 sec.			

1 h. 6 m. 32 sec.
6 0

Semidiur. arc 4 h. 53 m. time of sunset.

7 h. 7 m. time of sunrise.

2. Required the time of the sun's rising and setting at St. Petersburg, when the declination is 23° 28′ N. *Ans.* Sun rises at 2 h. 46 m. and sets at 9 h. 14 m.

PROBLEM XXV.

To find, by the aid of the Nautical Almanac, the time of the moon's rising or setting, on a given astronomical day.

Find the time of the moon's passage over the meridian of the given place by prob. XXIII.

To, or from the time of the passage, according as the moon's setting or rising is required, add or subtract 6 hours, and reducing the resulting time to the meridian of Greenwich, take from the Nautical Almanac the moon's declination corresponding to the whole hour nearest to this reduced time. With the latitude of the place and this declination, find the semidiurnal arc, by the last problem, and apply it to the time of the moon's passage over the meridian, by subtracting for the rising, or adding for the setting; the result will be the *approximate* time of rising or setting.

Find the moon's declination for the approximate time of rising or setting, reduced to the meridian of Greenwich, and with this declination again calculate the semidiurnal arc.

Take the difference between the times of the moon's passage over the meridian of Greenwich, on the given day and the next preceding or next following one, according as the rising or setting is required. Then, as 24 hours : the semidiurnal arc, last found :: this difference : a fourth term. This fourth term will be a correction which, added to the semidiurnal arc last found, will give the corrected semidiurnal arc.

Apply the corrected semidiurnal arc to the time of the passage over the meridian of the given place, by subtracting for the rising or adding for the setting, and the result will be the required time, very nearly.

Note. When it is required to make a number of consecutive calculations of the moon's rising or setting, for any particular place, they may be much abbreviated by little expedients which will be likely to present themselves to the computer, but which it would be troublesome to specify.

Exam. 1. Required the time of the moon's rising at Philadelphia, on the 13th of May, 1836.

		h.	m.
Passage over mer. of Greenwich	tab. LV,	23	7.0
Reduction	add	0	9.3
Passage over mer. of Phila.	- - - - -	23	16.3
Subtract	- - - - - - - .	6	0
		17	16.3
Diff. of long.	- - - - - - -	5	1
Time at Greenwich	- - - - -	22	17

Moon's declination on the 13th at at 22 h. is (tab. LV), 14° 10′ N.

Lat. - - -	39°	57′ N. - - -	tan.	9.92304
Declin. - -	14	10 N. - -	tan.	9.40212
Asc. diff. - -	12	12 - - -	sin.	9.32516
		4		
	48 m.	48 sec.		
	6 h.	0		
Semidiur. arc	6 h.	49 m.		

	h.	m.
Moon's passage over merid. of Phila. - - -	23	16
Semidiur. arc - - - - - - - -	6	49
Approx. time of moon's rising - - - -	16	27
Diff. of long. - - - - . - - -	5	1
Time at Greenwich - - - - - -	21	28

Moon's declin. at 21 h. 28 m. is 14° 4′ N.

Lat. - - -	39°	57′ N. - -	tan.	9.92304
Declin. - -	14	4 N. - -	tan.	9.39892
Asc. diff. - -	12	7		9.32196
		4		
	48	28		
	6 h. 0	0		

h. h. m. m. m.
24 : 6 48.5 :: 43 : 12.2

Semidiur. arc	6 h.	48.5 m.
Correction	0	12.2
Cor. Semidiur. arc	7	0.7

Moon's passage over mer. of Phila.	23	16.3
Correct semidiur. arc	7	0.7
Time of moon's rising	16	16

2. Required the time of the moon's setting at Washington on the 16th of May, 1836. *Ans.* 8 h. 29 m. P. M.

PROBLEM XXVI.

To find the Longitude and Altitude of the Nonagesimal Degree of the Ecliptic, for a given time and place.

Find the reduced latitude of the place by problem XIII: and when it is *north, subtract* it from 90°, but when it is *south, add* it to 90°, for the reduced distance of the place from the north pole. Take *half the difference* between this quantity and the obliquity of the ecliptic: also, *half the* sum of the same quantities. From the cosine of the half difference, subtract the cosine of the half sum, and call the result, logarithm A. From the tangent of the half difference, with the index increased by 10, subtract the tangent of the half sum, and call the result, logarithm B. Also, call the tangent of the half sum, logarithm C.

For the given time, reduced to mean time at Greenwich, find the sun's mean longitude and the argument N, from tables VII, VIII, XI, and XII. To the sun's mean longitude, increased by 2°, apply, according to its sign, the nutation in right ascension, taken from table XVI, with argument N, and it will give the sun's mean longitude reckoned from the true equinox.

To the sun's mean longitude from the true equinox, add the mean time of day, at the given place, expressed astronomically and reduced to degrees, and reject 360° from the sum, when it exceeds that quantity. The result will be the *right ascension of the mid-heaven.**

From the right ascension of the mid-heaven, subtract 90°, the former being first increased by 360°, when necessary, and call *half* the remainder R.

To the logarithm A, add the tangent of R, and the result will be the tangent of an arc E, which must be taken according to the sign, but less than 180°. To the tangent of E, add the logarithm of B, rejecting ten from the index, and the result will be the tangent of an arc F, which must also be taken according to the sign, and less than 180°. The sum of the arcs E and F, and 90°, rejecting 360°, when the sum exceeds that quantity, will be the longitude of the nonagesimal degree.

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* When the sun's true longitude has been previously calculated for the same time for which the right ascension of the mid-heaven is wanted, it is evident the tabular mean longitude and the argument N, are already known.

It may also be observed, that the right ascension of the mid-heaven is equal to the sum of the sun's true right ascension, and the *apparent* time expressed astronomically and reduced to degrees; 360° being rejected when the sum exceeds that quantity.

Add together the logarithm C, the cosine of E, and the arithmetical complement of the cosine of F, and reject ten from the index: the result will be the tangent of *half* the altitude of the nonagesimal degree.

Note 1. The above rule, which differs but little in substance from that given by Dr. Bowditch in his *Practical Navigator*, is general for all places, except within the north polar circle. And the only difference there, is, that for the longitude of the nonagesimal degree 90° must be added to the arc E, and the arc F subtracted from the sum.

2. When the longitude and altitude of the nonagesimal degree are required, at any given place for several different times in the same day, which is generally the case, the same logarithms A, B, and C, when they have been once found, will answer for all the other operations. Indeed, the obliquity of the ecliptic changes so slowly, that except great accuracy is required, the same logarithms may be used in calculations for a time several years distant from the time for which they were obtained.

3. The last part of the above rule gives the distance of the zenith of the place from the north pole of the ecliptic, which is not always the real altitude of the nonagesimal. Generally in the southern hemisphere, and frequently in the northern hemisphere, near the equator, it is the supplement of the altitude. But it simplifies the rule for the parallaxes, to which this problem is preliminary, and produces no error, to use the same term in all cases.

Exam. 1. Required the longitude and altitude of the nonagesimal degree of the ecliptic, at Philadelphia, on the 27th of August, at 7 h. 30 m. 21 sec. A. M., mean time, the obliquity of the ecliptic being then 23° 27′ 55″.

The reduced latitude of Philadelphia, found by problem XIII, is 39° 45′ 43″ N., and this taken from 90°, leaves the polar distance 50° 14′ 17″; the difference and sum of this quantity and the obliquity of the ecliptic are 26° 46′ 22″; and 73° 42′ 12″; half difference 13° 23′ 11″; half sum 36° 51′ 6″.

½ diff. 13° 23′ 11″	cos.	9.98803	tan.	+ 10,	19.37654
½ sum 36 51 6	cos.	9.90319	tan.	C.	9.87478
	A.	0.08484		B.	9.50176

The sun's longitude taken from the tables, for the given time, and increased by 2°, is 5ˢ 5° 24′ 38″, and the argument N is 71. The nutation, taken from table XVI, with argument N, is + 7″. Hence, the sun's mean longitude from the true equinox is 5ˢ 5° 24′ 45″, or 155° 24′ 45″. The given time of day expressed astronomically, is 19 h. 30 m. 21 sec.; which, in degrees, is 292° 35′ 15″.

Given time in degrees - - - 292° 35' 15"
Sun's mean long. - - - - 155 24 45

Right ascen. mid-heaven - - - 88 0 0
90 0 0

2)358 0 0

R. 179 0 0

A. 0.08484
R 179° 0' 0" tan. — 8.24192

E 178 47 4 tan. — 8.32676 cos. — 9.99990
B. 9.50176 C. 9.87478

F 179 36 50 tan. — 7.82852 Ar. Co. cos. — 0.00001
90 0 0

½ alt. non. 36° 50' 47" tan. 9.87469

88 23 54 long. nonages.

73 41 34 alt. nonages.

2. Required the longitude and altitude of the nonagesimal degree of the ecliptic at Philadelphia, on the 27th of August, 1821, at 8 h. 53 m. 20 sec. A. M. mean time. *Ans.* Long. 105° 2' 18", and alt. 72° 43' 32".

3. Required the longitude and altitude of the nonagesimal degree, at Philadelphia, on the 27th of August, 1821, at 10 h. 14 m. A. M. apparent time. *Ans.* Long. 121° 21' 25", and alt. 69° 30' 44".

PROBLEM XXVII.

The Longitude and Altitude of the Nonagesimal Degree of the Ecliptic, and the Moon's True *Longitude, Latitude, Equatorial Parallax, and Horizontal Semidiameter being given, to find the* Apparent *Longitude and Latitude as affected by parallax, and the Augmented Semidiameter of the Moon, for a given place.*

Find the reduction of parallax, by problem XIII, and subtract it from the equatorial parallax; and in *eclipses of the sun,* subtract from the remainder, the sun's parallax, which is 9", or more accurately 8.6". Call the last remainder the *Reduced* parallax. In *occultations of a fixed star,* the first remainder is the reduced parallax.

Subtract the longitude of the nonagesimal from the moon's longitude, increasing the latter by 360°, if necessary; then, if the remainder is *less* than 180°, it is the moon's distance to the *east* of the nonagesimal, which call D; but if the remainder is *greater* than 180°, subtract it from 360°, and the second remainder will be D, the moon's distance to the *west* of the nonagesimal. When the moon's latitude is *north*, *subtract* it from 90°, but when it is *south*, *add* it to 90°; the difference or sum will be the moon's distance from the north pole of the ecliptic, which call d. Call the altitude of the nonagesimal h, and the reduced parallax P.

Of the two following methods of finding the apparent longitude and latitude it may be observed, that the first is general, and may be used either in eclipses or occultations. The second is applicable *only in eclipses of the sun*, or when it is known that the apparent latitude is small. It is more concise than the first, and though not quite so accurate, yet the errors will seldom exceed two or three tenths of a second. In working by either method, the student must observe, that when logarithms are directed to be added together, the tens in the resulting index are to be rejected. When the logarithm of an arc is to be taken, the arc must first be reduced to seconds; and when an arc is found, corresponding to a logarithm, it is in seconds.

FIRST METHOD.

Which may be used either in Eclipses of the Sun, or in Occultations.

Add together the logarithm of P, the sine of h, and the arithmetical complement of the sine of d, and call the resulting logarithm c. To the logarithm c, add the sine of D, and the result will be the logarithm of an arc u. Add together the logarithm c, and the sine of $(D + u)$, and the result will be the logarithm of an arc u'. Add together the logarithm c, and the sine of $(D + u')$, and the result will be the logarithm of p, the parallax in longitude. Except when great accuracy is required, the last operation need not be performed, and p may be placed instead of u'.

Add p to the moon's true longitude, when the moon is to the *east* of the nonagesimal, but *subtract*, when it is to the *west*, and the result will be the *apparent* longitude.

When the apparent latitude is necessarily small, as in *eclipses of the sun*, add together the logarithm of P, and the cosine of h, and the result will be the logarithm of an arc x. But in *occultations*, add together the logarithm of P, the cosine of h, and the sine of d, and the result will be the logarithm of an arc v. To d, add v, attending to the sign of the latter. Then add together the logarithm of v, the sine of $(d + v)$, and the arithmetical complement of the sine of d, and the result will be the logarithm of the arc x.

To d, add x, attending to the sign of the latter. Then add together the logarithm of P, marked negative, the sine of h, the cosine of $(D + \frac{1}{2} p)$, and the cosine of $(d + x)$, and the result will be the logarithm of an arc z. The arc z, applied according to its sign, to the sum of d and x, will give the *apparent polar distance*. And the difference between this and 90°, will be the *apparent latitude*, which will be *north* or *south*, according as the polar distance is *less* or *greater*, than 90°.

The sum of x and z, regard being had to their signs, will be the parallax in latitude.

Add together the logarithm of the moon's horizontal semidiameter, the sine of the apparent polar distance, the sine of $(D + u)$, the arithmetical complement of the sine of d, and the arithmetical complement of the sine of D, and the result will be the logarithm of the *augmented* semidiameter.

SECOND METHOD.

Which can only be used when the Apparent Latitude is small, as in Eclipses of the Sun.

Add together, the logarithm of P, the cosine of h, and the arithmetical complement of the sine of d, and the result will be the logarithm of an arc x. Add together, the logarithm of x, the tangent of h, and the sine of D, and the result will be the logarithm of an arc u. Add together, the logarithm of u, the sine of $(D + u,)$ and the arithmetical complement of the sine of D, and the result will be the logarithm of p, the parallax in longitude. Take the sum of d and x, attending to the sign of the latter. Then, add together the logarithm of p, the logarithm of the difference between $(d + x)$ and 90°, the arithmetical complement of the logarithm of u, and the arithmetical complement of the sine of d, and the result will be the logarithm of the *apparent* latitude, which will be *north* or *south*, according as $(d + x)$, is *less* or *greater* than 90°.

The parallax in longitude, p, *added* to the moon's true longitude, when the latter is *greater* than the longitude of the nonagesimal, but *subtracted*, when it is *less*, gives the *apparent* longitude.

Add together, the logarithm of the moon's horizontal semidiameter, the sine of $(D + u)$, the arithmetical complement of the sine of d, and the arithmetical complement of the sine of D, and the result will be the logarithm of the *augmented* semidiameter.*

Note. In eclipses of the sun, it is not strictly the apparent longitude and latitude of the moon that are found by the preceding rules, but the values of those quantities, including the sun's parallax in longitude and latitude, which are the values wanted in the calculation.

Exam. 1. About the time of beginning of the eclipse of the sun, on the 27th of August, 1821, the longitude of the nonagesimal degree, at Philadelphia, was 88° 23′ 54″, the altitude 73° 41′ 34″, moon's true longitude 152° 31′ 1″, true latitude 0° 11′ 38″ N., equatorial parallax 55′ 18″, and semidiameter 15′ 3″; required the apparent longitude and latitude, and the augmented semidiameter.

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* The rules in the first method, are deduced from formulæ C, G, and H, of articles 54, 56, and 57, chap. X. Those in the second, from C, F, and H, of articles 54, 55, and 57.

Equat. par.	55°	18′	Moon's long.	152°	31′	1″
Reduction		5	Long. nonag.	88	23	54
	55	13	$D =$	64	7	7
Sun's paral.		9	$h =$	73	41	34
$P =$	55	4	$d =$	89	48	22

BY THE FIRST METHOD.

P			3304″	log.	3.51904
h	73°	41′	34″	sin.	9.98217
d	89	48	22	Ar. Co. sin.	0.00000
				c.	3 50121
D	64	7	7	sin.	9.95410
u			2853″	log.	3.45531
				c.	3.50121
$D + u$	64	54	40	sin.	9.95696
u'			2872″	log.	3.45817
				c.	3.50121
$D + u'$	64	54	59	sin.	9.95698
p		47′	52.1″	log.	3.45819
True long.	152	31	1		
App. long.	153	18	53.1		
P				log.	3.51904
h	73°	41′	34″	cos.	9.44838
x		15	27.7	log.	2.96742
P				log. —	3.51904
h				sin.	9.98217
$D + \frac{1}{2}p$	64	31	3	cos.	9.63371
$d + x$	90	3	49.7	cos. —	7.04434
z			1.5	log.	0.17926
Ap. pol. dist.	90	3	51.2		
	90	0	0		
Ap. lat.	0	3	51.2 S.		

Hor. semidiam.	903″	log.	2.95569
App. pol. dist.	90° 3′ 51″	sin.	10.00000
D + u		sin.	9.95696
d		Ar. Co. sin.	0.00000
D		Ar. Co. sin.	0.04590
Augmented semidiam.	15′ 9″	log.	2.95855

BY THE SECOND METHOD.

P			3304″	log.	3.51904
h	73°	41′	34″	cos.	9.44838
d	89	48	22	Ar. Co. sin.	0.00000
x		15	27.7	log.	2.96742
h	73	41	34	tan.	10.53379
D	64	7	7	sin.	9.95410
u		47	33	log.	3.45531
D				Ar. Co. sin.	0.04590
D + u	64	54	40	sin.	9.95696
p		47	52	log.	3.45817
$(d + x) - 90°$,		3	49.7	log.	2.36116
u				Ar. Co. log.	6.54469
d				Ar. Co. sin.	0.00000
App. lat.		3	51.2 S.	log.	2.36402

Moon's true long.	152°	31′	1″
p		47	52
App. long.	153	18	53

Hor. semidiam.	903″	log.	2.95569
D + u		sin.	9.95696
d		Ar. Co. sin.	0.00000
D		Ar. Co. sin.	0.04590
Augmen. semid.	15′ 9″	log.	2.95855

2. Given the longitude of the nonagesimal 67° 29′ 8″, the altitude 57° 56′ 36″, the moon's true longitude 3ˢ 18° 27′ 35″, latitude 4° 5′ 30″ S., reduced parallax 61′ 1″, and horizontal semidiameter 16′ 40″; to find the moon's apparent longitude, latitude, and augmented semidiameter.

D = 40° 58′ 27″, h = 57° 56′ 36″, d = 94° 5′ 30″, P = 61′ 1″.

P				3661″	log.	3.56360
h		57°	56′	36″	sin.	9.92815
d		94	5	30	Ar. Co. sin.	0.00111
					c.	3.49286
D		40	58	27	sin.	9.81672
u			34	0	log.	3.30958
					c.	3.49286
$D+u$		41	32	27	sin.	9.82161
u'			34	23	log.	3.31447
					c.	3.49286
$D+u'$		41	32	50	sin.	9.82167
p			34	23.1	log.	3.31453
True long.	3s	18	27	35		
App. long.	3	19	1	58.1		
P					log.	3.56360
h		57	56	36	cos.	9.72490
d		94	5	30	sin.	9.99889
v			32	18	log.	3.28739
$d+v$		94	37	48	sin.	9.99858
d					Ar. Co. sin.	0.00111
x			32	16.8	log.	3.28708
P					log.	— 3.56360
h					sin.	9.92815
$D+\frac{1}{2}p$		41	15	39	cos.	9.87605
$d+x$		94	37	46.8	cos.	— 8.90696
z		+	3	8.3	log.	2.27476
Ap. pol. dist.		94	40	55.1		
		90	0	0		
App. lat.		4	40	55.1		
Hor. semidiam.				1000″	log.	3.00000
App. pol. dist.		94°	40′	55″	sin.	9.99855
$D+u$					sin.	9.82161
d					Ar. Co. sin.	0.00111
D					Ar. Co. sin.	0.18328
Augm. semidiam.			16′	50.8″	log.	3.00455

3. About the middle of the eclipse of the sun, on the 27th of August, 1821, the longitude of the nonagesimal, at Philadelphia, was 105° 2′ 18″, the altitude 72° 43′ 32″, moon's true longitude 153° 13′ 52″, latitude 0° 7′ 42″ N. reduced parallax 55′ 12″, and semidiameter 15′ 3″; required the apparent longitude and latitude, and the augmented semidiameter. *Ans.* App. long. 153° 53′ 27″; app. lat. 0° 8′ 44″ S.; augm. semidiam. 15′ 12.2″.

4. About the end of the eclipse of the sun, on the 27th of August, 1821, the longitude of the nonagesimal, at Philadelphia, was 121° 21′ 25″, altitude 69° 30′ 44″, moon's true longitude 153° 56′ 15″, latitude 0° 3′ 47″ N., reduced parallax 55′ 10″, and semidiameter 15′ 3″; required the apparent longitude and latitude, and the augmented semidiameter.

Ans. App. long. 154° 24′ 21″; app. lat. 0° 15′ 40″ S.; augm. semidiam. 15′ 14.4″.

PROBLEM XXVIII.

To find from the Tables, the Time of New or Full moon, for a given Year and Month.

For New Moon.

Take from table III, the mean new moon in January, for the given year, and the arguments I, II, III, and IV. Take from table IV, as many lunations, and the corresponding arguments I, II, III, and IV, as the number* of the given month exceeds a unit, and add these quantities to the former, rejecting the ten thousands in the first two arguments, and the hundreds in the other two. Take the number of days corresponding to the given month, from the *second* or *third* column of table V, according as the given year is a *common* or a *bissextile* year, and subtract it from the sum, in the column of mean new moon, the remainder will be the tabular time of mean new moon in the given month. If the number of days, taken from table V, is greater than the sum of the days in the column of mean new moon, as will sometimes be the case, one lunation more than is directed above, with the corresponding arguments, must be added.

With the arguments I, II, III, and IV, take the corresponding equations from table VI, and add them to the time of mean new moon; the sum will be the *Approximate* time of new moon, expressed in mean time at Greenwich.

For the approximate time of new moon, calculate by problems VII and XI, the true longitudes and hourly motions in longitude of the sun and moon. Take the difference between the longitudes, and also between the hourly motions. Then, as the difference between the hourly motions : the difference between the longitudes : : 60 minutes : the correction. The correction, *added* to the approximate time of new moon, when the sun's longitude is *greater* than the moon's, but *subtracted*, when it is *less*, will give the *true* time of new moon, expressed in mean time at Greenwich. This time may be changed to apparent time, at any given meridian, by problems VI and V.

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* The numbers for the months are, for January, 1, for February, 2, for March, 3, &c.

For Full Moon.

When the time of mean new moon in January of the given year is on or after the 16th, *subtract* from it, and the arguments I, II, III, and IV, a half lunation, with the corresponding arguments, taken from table IV, increasing when necessary, either or both of the first two of the former by 10,000, and of the two latter by 100; but *add* them, when the time is before the 16th. The result will be the tabular time of mean full moon in January, and the corresponding arguments. Proceed to find the approximate time of full moon, in the same manner as directed for the new moon.*

Calculate the true longitudes and hourly motions in longitude of the sun and moon, for the approximate time of full moon. Subtract the sun's longitude from the moon's, and call the remainder R. Also, subtract the hourly motion of the sun from that of the moon. Then, as the difference of the hourly motions : the difference between R and VI signs : : 60 minutes : the correction. The correction, *added* to the approximate time of full moon, when R is *less* than VI signs, but *subtracted* when it is *greater*, will give the *true* time of full moon.

Exam. 1. Required the time of new moon in August, 1821, expressed in apparent time at Philadelphia.

	M. New Moon.			I.	II.	III.	IV.
	d.	h.	m.				
1821,	2	17	59	0092	7859	80	78
8 lun.	236	5	52	6468	5737	22	93
	238	23	51	6560	3596	02	71
Days,	212						
August,	26	23	51				
I.		0	54				
II.		2	13				
III.			9				
IV.			10				
August,	27	3	17	Approximate time.			

Sun's true long. found for the approx. time, is	5s	3°	57′	12″
Moon's do.	5	3	56	43
Difference, - - - - - - - -			0	29

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* When the half lunation and arguments are to be *added*, the addition may be left till the proper number of lunations, with their corresponding arguments, are placed under, and thus make one addition serve.

Moon's hourly motion in long. is	30′	55″
Sun's do.	2	25
Difference,	28	30

m. m. sec.

As 28′ 30″ : 29″ :: 60 : 1 1, the correction.

		d.	h.	m.	sec.
Approx. time of new moon,	August,	27	3	17	0
Correction,			+	1	1
True time, in mean time at Greenwich,		27	3	18	1
Equation of time,			—	1	19
Apparent time at Greenwich,		27	3	16	42
Diff. of meridians,			5	0	40
Apparent time at Philadelphia,		26	22	16	2

2. Required the time of full moon in July, 1823, expressed in apparent time at Philadelphia.

	M. New Moon.			I.	II.	III.	IV.
	d.	h.	m.				
1823,	11	0	20	0304	5787	61	55
½ lun.	14	18	22	404	5359	58	50
6 lun.	177	4	24	4851	4303	92	95
	202	23	6	5559	5449	11	0
Days,	181						
July,	21	23	6				
I.		2	55				
II.		13	7				
III.			5				
IV.			20				
July,	22	15	33	Approximate time.			

Moon's true long. found for the approx. time, is	9ˢ	29°	24′	51″
Sun's do.	3	29	25	23
R.	5	29	59	28
	6	0	0	0
Diff.			0	32

Moon's hourly motion in long. is - - - -	29′ 34″
Sun's - - - - - - - - - -	2 23
Difference, - - - - - - - - - -	27 11

m. m. sec.
As 27′ 11″ : 32″ :: 60 : 1 11, the correction.

		d.	h.	m.	sec.
Approximate time of full new moon,	July,	22	15	33	0
Correction, - - - - - - - -			+	1	11
True time, in mean time at Greenwich, -		22	15	34	11
Equation, - - - - - - - - -			−	6	2
Apparent time at Greenwich, - - - -		22	15	23	9
Diff. of meridian, - - - - - -			5	0	40
Apparent time at Philadelphia, - -		22	10	27	29

3. Required the time of new moon in July, 1821, expressed in apparent time at Philadelphia. *Ans.* 28 d. 9 h. 10 m. 4 sec. P. M.

4. Required the time of full moon in July, 1821, expressed in apparent time, at Philadelphia. *Ans.* 14 d. 11 h. 17 m. 53 sec.

PROBLEM XXIX.

To determine what Eclipses may be expected to occur in any given year, and the times nearly, at which they will take place.

For the Eclipses of the Sun.

Take, for the given year, from table III, the time of mean new moon in January, the arguments and the number N.* If the number N differs less than 53, from 0, 500, or 1000, an eclipse of the sun may be expected at that new moon. If the difference is less than 37, there *must* be one. When the difference is between 37 and 53, there is a doubt, which can only be removed by calculation.

If an eclipse may or must occur in January, calculate the approximate time of new moon by problem XXVIII, and it will be the time, nearly, at which the eclipse will take place, expressed in mean time at Greenwich. This time may be reduced to the meridian of any other place by problem V.

Look in column N of table IV, and, excluding the number belonging to the half lunation, seek the first number that, added to the number N of the given year, will make the sum come within 53, of 0, 500, or 1000. Take the corresponding lunations and arguments, and this number N, and add them to the similar quantities for the given year. Take from the *second* or *third* column of table V, according as the given year is *common* or *bissextile*, the number of days next less than the sum of the days in the column of mean new moon, and subtract it from the time in that column; the remainder will be the tabular time of mean new moon in the month corresponding to the days, taken from table V. At this new moon an eclipse of the sun may be expected; and if the sum of the numbers N, differs less than 37 from the numbers mentioned above, there *must* be one. Find the time nearly, of the eclipse, by calculating the approximate time of new moon as directed above.

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* The number N, in this table, designates the sun's mean distance from the moon's ascending node, expressed in thousandth parts of the circle.

If there are any other numbers in the column N, of table IV, that, when added to the number N of the given year, will make the sum come within the limit 53, proceed in a similar manner to find the times of the eclipses.

Note. When the time at which an eclipse of the sun will take place is thus found, nearly, and reduced to the meridian of a given place in north latitude, if it comes during the day time, and if the sum of the numbers N, or the number N itself when the eclipse is in January, is a little above 0, or a little less than 500, there is a *probability* that the eclipse will be visible at the given place. When the number N in January, or the sum of the numbers N, in other months, is more than 500, the eclipse will seldom be visible in northern latitudes, except near the equator.

For the Eclipses of the Moon.

When the time of new moon in January of the given year is on or after the 16th, *subtract* from it, from the arguments, and the number N, a half lunation, the corresponding arguments, and the number N; but when it is before the 16th, *add* them. The results will be the time of mean full moon in January, and the corresponding arguments, and number N. Proceed to find the times at which eclipses of the moon *may* or *must* occur, exactly as directed for the sun, except that the limits 35 and 25, must be used instead of 53 and 37.

Note. In an eclipse of the moon, when the time is found nearly, and reduced to the meridian of a given place, if it comes in the night, it will be visible at that place.

Exam. 1. Required the eclipses that may be expected in the year 1822, and the times nearly, at which they will take place.

For the Eclipses of the Sun.

	M. New Moon.			I.	II.	III.	IV.	N.
	d.	h.	m.					
1822,	21	15	32	0602	7182	78	66	930
1 lun.	29	12	44	808	717	15	99	85
	51	4	16	1410	7899	93	65	15
	31							
Feb.	20	4	16					
I.		7	38					
II.		19	29					
III.			13					
IV.			11					
Feb.	21	7	47	Mean time at Greenwich.				

As the sum of the numbers N comes within 37 of 0, there must be an eclipse.

	M. New Moon.			I.	II.	III.	IV.	N.
	d.	h.	m.					
1822,	21	15	32	0602	7182	78	66	930
7 lun.	206	17	8	5659	5020	7	94	596
	228	8	40	6261	2202	85	60	526
	212							
August,	16	8	40					
I.		1	24					
II.		0	40					
III.			16					
IV.			14					
August,	16	11	14	Mean time at Greenwich.				

As the sum of the numbers N comes within 37 of 500, there must be an eclipse,

	M. New Moon.			I.	II.	III.	IV.	N.
	d.	h.	m.					
1822,	21	15	32	0602	7182	78	66	930
½ lun.	14	18	22	404	5359	58	50	43
	6	21	10	0198	1823	20	16	887
1 lun.	29	12	44	808	717	15	99	85
	36	9	54	1006	2540	35	15	972
	31							
Feb.	5	9	54					
I.		6	52					
II.		0	20					
III.			4					
IV.			29					
Feb.	5	17	39	Mean time at Greenwich.				

As the sum of the numbers N, although it comes within 35 of 1000, does not come within 25, the eclipse may be considered doubtful. It may, however, be observed, that further calculation by the next problem would show that there will be a small eclipse.

	M. Full Moon.	I.	II.	III.	IV.	N.
	d. h. m.					
1822,	6 21 10	0198	1823	20	16	887
7 lun.	206 17 8	5659	5020	7	94	596
	213 14 18	5857	6843	27	10	483
	212					
August,	1 14 18	As the sum of the numbers N comes within 25 of 500, there must be an eclipse.				
I.	2 14					
II.	19 26					
III.	3					
IV.	26					
August,	2 12 27	Mean time at Greenwich.				

2. Required the eclipses that may be expected in 1823, and the times nearly, at which they will take place, expressed in mean time at Greenwich. *Ans.* One of the moon on the 26th of January, at 5 h. 24 m. P. M.; one of the sun on the 11th of February, at 3 h. 12 m. A. M.; one of the sun on the 8th of July, at 6 h. 50 m. A. M.; and one of the moon on the 23d of July, at 3 h. 32 m. A. M.

PROBLEM XXX.

To Calculate an Eclipse of the Moon.

Find the approximate time of full moon, by prob. XXVIII, and for this time calculate the sun's longitude, semidiameter, and hourly motion, and the moon's longitude, latitude, equatorial parallax, semidiameter, and hourly motions in longitude and latitude. Then find the true time of full moon as directed in prob. XXVIII, and reduce it to apparent time at the place for which the calculation is to be made. Call the reduced time, T.

For the Moon's Latitude at the True Time of Full Moon.

As 1 hour : correction for the time of full moon : : moon's hourly motion in latitude : correction of latitude. When the true time of full moon expressed in mean time at Greenwich, is *later* than the approximate time, the correction of latitude must be *added*, if the latitude is *increasing*,* but *subtracted*, if it is *decreasing*; but when the true time is *earlier* than the approximate time, the correction must be *subtracted*, if the latitude is *increasing*, but added, if it is *decreasing*. The result will be the moon's latitude at the true time of full moon.

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* When the moon's latitude is north, tending north, or south, tending south, it is *increasing*; but when it is north, tending south, or south, tending north, it is *decreasing*.

For the Semidiameter of the Earth's Shadow.

To the moon's equatorial parallax, add the sun's, which may be taken 9″, and from the sum subtract the semidiameter of the sun. Increase the result by a $\frac{1}{60}$ part, and it will be the semidiameter of the earth's shadow, which call S.

For the Inclination of the Moon's Relative Orbit.

To the arithmetical complement of the logarithm of the difference between the hourly motions in longitude of the moon and sun, add the logarithm of the moon's hourly motion in latitude, and the result will be the tangent of the inclination, which call I.

Add together the constant logarithm 3.55630, the cosine of I, and the arithmetical complement of the difference between the hourly motions of the moon and sun, in longitude, rejecting the tens in the index, and call the resulting logarithm R.

For the Time of the Middle of the Eclipse.

Add together the logarithm R, the logarithm of the moon's latitude at the true time of full moon, and the sine of I, rejecting the tens in the index, and the result will be the logarithm of an interval *t*, in seconds of time, which, *added* to T, when the latitude is *decreasing*, but *subtracted* when it is *increasing*, will give the time of the middle of the eclipse.

For the Times of Beginning and End.

To the logarithm of the moon's latitude at the true time of full moon, add the cosine of I, rejecting the tens in the index, and the result will be the logarithm of an arc, which call *c*. Call the moon's semidiameter, *d*.

To, and from, the sum of S and *d*, add and subtract *c*. Then add together the logarithms of the results, $S + d + c$ and $S + d - c$, divide the sum by 2, and to the quotient add the logarithm R, and the result will be the logarithm of an interval *x*, in seconds of time, which subtracted from, and added to, the time of the middle, will give the times of the beginning and end.

Note. If *c* is equal to, or greater than the sum of S and *d*, there can not be an eclipse.

For the times of Beginning and End of the Total Eclipse.

To and from the difference of S and *d*, add and subtract *c*. Then add together the logarithms of the results, $S - d + c$, and $S - d - c$, divide the sum by 2, and to the quotient add the logarithm R, and the result will be the logarithm of an interval *x*′, in seconds of time, which subtracted from, and added to, the time of the middle, will give the times of the beginning and end of the total eclipse.

Note. When *c* is greater than the difference of S and *d*, the eclipse cannot be total.

For the Quantity of the Eclipse.

Add together the constant logarithm 0.77815, the logarithm of $(S + d - c)$, and the arithmetical complement of the logarithm of d, rejecting the tens in the index, and the result will be the logarithm of the quantity of the eclipse, in digits.

Note 1. In partial eclipses of the moon, the southern part of the moon is eclipsed when the latitude is north, and the northern part when the latitude is south.

2. When the eclipse commences before sunset, the moon rises about the same time the sun sets. To obtain the quantity of the eclipse nearly, at the time the moon rises, take the difference between the time of sunset and the middle of the eclipse. Then, as 1 hour : this difference : : difference between the hourly motion of the moon and sun, in longitude : a fourth term. Add together the squares of this fourth term and of the arc c, both in seconds, and extract the square root of the sum. Use this root instead of c, in the above rule, and it will give the quantity of the eclipse at the time of the moon's rising, very nearly. When the eclipse ends after sunrise in the morning, the quantity at the time of the moon's setting may be found in the same manner, only using sunrise instead of sunset.

Exam. 1. Required to calculate, for the meridian of Philadelphia, the eclipse of the moon, in July, 1823.

The approximate time of full moon, is July 22, at 15 h. 33 m.

Sun's longitude at that time, -	3^s	29°	25′	23″
Do. hourly motion, - -			2	23
Do. semidiameter, - - -			15	46
Moon's longitude, - -	9	29	24	51
Do. latitude, - - -			9	10 N.
Do. equatorial parallax,			54	1
Do. semidiameter, - d.			14	43
Do. hor. mot. in long. -			29	34
Do. do. in lat. - -			2	43, tending north.

		d.	h.	m.	sec.
Approx, time of full moon, July, - - -		22	15	33	0
Correct. found by prob. XXVIII. - -			+	1	11
True time, in mean time at Greenwich,		22	15	34	11
Equat. of time, - - - - - - - -			—	6	2
Apparent time at Greenwich, - - -		22	15	28	9
Diff. of Long. - - - - - - - -			5	0	40
Apparent time at Philadelphia,	T.	22	10	27	29

m. m. sec.
As 60 : 1 11 : : 2′ 43″ : 3″, the correct. of lat.

Moon's lat. at approx. time, - - - - - 9′ 10″ N.
Correction, - - - - - - - - - + 3

Moon's lat. at true time, - - - - - 9 13 N.

Moon's equatorial parallax, - - - - - 54′ 1″
Sun's do. - - - - - - 9

Sum, - - - - - - - - - 54 10
Sun's semidiameter, - - - - - - - 15 46

38 24
Add - - - - - - - - - - - 0 38

Semidiam. of earth's shadow, - - - S. 39 2

Moon's hor. mot. less sun's, 1631″ Ar. Co. log. 6.78755
Moon's hor. mot. in lat. 163 - - log. 2.21219

I - - - - 5° 42′ - - tan. 8.99974

3.55630
I - - - 5° 42′ - - - cos. 9.99785
Moon's hor. mot. less sun's - Ar. Co. log. 6.78755

R. 0.34170
Moon's lat. - - - 553″ - log. 2.74272
I - - - - 5° 42′ - - sin. 8.99704

t 121 sec. = 2 m. 1 sec. - - log. 2.08146
T 10 h. 27 m. 29 sec.

Middle, 10 h. 25 m. 28 sec.

Moon's lat. - - - - - - - log. 2.74272
I - - - - - - - - - cos. 9.99785

c - - 550″ = 9′ 10″ - - - log. 2.74057

$S + d + c$ - - 3775″ - - log. 3.57692
$S + d - c$ - - - 2675 - - - log. 3.42732

2)7.00424

3.50212
R. 0.34170

sec. h. m. sec.
x = 6980 = 1 56 20 - - - - - - log. 3.84382

	h.	m.	sec.
Middle,	10	25	23
x	1	56	20
Beginning,	8	29	8
End,	12	21	48 A. M. of 23d day

$S-d+c$ - - 2009″ - - log. 3.30298
$S-d-c$ - - - 909 - - log. 2.95856

2)6.26154

3.13077
R. 0.34170

sec. m. sec.
$x' = 2968 = 49\ 28$ - - - - - log. 3.47247

	h.	m.	sec.
Middle,	10	25	28
x'		49	28
Beginning of the total eclipse,	9	36	0
End do.	11	14	56

0.77815
$S+d-c$ - - - - - - - - log. 3.42732
d - - - - 883″ Ar. Co. log. 7.05404

Digits eclipsed, 18.2 - - log. 1.25951

2. Required to calculate, for the meridian of Philadelphia, the eclipse of the moon, on the 2d of August, 1822.

Ans. Moon rises about sunset, 8½ digits eclipsed,
Ecliptic opposition, - - - - - 7h. 16m.
Middle, - - - - - - - - 7 23
End, - - - - - - - - - 8 55
Digits eclipsed 9, on moon's northern limb.

3. Required to calculate, for the meridian of Philadelphia, the eclipse of the moon, in January, 1824.

Ans. Beginning, January 16th, at - - - - 2 17 A. M.
Middle, - - - - - - - - 3 43
Ecliptic opposition, - - - - - 3 49
End, - - - - - - - - 5 8
Digits eclipsed 9.4, on moon's northern limb.

PROBLEM XXXI.

To Project an Eclipse of the Moon.

Find the true time of full moon, the moon's latitude at that time, the semidiameter of the earth's shadow, the sun's hourly motion, and the moon's semidiameter and hourly motions in longitude and latitude, as directed in the last problem.

To the moon's hourly motion from the sun in longitude* add 9″, and it will give the *moon's hourly motion from the sun, on the Relative Orbit,* with sufficient accuracy for a construction.

Draw any right line AB, *Fig.* 56, for a part of the ecliptic, and in it take a point C. Take the semidiameter of the earth's shadow from a scale† of equal parts, and with the centre C describe a circle RST, to represent a section of the earth's shadow. Through C draw KCL, perpendicular to AB. Take the moon's latitude from the scale, and set it on the line KL, from C to M, *above* AB, when the latitude is *north*, but *below*, when it is *south*. Then M will be the moon's place at the time of ecliptic opposition, or full moon. Draw M*b* parallel to AB, and to the *left* of KL, and make it equal to the moon's hourly motion from the sun in longitude. Make *bc* perpendicular to M*b*, and equal to the moon's hourly motion in latitude, drawing it *above* M*b*, when the latitude is *tending north*, but *below*, when it is *tending south*. Through M and *c* draw the indefinite right line PQ, and it will be the moon's relative orbit.

Make the proportion, as 60 minutes : minutes and seconds of the true time of full moon :: moon's hourly motion from the sun on the relative orbit : a fourth term. Take this fourth term from the scale, and lay it on the relative orbit from M to the *right hand*, and it will give the moon's place at the whole hour next preceding the time of full moon. Place the number of the hour to the point thus obtained. Then commencing at this point, and with the moon's hourly motion from the sun on the relative orbit, in the dividers, set off equal spaces on the orbit, on each side of the point, and thus obtain the moon's places at some of the whole hours, preceding and following the hour mentioned above. Put the numbers of the hours to these places. Divide each hour space into four equal parts, for quarters, and these into five-minute or minute spaces.

Through C draw SCT, perpendicular to PQ, and the intersection F will be the moon's place at the middle of the eclipse. With the centre C, and a radius equal to the sum of the semidiameters of the earth's shadow and moon, describe arcs cutting PQ in D and H, the moon's places at the beginning and end of the eclipse. With the same centre, and a radius equal to the difference of the semidiameters of the earth's shadow and moon, provided this difference is greater than CF, describe other arcs, cutting PQ in E and G, the moon's places at the beginning and end of the total eclipse. If the difference of the semidiameters of the earth's shadow and moon is less than CF, the eclipse will not be total.

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* Which is the difference of their hourly motions in longitude.

† A suitable scale is one of ten minutes to an inch. It may also be observed, that it is most convenient to reduce the seconds, in the quantities to be taken off, to decimals of a minute.

From the divisions on the relative orbit, the times at which the moon is at the points D, F, and H, and consequently the times of the beginning, middle, and end of the eclipse, are easily estimated. In like manner, when the eclipse is total, the beginning and end of the total eclipse are determined from the points E and G.

With the moon's semidiametet for a radius, and the centres D, F, and H, describe circles to represent the moon at the beginning, middle, and end of the eclipse.

Take the distance NS, when the latitude is north, but UT, when the latitude is south, and measure it on the scale. Then, as the moon's semidiameter : this distance : : 6 digits : the digits eclipsed.

Note. The quantities used in constructing an eclipse are frequently called the *Elements.* It is convenient to have them collected in order, before commencing the construction. The true time of full moon, which is one of the elements, may be expressed either astronomically or in common reckoning; the former is perhaps the most convenient.

EXAM. 1. Required to construct the eclipse of the moon, in July, 1823, taking the time for the meridian of Philadelphia.

The elements for this construction, the most of which have been found in the first example of the last problem, are as follows:

Elements Collected.

True time of full moon, July 22d,	10 h.	27 m.	23 sec.
Semidiam. of earth's shadow,	39′	2″	=39.03′
Moon's latitude, *north,*	9	13	= 9.22
Moon's hor. mot. from sun, in long.	27	11	= 27.18
Moon's hor. mot. in lat. *tending north,*	2	43	= 2.72
Moon's hor. mot. from sun, on rel. orb.	27	20	= 27.33
Moon's semidiameter,	14	43	= 14.72
Sum of semidiam. of earth's shadow and moon,	53	45	= 53.75
Difference of do.	24	19	= 24.32

Draw AB, *Fig.* 56, take the point C, and through it draw KCL, perpendicular to AB. Take the moon's latitude, 9.22, from the scale, and lay it on KL, from C to M, above AB, because the latitude is north. Draw M*b* parallel to AB, and make it equal to 27.18, the moon's hourly motion from the sun in longitude. Draw *bc* perpendicular to M*b*, on the upper side, because the latitude is tending north, and make it equal to 2.72, the moon's hourly motion in latiude. Through M and *c* draw the relative orbit, PQ.

As 60 minutes : 27 m. 23 sec. the minutes and seconds of the true time of full moon : : 27.33, the moon's hourly motion from the sun on the relative orbit : 12.47, the fourth term. Take this fourth term and lay it on the relative orbit, from M to 10, the moon's place at the 10th hour, in this example. Take 27.33, the moon's hourly motion from the sun on the relative orbit, and lay it from 10 to 9, and 9 to 8, and on the other side, from 10 to 11, 11 to 12, and 12 to 13, for the moon's places at those hours. Divide the hour spaces into quarters, and these into five minute or minute spaces.

Through C draw SCT, perpendicular to PQ, intersecting it in F, which will be the moon's place at the middle of the eclipse. With the radius 53.75, the sum of the semidiameters of the earth's shadow and moon, and the centre C, describe arcs, cutting the relative orbit in D and H, the moon's places at the beginning and end of the eclipse. With the radius 24.82, the difference of the semidiameters of the earth's shadow and moon, describe arcs, cutting the relative orbit in E and G, the moon's places at the beginning and end of the total eclipse. The times designated by the points D, F, H, E, and G, agree nearly with the beginning, middle, and end of the eclipse, and beginning and end of the total eclipse, found in the first example of the last problem.

With 14.72, the moon's semidiameter, for a radius, describe the circles about the centres D, F, and H. Take the distance NS, and measure it on the scale, and it will be found to be about 44.65. Then, 14.72 : 44.65 : : 6 digits : 18.2 digits, the quantity of the eclipse.

2. Construct the eclipse of the moon, mentioned in the second example of the last problem, and the results will be found to agree nearly with the answer there given.

3. Construct the eclipse of the moon, mentioned in the third example of the last problem.

PROBLEM XXXII.

To Project an Eclipse of the Sun, for a given place.

Calculate the approximate time of new moon by prob. XXVIII, and for that time calculate the sun's longitude, semidiameter, and hourly motion, and the moon's longitude, latitude, equatorial parallax, semidiameter, and hourly motions in longitude and latitude. Find the true time of new moon by prob. XXVIII, and reduce it to apparent time at the given place, expressing it astronomically. Also, find the moon's latitude at the true time of new moon, from the hourly motion in latitude, in the same manner as directed in prob. XXX, for finding the latitude at the true time of full moon. With the sun's longitude at the approximate time of new moon, neglecting the seconds, and taking the obliquity of the ecliptic 23° 28′, find the sun's declination by prob. VIII. Find the moon's hourly motion from the sun on the relative orbit, by adding 9″ to the difference of their hourly motions in longitude. Find the reduced latitude of the place and the reduced parallax, by prob. XIII. From the moon's reduced parallax subtract the sun's parallax, which may be taken 9″, and the remainder will be the *Semidiameter of the Circle of Projection.*

Draw a right line AB, *Fig.* 57, and in it take a point C. Take the semidiameter of the circle of projection from a scale of equal parts, and with the centre C, describe, on the *upper* side of AB, the semicircle ADB, to represent the northern half of the circle of projection. When the latitude of the place is *south*, the whole circle must be described. Through C, and perpendicular to AB, draw the line TCY, to represent the *universal meridian*. With a sector,* opened to the radius AC or CB, set off from D, the arcs DV, DR, each equal to the obliquity of the ecliptic, which may be taken 23° 28′; join RV, and on it describe the semicircle RTV. With the sector, opened to the radius OV or OR, make the arc VU, equal to the sun's longitude. When the longitude exceeds VI signs, take VI signs from it, and set off the remainder from R, round towards V. Draw UW perpendicular to RV, and through W draw CWL, and it will be the projection of the circle of latitude which passes through the moon at the time of new moon. Take the moon's latitude from the scale, and lay it on CL, from C to M, *above* AB, when it is *north*, but on LC produced, *below* AB, when it is *south*. Then M will be the moon's place at the true time of ecliptic conjunction. From M, draw *Mb* perpendicular to CL, to the left hand, and make it equal to the moon's hourly motion from the sun in longitude. Draw *bc* perpendicular to *Mb*, *above*, when the latitude is tending *north*, but *below*, when it is tending *south*, and make it equal to the moon's hourly motion in latitude. Through M and *c* draw the moon's relative orbit, PQ. Make the proportion, as 60 minutes : minutes and seconds of the true time of new moon : : moon's hourly motion from the sun on the relative orbit : a fourth term. Take this fourth term from the scale, and lay it on the relative orbit, from M to the right hand, and it will give the moon's place at the whole hour next preceding the time of new moon. Take the moon's hourly motion from the sun on the relative orbit, from the scale, and with it lay off equal spaces on each side of the moon's place, just found, and thus obtain the moon's places for four or five other hours, contiguous to the time of new moon, some of them preceding and some following it. When the time of new moon is several hours before noon, there should be more places found for hours preceding the time of new moon than for the hours following it, and the *contrary*, when the time of new moon is several hours past noon. To each of the moon's places, thus found, put the number of the hour.

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* For the manner of using the sector, see the note at the bottom of page 127. To what is there said respecting the manner of using it, may be added, that when an arc greater than 60° is to be laid off, it may be done by applying the radius of the circle as a chord to the arc, as many times successively as 60° is contained in the arc to be laid off, and then with the sector, laying off from the last point, an arc equal to the remainder. When a very small arc is to be laid off with a sector, it is better to add some constant arc to it, for instance 10°. Then taking the chord of the sum from the sector, lay it on the arc, from the given point to a second one, and taking the chord of the arc which was added, set it from the second point backwards, towards the first. The arc, intercepted between the last point and the given one, will be the arc which was to be laid off.

With a sector opened to the radius AC or CB, set off arcs equal to the reduced latitude of the place, from A to E and B to F, on the semicircle *above* AB when the latitude is *north*, but *below*, when it is *south*, and join EF. With the sector, opened to the same radius, make the arcs EG, EI, FH, and FK, each equal to the sun's declination, and join GH and IK. Bisect *vw* in N, and through N draw 6 N 18, parallel to EF. Make N 6 and N 18, each equal to E*r* or *r*F, and on 6 N 18 describe the semicircle 6 Y 18. With the centre N, and radius N*v* or N*w*, describe the circle *uvxw*. Take the intervals between noon and each of the hours marked on the relative orbit, and convert them into degrees, allowing 15° to each hour, and they will be the hour angles from noon. With the sector, opened to the radius N 6 or N 18, lay off from Y, on the semicircle 6 Y 18, the arc being produced above 6 N 18, when necessary, arcs equal to each of the hour angles, laying them to the *right*, when the hours are in the *forenoon*, but to the *left*, when they are in the *afternoon*, and at the extremity of each arc place the number of degrees which it contains. From these points, draw lines parallel to the universal meridian DY. Also, from the same points, draw lines to the centre N, *intersecting* the circle *uvxw*; and when the sun's declination is *south*, produce them to *meet* the same circle on the other side of N. From the points in which these lines *intersect* the circle *uvxw*, when the sun's declination is *north*, but from the points in which, being produced, they *meet* it, when the declination is *south*, draw lines parallel to EF, to meet respectively the corresponding lines, drawn parallel to the universal meridian; and the points in which they meet will be the sun's places, on the circle of projection, at the hours to which the lines correspond. At each of these points place the number of the hour to which it belongs. The points 6 and 18 are always the sun's places at those hours. When the declination is *north*, the point *v* is the sun's place at noon, designated by 0; but when the declination is *south*, the point *w* is the sun's place at noon. From the places of the moon at the hours, marked on the relative orbit, draw lines parallel to AB or EF, to meet the lines, produced, if necessary, which are parallel to the universal meridian, and pass through the sun's places at the same hours, in the points S.

Draw a right line AC, *Fig.* 58, and in it take a point *s*. Take the distance from each of the points S, in *Fig.* 57, to the corresponding place of the moon on the relative orbit, and lay it on AC, from *s*, to the *right* or *left*, according as the moon's place is to the *right* or *left* of the point S, and at the extremity of each distance, put the number of the hour to which the distance corresponds. Through each of these points draw lines perpendicular to AC. This may be most conveniently done, by drawing through one of the points a perpendicular line, and then, parallel to this, drawing lines through the others. Take from *Fig.* 57, the distances from the sun's place at each of the hours, marked on the relative orbit, to the corresponding point S, and place them on the perpendiculars, from the same numbers on the line AC, in *Fig.* 58, *above* or *below* AC, according as the point S in *Fig.* 57, is *above* or *below* the sun's place. At the extremities of these distances, place the same numbers that are on the line AC. Join each adjacent two of these extremities, and the broken line thus formed will be a near representation of the moon's *apparent*, relative orbit, and the points on it will be the moon's places at the hours denoted by their numbers.

With the centre *s*, and a radius equal to the sum of the semidiameters of the sun and moon, describe arcs, cutting the apparent relative orbit in B and E, which will be the moon's places at the beginning and end of the eclipse. With the centres B and E, and a radius greater than half the distance of these points, describe two arcs, cutting each other in *a*. Lay the edge of a ruler from *s* to *a*, and draw the line D*s*G*a*, intersecting the apparent orbit in G, which will be the moon's place at the time of greatest obscuration. From the moon's place on the apparent orbit at the whole hour next following the end of the eclipse, draw a right line LN in any convenient direction, and taking any short distance in the dividers, lay it over 12 times, from L to the point M. Then LM is to be considered as representing an hour, divided into parts of 5 minutes each, which must be reckoned from L towards M. Join M and each of the hour points on the apparent orbit. From the points B, G, and E, draw the lines B*c*, G*h*, and E*f*, respectively parallel to the lines joining M and the hours next *following* those points, and meeting the lines joining M and the hours next preceding the same points, in the points *c*, *h*, and *f*. Draw *cb*, *hg*, and *fe*, respectively parallel to lines joining L, and the hours next *preceding* the points B, G, and E. Then the minutes corresponding to *b*, connected with the hour next preceding B, those corresponding to *g*, connected with the hour next preceding G, and those corresponding to *e*, connected with the hour next preceding E, will be the times of the beginning, greatest obscuration, and end of the eclipse.

If a circle, described about the centre *s*, with a radius equal to the difference of the semidiameters of the sun and moon, cuts the apparent orbit, the eclipse will be annular or total; *annular*, when the sun's semidiameter is *greater* than the moon's; *total*, when it is *less*. The beginning or end of the annular or total eclipse, when either has place, may be found in the same manner as the beginning or end of the eclipse, taking the points in which the circle cuts the apparent orbit.

About the centres *s* and G, with radii respectively equal to the semidiameters of the sun and moon, describe circles to represent those bodies. Take the distance DH, and applying it to the scale, obtain its measure. Then, as the sun's semidiameter : measure of DH :: 6 digits : digits eclipsed.

Take the interval between the beginning of the eclipse and noon, and convert it into degrees. With the sector opened to the radius N 6, or N 18, *Fig.* 57, lay off from Y, on the semicircle 6 Y 18, the arc being produced if necessary, an arc containing this number of degrees, laying it to the *right* or *left*, according as the time of beginning is *before* or *after* noon, and proceed to find the sun's place on the circle of projection for the time of beginning, in the same manner as directed above, for other times. Mark this place of the sun with the letter *n*, and join C*n*. Make the angle C*s*V, *Fig.* 58, equal to the angle BC*n*, *Fig.* 57, and join *s*B. Then *v* will represent the sun's vertex at the beginning of the eclipse, *z* the place at which the eclipse commences, and the angle V*s*B, the angular distance of this point from the sun's vertex.

Note. The times of beginning, &c. obtained by projection, are only approximate values. But when the construction is carefully made, they will seldom err more than one or two minutes.

Exam. 1. Required the times, &c. of the eclipse of the sun of August 27th, 1821, at Philadelphia.

The different elements necessary for the construction are easily found, and are as follows:—

Elements Collected.

True time of new moon, August, 26 d. 22 h.	15 m.	56 sec.		
Semidiameter of the circle of projection,		55′	1″	=55.02′
Sun's longitude,	153°	57′		
Sun's declination, *north*,	10	4		
Moon's latitude, *north*,		3	41	= 3.68
Moon's hor. mot. from sun, in long.		28	30	= 28.5
Moon's hor. mot. in lat. *tending south*,		2	51	= 2.85
Moon's hor. mot. from sun, on rel. orb.		28	39	= 28.65
Fourth term,		7	36	= 7.6
Sun's semidiameter,		15	52	= 15.87
Moon's do.		15	3	= 15.05
Sum of semidiameters of sun and moon,		30	55	= 30.92
Latitude of Philadelphia, reduced,	39	46	N.	

Draw AB, *Fig.* 57, and take the point C. Take 55.02′, the semidiameter of the circle of projection, from a scale of equal parts, and with the centre C, describe the semicircle ADB. Through C, and perpendicular to AB, draw the universal meridian TCY, cutting ADB in D. With a sector opened to the radius AC or CB, make the arcs DR, DV, each equal to 23° 28′, the obliquity of the ecliptic; join RV, and on it describe the semicircle RTV. With the sector opened to the radius OR or OV, make the arc VTU equal to 153° 57′, the sun's longitude. Draw UW perpendicular to RV, and through W draw CWL. Take 3.68′, the moon's latitude, from the scale, and lay it on CL, from C to M, above AB, because the latitude is north. Draw M*b* perpendicular to CL, and make it equal to 28.5′, the moon's hourly motion from the sun in longi-

tude. Draw *bc* perpendicular to M*b*, below M*b*, because the latitude is tending south, and make it equal to 2.85′ the moon's hourly motion in latitude. Through M and *c* draw the moon's relative orbit PQ. Take 7.6′, the fourth term, from the scale, and lay it on the relative orbit, from M to XXII, the moon's place at that hour. Take 28.65′, the moon's hourly motion from the sun on the relative orbit, from the scale, and set it over from XXII, backwards to XXI, XX, and XIX, and forwsrds to XXIII, for the moon's places at those hours. With the sector opened to the radius AC or CB, make the arcs AE and BF, each equal
the reduced latitude of Philadelphia. With the sector open
radius, make the arcs EG, EI, FH, and FK, each equal
sun's declination, and join GH and IK, intersecting the univ
in *w* and *v*. Bisect *vw* in N; through N draw 6 N 18, p
and make N 6 and N 18 each equal to *r*E or *r*F. With
and radius N 6 or N 18, describe the semicircle 6 Y 18,
same centre, and radius N*v* or N*w*, describe the circle *uvxw*
vals between noon and the hours, marked on the relative o
3, 4, and 5 hours, and these in degrees are 15°, 30°, 45°,
With the sector opened to the radius N 6 or N 18, lay off
the semicircle 6 Y 18, all of them from Y to the right hand, because the hours are all in the forenoon. From the points 15, 30, 45, 60, and 75, which are the extremities of the arcs, draw the lines 15, 23; 30, 22; 45, 21; 60, 20; and 75, 19, parallel to the universal meridian TY; and from the same points draw lines to the centre N, not producing them, because the sun's declination is north. From the points in which the lines N 15, N 30, N 45, N 60, and N 75, intersect the circle *uvxw*, draw lines parallel to EF, respectively meeting the lines 15, 23; 30, 22; 45, 21; 60, 20; and 75, 19, in the points 23, 22, 21, 20, and 19, which are the sun's places at those hours. From the points XIX, XX, XXI, XXII, and XXIII, draw, parallel to AB or EF, the lines XIXS, XXS, XXIS, XXIIS, and XXIIIS, meeting the lines 75, 19; 60, 20; 45, 21; 30, 22; and 15, 23, in the points S. Draw AC, *Fig.* 58, and in it take the point *s*. Take the distances SXIX, SXX, SXXI, SXXII, and SXXIII, *Fig.* 57, and set them on the line AC, *Fig.* 58, from *s* to 19, 20, 21, 22, and 23, placing the first three to the right of *s*, because the moon's places at those hours are to the right of the corresponding points S, and the other two to the left, because the moon's places are at those hours to the left of the corresponding points S. Draw 21, XXI, perpendicular to AC, and parallel to it, draw 19, XIX; 20, XX; 22, XXII; and 23, XXIII. Take the distances S19, S20, S21, S22, and S23, *Fig.* 57, and set them in *Fig.* 58, from 19 to XIX, 20 to XX, 21 to XXI, 22 to XXII, and 23 to XXIII, setting the first two above AC, because the points S are above the sun's places, and the others below, because the points S are below the sun's places. Join XIX, XX; XX, XXI; XXI, XXII; and XXII, XXIII, for the apparent relative orbit of the moon. Take 30.92′, the sum of the semidiameters of the sun and moon, from the

scale, and with the centre *s* describe arcs cutting the apparent orbit in B and E, the moon's places at the beginning and end. With the centres B and E, and a radius greater than half the distance between them, describe arcs cutting each other in *a;* and with the edge of a ruler, applied to *s* and *a*, draw the line D*s*GH, intersecting the apparent orbit in G, the moon's place at the greatest obscuration. From the point XXIII, in the apparent orbit, draw LN, and taking some short distance in the dividers, lay it over 12 times, from L to M, and number the divisions as in the figure. Join M, XIX ; M, XX ; M, XXI ; and M, XXII, and draw B*c* parallel to M, XX ; G*h* parallel to M, XXI ; and E*f* parallel to ML. Draw *cb* parallel to L, XIX ; *hg* parallel to L, XX ; and *fe* parallel to L, XXII. Then, attending to the rule, it is easy to perceive that the beginning of the eclipse is at 19 h. 31 m. ; the greatest obscuration at 20 h. 48 m. ; and the end at 22 h. 14 m. Take 15.87′, the sun's semidiameter from the scale, and with the centre *s* describe a circle to represent the sun, and with 15.05, the moon's semidiameter, taken from the scale, and the centre G, describe another circle, to represent the moon. The distance DH, applied to the scale, will be found to measure 22.9′. Then, 15.87′ : 22.9′ : : 6 digits : 8⅔ digits, the quantity of the eclipse. The interval between the time of beginning and noon is 4 h. 29 m. which in degrees is 67° 15′. With the sector opened to the radius N 6 or N 18, *Fig.* 57, lay off this arc on the semicircle 6 Y 18, from Y to the right hand, because the time is in the forenoon, and find *n*, the sun's place at that time, in the same manner as for other times. Join C*n*, and make the angle C*s*V, *Fig.* 58, equal to BC*n*, and join *s*B. The measure of the angle B*s*V is 26°, which is the angular distance of the point at which the eclipse commences from the sun's vertex to the right hand. In *Fig.* 59, is a reduced representation of the sun's and moon's discs, with the line *s*V placed in a vertical position.

2. Required to calculate the elements, and project an eclipse of the sun, for the latitude and meridian of Philadelphia, that will occur in February, 1831.

Elements.

True time of new moon, February,	11 d.	23 h.	57 m.	40 sec.
Semidiam. of circle of projection,		57′	24″	= 57.4′
Sun's longitude,	323°	18		
Sun's declination, *south*,	13	46		
Moon's latitude, *north*,		42	10	= 42.17
Moon's hor. mot. from sun, in long.		31	4	= 31.07
Moon's hor. mot. in lat. *tending south*,		3	4	= 3.07
Moon's hor. mot. from sun, on rel. orb.		31	13	= 31.22
Fourth term,		30	0	= 30.00
Sun's semidiameter,		16	14	= 16.23
Moon's do.		15	42	= 15.7
Sum of semidiameters,				31.93
Latitude of Philadelphia, reduced	39°	46′		

Result of Projection.

	d.	h.	m.	
Beginning, - - - -	12	11	7	A. M.
Greatest obscuration, - -		0	42	P. M.
End, - - - - -		2	11	"

Digits eclipsed $11\frac{3}{5}$ on sun's south limb.

Eclipse commences about 101° from the sun's vertex to the right hand.

Note. In the third volume of the Transactions of the American Philosophical Society, *New Series*, a simple method is given for obtaining, by construction, the times of beginning, &c. of an eclipse of the sun. But the improved methods now in use for obtaining the same results by calculation, render it unnecessary to introduce it here. The preceding method of projection is retained, as it serves to give the student a clearer view of the whole subject.

PROBLEM XXXIII.

To find for a given place certain auxiliary logarithms, and a natural number convenient in calculating an eclipse of the sun for that place.

Let ρ denote the distance from the given place to the earth's centre, φ' the geocentric latitude of the place, and ε the apparent obliquity of the ecliptic.

To log. x, taken from Table LXXII, with the geographic latitude as the argument, add the cosine of the geographic latitude, and the sum* will be the logarithm of $10\,\rho$ cos. φ'. Call this log. B. To log. B add the sine of ε, marking the sum *negative*,† and call it log. G. To log. y, taken from table LXXII, add the sine of the geographic latitude, and the sum will be the logarithm of $10\,\rho$ sin. φ'. Call this log. F. To log. F add the sine of ε, and call the sum log. A. To log. F add the cosine of ε, and, taking the natural number corresponding to the sum, call it f.

Note 1. If the logarithms, and the quantity f, are calculated for any given place, with the obliquity 23° 27′ 30″, they will serve in all calculations of eclipses of the sun for that place, when great accuracy is not desired. But when the calculation is required to be made with precision, the apparent obliquity at the time of the eclipse must be used in finding them.

2. Table LXXIV. contains, for several places, the logarithms A, B, and G, to three decimal figures, and the quantity f to two decimal figures. These may be used for those places, in the calculation of eclipses, when the places of the sun and moon have been obtained from the tables in this

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* Here, and throughout the rule, 10 is to be rejected from the index of the sum, when it is equal to, or greater than 10.

† Logarithms are now frequently marked *negative*, by placing a small *n* to the right of the right-hand figure, near the bottom.

work, and in all cases when only near approximate values of the times of beginning, &c. are required.

3. In *south* latitudes the logarithms F and A, and the quantity f, must each be marked *negative*.

EXAM. 1. Required the logarithms A, B, and G, and the quantity f, for Philadelphia, when the obliquity of the ecliptic is 23° 27′ 30″.

From table LXXII,	log. x,	1.00060	log. y,	0.99770
Lat. 39° 57′	cos.	9.88457	sin.	9.80762
	Log. B.	0.88517	Log. F.	0.80532
$\varepsilon = 23°\ 27'\ 30''$	sin.	9.59997	sin.	9.59997
	Log. G.	0.48514 n	Log. A.	0.40529
		Log. F.	0.80532	
ε - -		cos.	9.96254	
$f = +5.8595$ -			0.76786	

2. Required the logarithms A, B, and G, and the quantity f, for Boston, when the obliquity of the ecliptic is 23° 27′ 30″.

Ans. Log. A = 0.42620, log. B = 0.86930, log. G = 0.46927 n, and $f = +6.1486$.

PROBLEM XXXIV.

To calculate an Eclipse of the Sun for a given place, using the tables of the sun and moon contained in this work.

1. Find, by Prob. XXVIII, the approximate time of new moon, in mean time at Greenwich, and denote this time, or the time taken to the nearest whole hour,* by T. For the time T, calculate the sun's longitude, hourly motion, and semidiameter; also the moon's longitude, latitude, horizontal parallax, and hourly motions in longitude and latitude.

2. Take the difference† between the sun's and moon's longitudes, marking it *affirmative* when the moon's longitude is *greater* than the sun's, but *negative* when it is *less*. Take also the difference between the sun's parallax, 9″, and the moon's parallax. Multiply the difference of the longitudes, in seconds, by 10, and divide the product by the difference of the parallaxes, also in seconds; extending the quotient to two decimal figures, and denote it by p. Do the same with the moon's latitude, marked

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* When the odd minutes of the approximate time are near to 30, it is generally better to take T to represent the approximate time as found; but when they are near to 0 or 60, T may be put for the time taken to the nearest whole hour.

† It may sometimes occur that one of the longitudes is near to 0°, and the other near to 360°. When this is the case, the less longitude, increased by 360°, is to be considered the longitude.

affirmative or *negative*, according as it is *north* or *south*, and denote the quotient by q; with the difference of the hourly motions in longitude, denoting the quotient by p'; with the moon's hourly motion in latitude, denoting the quotient by q'; and with the sun's semidiameter, denoting the quotient by R.

3. To the sun's tabular mean longitude at the time T, increased by 2°, apply according to its sign the nutation in right ascension, taken from table XVI, with the argument N, and the result will be the sun's mean longitude from the true equinox. To this longitude add the mean time at the given place, expressed in degrees, corresponding to the Greenwich time T, and the sum will be the right ascension of the zenith of the given place at that time. Call this right ascension Z. From Z, increased by 360°, if necessary, subtract the sun's longitude, and call the remainder D.

4. Call the sun's longitude L′. To log. A, found by the last problem, or taken from table LXXIV, add cos. L′, and taking the natural number corresponding to the sum, call it a. To log. A add sin. L′, and taking the natural number corresponding to the sum, call it a''. To the constant logarithm 9.317, add cos. L′, rejecting 10 from the index of the sum, and call the result log. C.

5. The quantities p', q', R, a, a'', and log. C, may be considered constant during the eclipse. But the values of p, q, Z, and L′, and consequently D, are only true for the time T. To find them for any other time differing from the time T, by an interval t, proceed thus:—As 1 hour : t : : p' : the change of p, in the time t, *affirmative*. As 1 hour : t : : q' : the change of q in time t, with the *same sign* as q'. As 1 hour : t : : 15° 2.5′ : change of Z, in time t, *affirmative*. And as 1 hour : t : : 2.5′ : change of L′, in time t, affirmative. Or the change of Z may be found by adding together the arcs in table LXXI*, corresponding to the hours, minutes, and seconds of the time t. And the change of L′ may be found by adding together the sun's motion in longitude, taken from tables XI and XII, for the hours, minutes, and seconds of the time t. The changes of the values, applied respectively to p, q, Z, and L′, by *adding* when the given time is *later* than T, but by *subtracting* when it is *earlier*, will give their required values.

6. Find log. B, log. G, and the quantity f, by the last problem, or take them from table LXXIV. Using the values of D, Z, p, and q, at the time T, to log. B add sin. D, and taking the natural number corresponding to the sum, call it b. Do the same with log. B and cos. D, calling the resulting natural number b'; with log. G and sin. Z, calling the result g; and with log. G and cos. Z, calling the result g'. To the logarithm of g add log. C, and call the natural number corresponding to the sum, c. Do the same with the logarithm of g' and log. C, calling the result c'. Attending to their signs, take the sum of a, b, and c, and call it u; the sum of f and g, and call it v; and the sum of b' and c', and call it b''. With b'' as the argument, take the value of u' from table LXXIII; and,

with g' as the argument, take the value of v' from the same table. Attending to the signs, subtract u from p, calling the remainder x; v from q, calling the remainder y; u' from p', calling the remainder x'; and v' from q', calling the remainder y'.

7. Using table LXXIX, add together the squares of x' and y', and call the square root of the sum n. Attending to the signs, add together the products of x by x', and y by y'. Multiply the sum by 60, and dividing the product by the square of n, the quotient will be an interval t, in minutes of time; which, subtracted from T, attention being given to its sign, will give the *approximate time of greatest obscuration*, in mean time at Greenwich.

8. Call the approximate time of greatest obscuration T′, and for this time, find, by art. 5, the values of p, q, Z, and D; and by art. 6, the values of x, y, x', and y'. Then, proceeding as in the last article, find an interval t, in minutes and tenths of a minute. Attending to the sign, subtract t from T′, and the remainder will be the *true time** *of greatest obscuration.*

9. Divide the sum of the products x by x', and y by y', by n, and call the quotient n'. From the sum of the squares of x and y, subtract the square of n', and taking the square root of the remainder, call it m. Then will m be the nearest distance of the centres of the sun and moon, expressed in the same parts as the quantities R, p, &c., that is in such parts as the difference of the parallaxes, in seconds, contains 10. To a'' add the value of b' at the time T′, and call the sum E. With the argument E at the top and the sun's semidiameter at the side, take the correction of R from table LXXVI, and subtracting it from R, call the remainder R′. To R′ add 2.73 and call the sum k. Subtract m from k, multiply the remainder by 6, and divide the product by R′. The quotient will be the *digits eclipsed;* on the *north* or *south* limb, according as the value of y is *affirmative* or *negative*. When m is equal to or greater than k, the eclipse will not be visible at the given place.

From the square of k, subtract the square of m, and call the square root of the remainder h. Then, taking the value of n at the time T′, as $n : h :: 1$ hour : an interval of time, which subtracted from the true time of greatest obscuration, and added to it, will give the *approximate times of the beginning and end of the eclipse.*

10. Taking now T′ equal to the approximate time of beginning, find for this time, the values of x, y, x' and y'; also by the last article find R′,

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* The expression, *true time*, is to be taken here and in the subsequent part of the rule, in a relative sense; as only implying that the time found has an accuracy corresponding with that of the tables, from which the places of the sun and moon have been obtained, and of the number of decimals used in the calculation. With reference to a more exact determination, with more accurate data, they are *near approximate times*. They may frequently be in error to the amount of two or three tenths of a minute; and sometimes, perhaps, though seldom, to the amount of half a minute.

and thence the value of k. Subtract the square of m, which is constant, from the square of k, and call the square root of the remainder h. Add together the squares of x and y, and from the sum subtract the square of m, and call the square root of the remainder h'. With the values of x' and y', find n, as in art. 7. Then, as n : diff. of h and h' :: 60 minutes : a correction in minutes and tenths of a minute, which being added to T', or subtracted from it, according as h' is greater or less than h, gives the *true time of beginning*.

11. The *true time of end* is found in exactly the same manner, except that the correction is to be subtracted from T', the approximate time of end, when h' is greater than h, but added to it, when h' is less than h.

12. From the values of u, v, x, and y, at the approximate time of beginning, find by means of their hourly changes u', v', x', and y', at the same time, their values at the true time of beginning. Taking these values, to the logarithm of x, add the arithmetical complement of the logarithm of y, and the sum will be the tangent of an arc P. The arc P must be taken *less* or *greater* than 90°, according as y is *affirmative* or *negative*, and it must be marked with the same sign as x. Proceed in the same manner with u and v, to find an arc Q, taking it *less* or *greater* than 90°, according as v is *affirmative* or *negative*, and marking it with the same sign as u. Attending to the signs subtract P from Q, and call the remainder V. Then will V express the distance on the circumference of the sun's disc, from the vertex to the point at which the eclipse commences; reckoned from the vertex to the *west*, when V is *affirmative*, but to the *east*, when V is *negative*. In a similar manner the point at which the eclipse ends, may be found.

Note 1. In an annular or a total eclipse, the time at which it begins or ceases to be annular or total, may be found in the same manner as the time of beginning or end of the eclipse, except that k must be taken equal to the difference of R' and 2.73, instead of their sum.

2. The above rule is formed from the formula investigated in the appendix to Part I., for calculating an eclipse of the sun, using longitudes and latitudes of the sun and moon. The different terms of the formula were multiplied by 10, in order to lessen the number of entirely decimal numbers.

Exam. It is required to calculate, for Philadelphia, the eclipse of the sun of May 15th, 1836.

Approx. time of new moon, 15 2 8, Greenwich mean time.

At time T = 15 d. 2 h. 0 m.

Sun's longitude,	54°	42'	8"
" hourly motion,		2	25
" semidiameter,		15	50
" tabular mean long. + 2°	53	19	39

Argument N = 862

Moon's longitude,	54	38	55
" latitude,	0	25	27 N.
" hourly mot. in longitude,		29	57
" " " latitude,	+	2	45
" equatorial hor. parallax,		54	24

Moon's longitude,	54°	38′	55″
Sun's "	54	42	8
Diff. of longitude,	—	3	13 = — 193″

Moon's hourly mot. in long.	29′	57″
Sun's " " "	2	25
Diff. of hourly mot. in long.	27	32 = 1652″

Moon's equatorial parallax,	54′	24″
Sun's " "		9
Diff. of parallaxes,	54	15 = 3255″

$$p = \frac{10 \times -193}{3255} = -0.59\,; \quad q = \frac{10 \times +1527}{3255} = +4.69$$

$$p' = \frac{10 \times 1652}{3255} = +5.08\,; \quad q' = \frac{10 \times +165}{3255} = +0.51$$

$$R = \frac{10 \times 950}{3255} = 2.92.$$

Sun's tab. mean long. + 2°,	53°	19′	39″
Nut. in right ascen. tab. XVI,		—	13
Sun's mean long. from true equinox,	53	19	26
Mean time at Phila. in degrees,	314	50	0
Z =	8	9	26
Sun's longitude,	54	42	8
D =	313	27	18

From tab. LXXIV,	Log. A. 0.405		Log. A. 0.405
L′ = 54° 42′	cos. 9.762*		sin. 9.912
$a = +1.47$	0.167	$a'' = +2.07$	0.317

Constant log. 9.317
L′ cos. 9.762

Log. C. 9.079 From table LXXIV., $f = +5.86$.

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* The sines, cosines, &c., may be taken from tab. LXXVII, and the natural numbers corresponding to the sums, from tab. LXXVIII.

	Log. B.	0.885		Log. B.	0.885
$D = 313° 27'$	sin.	9.861 *n*		cos.	9.837
$b = -5.57$		0.746 *n*	$b' = +5.27$		0.722
	Log. G.	0.485 *n*		Log. G.	0.485 *n*
$Z = 8° 9.5'$	sin.	9.151		cos.	9.996
$g = -0.43$		9.636 *n*	$g' = -3.03$		0.481 *n*
	Log. C.	9.079		Log. C.	9.079
$c = -0.05$		8.715 *n*	$c' = -0.36$		9.560 *n*
			$b'' = +4.91$		

$u = -4.15$; $u' = +1.28$
$v = +5.43$; $v' = -0.80$
$x = +3.56$; $x' = +3.80$
$y = -0.74$; $y' = +1.31$

x'	sq	14.44	$xx' = +13.53$
y'	sq	1.72	$yy' = -0.97$
$n = 4.02$	sq	16.16	sum $+12.56$

$$t = \frac{+12.56 \times 60}{16.16} = +47 \text{ m.}$$

	h.	m.
$T =$	2	0
$t =$	+	47
	1	13 = approx. time of greatest obscur.

	h.	m.
At $T' =$	1	13

$p = -4.57$ $Z = 356° 23$
$q = +4.29$ $L' = 54 40$

	Log. B.	0.885		Log. B.	0.885
$D = 301° 43'$	sin.	9.930 *n*		cos.	9.721
$b = -6.53$		0.815 *n*	$b' = +4.04$		0.606
	Log. G.	0.485 *n*		Log. G.	0.485 *n*
Z - - -	sin.	8.803 *n*		cos.	9.999
$g = +0.19$		9.288	$g' = -3.05$		0.484 *n*
	Log. C.	9.079		Log. C.	9.079
$c = +0.02$		8.367	$c' = -0.37$		9.563 *n*
			$b'' = +3.67$		

$$\begin{array}{lcl} u = -5.04 & ; & u' = +0.96 \\ v = +6.05 & ; & v' = -0.80 \\ x = +0.47 & ; & x' = +4.12 \\ y = -1.76 & ; & y' = +1.31 \end{array}$$

$$\begin{array}{lrll} x' & sq & 16.97 & xx' = +1.93 \\ y' & sq & 1.72 & yy' = -2.31 \\ \hline n = 4.32 & sq & 18.69 & \quad -0.38 \end{array}$$

$$t = \frac{-0.38 \times 60}{18.69} = -1.2 \text{ m.}$$

$$n' = \frac{-0.38}{4.32} = -0.09.$$

$$\begin{array}{lrr} & \text{h.} & \text{m.} \\ T' = & 1 & 13 \\ t = & - & 1.2 \\ \hline & 1 & 14.2 \end{array}$$

1 14.2 = true time of greatest obscur.

$$\begin{array}{r} E = a'' + b' = 6.11 \\ \text{Cor. of } R = 0.03 \\ R' = 2.89 \\ k = 5.62 \end{array}$$

$$\begin{array}{lrl} x & sq & 0.22 \\ y & sq & 3.10 \\ \hline & & 3.32 \\ n' & sq & 0.01 \\ \hline m = 1.82 & sq & 3.31 \end{array}$$

$$\frac{(5.62 - 1.82) \times 6}{2.89} = 7.9$$ = digits eclipsed on sun's south limb.

$$\begin{array}{lrl} k & sq & 31.58 \\ m & sq & 3.31 \\ \hline h = 5.32 & sq & 28.27 \end{array}$$

$$\begin{array}{ccccccccc} & & & & \text{m.} & & \text{h.} & & \text{m.} \\ 4.32 & : & 5.32 & :: & 60 & : & 1 & & 13.9 \end{array}$$

$$\begin{array}{llllll} \text{h.} & \text{m.} & \text{h.} & \text{m.} & \text{h.} & \text{m.} \\ 1 & 14.2 - & 1 & 13.9 = & 0 & 0, \end{array}$$ approx. time of beginning,

$$\begin{array}{llllll} 1 & 14.2 + & 1 & 13.9 = & 2 & 28, \end{array}$$ " " end.

$$\begin{array}{lcc} & \text{h.} & \text{m.} \\ \text{At } T' = & 0 & 0 \end{array}$$

$$\begin{array}{lcl} p = -10.75 & ; & Z = 338^\circ \; 4' \\ q = +\; 3.67 & ; & L' = \;\; 54 \;\; 37 \end{array}$$

$$\begin{array}{lrl} & \text{Log. B.} & 0.885 \\ D = 283^\circ\, 27' & \text{sin.} & 9.988\,n \\ \hline b = -7.46 & & 0.873\,n \end{array}$$

$$\begin{array}{lrl} & \text{Log. B.} & 0.885 \\ & \text{cos.} & 9.366 \\ \hline b' = +1.78 & & 0.251 \end{array}$$

	Log. G	0.485 n		Log. G	0.485 n
Z	sin.	9.572 n		cos.	9.967
$g = +1.14$		0.057	$g' = -2.83$		0.452 n
	Log. C.	9.079		Log. C.	9.079
$c = +0.14$		9.136	$c' = -0.34$		9.531 n
			$b'' = +1.44$		

$u = -5.85$;	$u' = +0.38$	$E = 3.85$
$v = +7.00$;	$v' = -0.74$	$R' = 2.90$
$x = -4.90$;	$x' = +4.70$	$k = 5.63$
$y = -3.33$;	$y' = +1.25$	

k	sq 31.70	x	sq 24.01	x'	sq 22.09
m	sq 3.31	y	sq 11.09	y'	sq 1.56
$h = 5.33$	sq 28.39	sum	35.10	$n = 4.86$	23.65
$h' = 5.64$		m	sq 3.31		
Diff. = 0.31		$h' = 5.64$	sq 31.79		

m. m.
4.86 : 0.31 : : 60 : 3.8

h. m. m. h. m.
0 0 + 3.8 = 0 3.8, true time of beginning.

h. m.
At T' = 2 28

$p = +1.78$; $Z = 15° 11'$
$q = +4.93$; $L' = 54 43$

	Log. B.	0.885		Log. B.	0.885
D = 320° 28'	sin.	9.804 n		cos.	9.887
$b = -4.89$		0.689 n	$b' = +5.92$		0.772
	Log. G.	0.465 n		Log. G.	0.485 n
Z	sin.	9.413		cos.	9.985
$g = -0.79$		9.898 n	$g' = -2.95$		0.470 n
	Log. C.	9.079		Log. C.	9.079
$c = -0.09$		8.977 n	$c' = -0.35$		9.549 n
			$b'' = +5.57$		

$u = -3.51$ $u' = +1.46$ $E = 7.99$
$v = +5.07$ $v' = -0.77$ $R' = 2.88$
$x = +5.29$ $x' = +3.62$ $k = 5.61$
$y = -0.14$ $y' = +1.28$

k	sq 31.47	x	sq 27.98	x'	sq 13.10
m	sq 3.31	y	sq 0.02	y'	sq 1.64
$h = 5.31$	28.16	sum	28.00	$n = 3.84$.	14.74
$h' = 4.97$		m	sq 3.31		
Diff. = 0.34		$h' = 4.97$	24.69		

m. m.
3.84 : 0.34 : : 60 : 5.3

h. m. m. h. m.
2 28 + 5.3 = 2 33.3, true time of end.

h. m.
At true time of beginning, 0 3.8

$u = -5.83$ $x = -4.60$
$v = +6.95$ $y = -3.25$

x	log. 0.663 n	u	log. 0.766 n
y	Ar. Co. " 9.488 n	v	Ar. Co. " 9.158
P = −125° 13′	tan. 0.151	Q = −40° 0′	tan. 9.924 n

Q = − 40° 0′
P = − 125 13
V = + 85 13

Hence, expressing the times in mean time at Philadelphia, we have,

	h. m.
Beginning of the eclipse at	7 3.1 A. M.
Greatest obscuration "	8 13.5
End "	9 32.6

Digits eclipsed 7.9 on south limb.
Eclipse begins 85° 13′ from sun's vertex, to the right.

PROBLEM XXXV.

To calculate various circumstances of an Eclipse of the Sun, for the earth in general.

This problem will be rendered more explicit to the student by separating it into parts, and illustrating each part by an example. We shall take for this purpose the eclipse, calculated in the last problem, and use such of the quantities found there as may be requisite.

1. *To find the times of beginning and end of the General and Central Eclipses.*

Add together the squares of p' and q', and taking the square root of the sum, call it n. Taking for p and q the values which they have at the time T, add together the products of p by p', and q by q'. Multiply the sum by 60, and divide the product by the square of n. The quotient will be an interval in minutes of time, which, being subtracted from T, will give the *time of the middle of the general eclipse.* Call this time T′.

Find, by art. 5, of last problem, the values of p and q, at the time T′, and call these values a and b, respectively. Add together the squares of a and b, and call the square root of the sum m', marking it with the same sign as b. Taking k, equal to $(R + 2.73)$, from the square of $(10 + k)$, subtract the square of m', and call the square root of the remainder h. Then, as $n : h :: 1$ hour : an interval of time; which, being subtracted from T′, and added to it, will give the times of *beginning* and *end* of the general eclipse. When the square of m' exceeds the square of $(10 + k)$, there will not be an eclipse any where on the earth.

Subtract the square of m' from 100, and call the square root of the remainder h'. Then, as $n : h' :: 1$ hour : an interval of time, which, being subtracted from the time T′, will give the times at which the eclipse commences and ceases to be central somewhere on the earth. When the square of m' exceeds 100, the eclipse cannot be central.

The quantities a, b, and m', are necessarily constant, and the quantity n may be considered constant during the eclipse. These quantities will be used in other parts of the problem.

We have from the calculation of the last problem,

d. h. m.
T = 15 2 0

$p = -0.59$ $p' = +5.08$ $k = 5.65$
$q = +4.69$ $q' = +0.51$

p' sq 25.81 $pp' = -3.00$
q' sq 0.26 $qq' = +2.39$
$n = 5.11$ sq 26.07 -0.61

$$t = \frac{-0.61 \times 60}{26.07} = -1.4 \text{ m.}$$

h. m.
T = 2 0
— 1.4
T′ = 2 1.4, time of middle of general eclipse.

The values of p and q, found for the time T′, give,

$a = -0.47$ $b = +4.70$

a	*sq* 0.22		(10 + *k*) = 15.65	*sq* 243.67
b	*sq* 22.09		*m′*	*sq* 22.31
m′ = + 4.72	*sq* 22.31		*h* = 14.88	*sq* 221.36

h. h. m.
5.11 : 14.88 : : 1 : 2 54.7

d. h. m. h. m. d. h. m.
15 2 0 — 2 54.7 = 14 23 6.7, time of beg. of general eclipse.
15 2 0 + 2 54.7 = 15 4 56.1, time of end of general eclipse.

	100.00		h. h. m.
m′	*sq* 22.31		5.11 : 8.81 : : 1 : 1 43.4
h′ = 8.81	*sq* 77.69		

h. m. h. m. h. m.
2 1.4 — 1 43.4 = 0 18.0, beginning of central eclipse.
2 1.4 + 1 43.4 = 2 44.8, end of central eclipse.

2. *To determine certain arcs and logarithms, wanted in subsequent parts of the problem.*

From the sun's longitude at the time T′, increased by 360°, if necessary, subtract 90°, and call the remainder L″. Then will L″ or L″ ± 180°, be the longitude of the zenith, for all those places that have the sun in the horizon at that time. To the constant logarithm 0.03746, add the tangent of L″, and the sum will be the tangent of an arc A, which must be taken in the same quadrant as L″. To the constant logarithm 9.63743, add the sine of L″, and the sum, rejecting 10 from the index, will be the tangent of an arc B, less than 90°, which must be marked affirmative or negative, according as the tangent is affirmative or negative. From the tangent of B, with the index increased by 10, subtract the tangent of A, and call the remainder log. *c*. From the constant logarithm 19.96544, subtract the cosine of B, and call the remainder log. *d*. According as A is *less* or *greater* than 180°, *add* to it, or *subtract* from it, 180°, and call the result A′. Call the arc B, taken with a contrary sign B′; and log. *c*, with a contrary sign log. *c′*. The arcs A′ and B′, the log. *c′*, and the log. *d*, which does not change its sign, correspond to the longitude L″ ± 180°, of the zenith of a place in the horizon. All these quantities may, without material error, be regarded as constant during the eclipse. If, however, greater accuracy is required, the values of A, B, and log. *c*, may be again computed, for a time an hour later or earlier than T′. Their small hourly changes of value will thus become known; and thence their values, and consequently those of A′, B′, and log. *c′*, may, for any time during the eclipse, be immediately obtained. The log. *d* may, in all cases, be considered constant during the eclipse.

To the logarithm of q' add the arithmetical complement of the logarithm of p', and the sum will be the tangent of an arc N, less than 90°, which must be marked affirmative or negative, according as the tangent is affirmative or negative. The arc N may be considered constant during the eclipse.

Add together the constant logarithm 1.77815, the logarithm of b, and the arithmetical complement of the logarithm of p', rejecting 10 from the index, when it amounts to 10, and call the result, marked negative when b is negative, log. G. This logarithm may be considered constant during the eclipse.

Let Z be the right ascension of the zenith of Greenwich. To the sun's mean longitude from the true equinox, at the time* T′, add the time T′, expressed in degrees, and the sum will be the value of Z, at that time. To find the value of Z at any other time differing from T′, by an interval t, take from table LXXI* the arcs corresponding to the hours, minutes, and seconds, in the interval t, and add their sum to the value of Z at the time T′, or subtract it from this value, according as the time is later or earlier than the time T′, and the result will be the required value of Z.

	h.	m.
At the time T′ =	2	1.4

Sun's longitude,	54°	42′
Subtract,	90	0
L″	324	42

		0.03746			9.63743
L″	tan.	9.85006 n	L″	sin.	9.76182 n
A = 322° 21′	tan.	9.88752 n	B = — 14° 5′	tan.	9.39925 n
			A	tan.	9.88752 n
		19.96544			
B	cos.	9.98675		log. c	9.51173
	log. d	9.97869			

$$A' = 142° 21' \;;\; B' = + 14° 5' \;;\; \log. c = 9.51173\, n$$

$q' = + 0.51$	log.	9.70757			1.77815
$p' = + 5.08$	Ar. Co. "	9.29414	$b = + 4.70$	log.	0.67210
			p'	Ar. Co "	9.29414
N = 5° 44′	tan.	9.00171			
				log. G.	1.74439

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* When the time T′ differs but a few minutes from the time T, the sun's mean longitude from the true equinox at the latter time, as found in the last problem, may be used.

Sun's mean longitude from true equinox at time T′ =	53°	19′
Time T′ in degrees, - - - - - - -	30	21
Z, at time T′ - - - - - - - -	83	40

3. *The latitude of the zenith of a place which has a given phenomenon of the eclipse in its horizon, being given, for a given time, at which the phenomenon occurs, to find the latitude and longitude of the place.*

Let λ'' represent the latitude of the zenith of the place. When the given time is *before* the time T′, to the value of λ'', add the arc B, and to the tangent of the sum[*] add log. *c*. The result will be the tangent of an arc F, which when $(\lambda'' + B)$ is less than 90°, must be taken less than 90°, and be marked with the same sign as the tangent; but when $(\lambda'' + B)$ is greater than 90°, the arc F must be taken greater than 90°, and must be marked with a *contrary* sign to that of the tangent. Attending to the signs, subtract F from A, and the remainder will be the right ascension of the zenith of the place, at the given time. Call this right ascension *z*. Add together the tangent of $(\lambda'' + B)$, the cosine of F, and log. *d*, and the sum, rejecting 20 from the index, will be the tangent of the geographic latitude of the place, which must be taken less than 90, and will be *north* or *south* according as the tangent is *affirmative* or *negative*. From the value of Z, found for the given time, and increased by 360°, if necessary, subtract *z*, and the remainder will be the longitude of the place, to the *west* of Greenwich. When the remainder exceeds 180°, its supplement to 360°, will be the longitude to the *east*.

When the given time is after the time T′, the proceeding is the same, except that A′, B′, and log. *c*′, are to be used instead of A, B, and log. *c*.

It sometimes occurs, especially near the time T′, that the value of λ'', when found as directed in subsequent parts of the problem, comes out greater than 90°. When this is the case, the numerical supplement[†] is to be regarded as the proper value of λ'', and then in finding the latitude and longitude of the place, the quantities A, B, &c., are to be used in a *contrary* manner; that is, A′, B′, and log. *c*′, are to be used if the time is before T′, but A, B, and log. *c*, if it is after T′.

Note. The determination of the right ascension and declination of the zenith from its longitude and latitude, as directed above, is according to the method given for this purpose by Professor Encke in the *Jahrbuch* for 1831. Putting ω = the obliquity of the ecliptic, and e = the earth's eccentricity, the constant logarithms used in finding A, B, &c., in the preceding article, were the arithmetical complement of cosine of ω, the tangent of ω, and $(\cos \omega + \log. \frac{1}{1 - e^2} + 10)$; the value of ω being assumed 23° 27.5′. By the introduction of $\log. \frac{1}{1 - e^2}$, in the constant logarithm used in obtaining log. d, we obtain at once the geographic instead of the geocentric latitude.

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[*] If the sum is *negative*, the tangent must have a *contrary* sign to that which it would have if the sum was affirmative. It may be observed that the sign of the sine or tangent of a *negative* arc is always *contrary* to that of the same arc, when affirmative, but the sign of the cosine of an arc is the same, whether the arc is affirmative or negative.

[†] By the *numerical supplement* of an arc is meant the result that is obtained by subtracting the arc, taken without regard to its sign, from 180°, and attaching to the remainder the sign of the arc.

Examples illustrating this article will occur in the next.

4. ***To find the places at which the general eclipse begins and ends, and also those at which it begins and ceases to be central.***

Let t be the interval expressed in minutes, between the time T, and the time of beginning or end of the eclipse. To log. G. add the arithmetical complement of the logarithm of t, and the sum will be the tangent of an arc G, less than 90°, which must be marked affirmative or negative, according as the tangent is affirmative or negative. From G, subtract the arc N, and the remainder will be the value of λ'' at the time of beginning. And adding N to G, the sum will be the value of λ'' at the time of end. With these values of λ'' find the latitudes and longitudes of the places, as directed in the last article.

By art. 1, t = 2 h. 54.7 m. = 174.7 minutes.

	log. G.	1.74439
t	Ar. Co. log.	7.75771
G = + 17° 38′	tan.	9.50210
N = + 5 44		
λ'' = + 11 54, at beginning.		
λ'' = + 23 22, at end.		

At beginning.

$\lambda'' + B$ = −2° 11′	tan.	8.58121 n	$\lambda'' + B$	tan.	8.58121 n
	log. c	9.51173		log. d	9.97869
			F	cos.	9.99996
F = − 0° 44′	tan.	8.09294 n			
A = 322 21			Lat. 2° 5′ S.	tan.	8.55986 n
z = 323 5					

At end.

$\lambda'' + B' = 37° 27'$	tan.	9.88420	$\lambda'' + B'$	tan.	9.88420
	log. c	9.51173 n		log. d	9.97869
			F	cos.	9.98697
F = — 13° 58'	tan.	9.39593 n			
A' = 142 21			Lat. 35° 17' N.	tan.	9.84986
z = 156 19					

Z, at time T' - - - - - -	83°	40'
Change of Z, during interval t, tab. LXXI*	43	48
Z, at time of beginning, - - -	39	52
Z, at time of end, - - - - -	127	28

Z - - -	39°	52'	Z - - -	127°	28'
z - - - -	323	5	z - -	156	19
Long. place of beg.	76	47 W.		331	9
				360	0
			Long. place of end	28	51 E.

Taking t = 1 h. 43.4 m., the interval between the time T', and the time at which the eclipse begins or ceases to be central, it will be found in like manner that the eclipse

Begins to be central in Long. 98° 9' W., Lat. 8° 1' N.
Ceases to be central in Long. 52 38 E., Lat. 44 53 N.

5. ***To find the times and places, at which the beginning, greatest obscuration and end of the eclipse, will have place in the horizon, at the same instant; that is, when there will only be a simple contact, and that in the horizon.***

Taking k with the same sign as m', add it to m', and subtract it from m', and call the sum and remainder s and r, respectively. If s and r are each numerically less than 10, that is, less than 10 without regard to the sign, they may both be used, and there will then be four times and places, at which the phenomenon will occur. If either is numerically greater than 10, it must be rejected, and there will be only two times and places.

Taking either of the quantities s and r, that is numerically less than 10, subtract its square from 100, and call the square root of the remainder h. Then, as $n : h :: 1$ hour : an interval t. Subtract t from T', and add it to T', and the results will be the times at which the phenomenon occurs.

Subtract h from 10, and add the logarithm of the remainder to the constant logarithm 18.69897, or when the remainder is less than a unit, to the constant logarithm 8.69897. Divide the result by 2, and the quotient will be the sine of an arc ½ G, which must be marked with the same sign as the quantity *s* or *r*, that has been used. Subtract N from G, and the remainder will be the value of λ'' at the time T — *t;* and add N to G, and the sum will be its value at the time T + *t*. With the values of λ'', proceed as directed in art. 3, to find the latitudes and longitudes of the corresponding places.

If the other quantity *s* or *r*, is also numerically less than 10, a similar proceeding will give two other times and places.

Note. When there can be only two places found, these places will be *northern* limits if *m'* is *negative*, but *southern* ones, if *m'* is affirmative. When four places are obtained the distinction of the northern from the southern will be obvious.

$m' = +\ 4.72$			100.00
$k = +\ 5.65$	r - -	sq	0.86
$s = +\ 10.37$	$h = 9.96$	sq	99.14
$r = -\ 0.93$			

h. h. m.		h.	m.
5.11 : 9.96 :: 1 : t = 1 56·9	T' =	2	1.4
	t =	1	56.9
		0	4.5, earliest time.
		3	58.3, latest do.

	10.00		
h =	9.96		8.69897
remainder	0·04	log.	8.60206
			2)17.30103
½ G = —	2° 34'	sin.	8.65051

G = — 5 8
N = + 5 44

λ'' = — 10 52, at time T' — *t*

λ'' = — 0 36, " T' + *t*

At time T' — *t*.

λ'' + B = — 24° 57'	tan.	9.66768 *n*	λ'' + B	tan.	9.66768 *n*
	log. *c*	9.51173		log. *d*	9.97869
			F	cos.	9.99497
F = — 8° 42'	*tan.*	9.17941 *n*			
A = 322 21			Lat. 23° 39' S.	tan.	9.64134 *n*
z = 331 3					

At time, $T' + t$.

$\lambda'' + B' = + 14° 41'$	tan. 9.41836	$\lambda'' + B'$	tan. 9.41836
	log. c' 9.51173 n		log. d 9.97869
	———	F	cos. 9.99843
$F = - 4° 52'$	tan. 8.93009 n		———
$A' = 142\ 21$		Lat. 13° 58′	tan. 9.39548
———			
$z = 147\ 13$			

Z, at time T' - - - - - -	83° 40′
Change of Z, in interval t - -	29 18
	———
Z, at time $T' - t$ - - - - -	54 22
	———
Z, at time $T' + t$ - - - -	112 58

Z, at time $T' - t$,	54° 22′	Z, at time $T' + t$	112° 58
z „ „	331 8	z, " "	147 13
	———		———
Long. 1st place	83 19 W.		325 45
			———
		Long. 2d place	34 15 E.

6. *To find the places which, at a given time, will have the beginning or end of the eclipse, in the horizon.*

Let t be the interval in minutes, between the time T' and the given time, and find the arc G, as in art. 4. As 60 minutes : t : : n : a fourth term. To the square of m', add the square of this fourth term, and call the square root of the sum, m. Take the difference between m and 10, and add it to k and subtract it from k. Then add together the logarithms of the sum and remainder, and the arithmetical complement of the logarithm of 40 m, increasing the index of the result by 10, if this is necessary to make it near to 20, and divide by 2. The quotient will be the sine of an affirmative arc $\frac{1}{2}$ K, less than 90°. When the given time is earlier than T', add the arc K to (G — N), and subtract it from (G — N), and the sum and remainder will both be values of λ''; and thus there will be two places that will have the phenomenon at the same time. But when the given time is later than T, add the arc K, to (G + N) and subtract it from (G + N), for the values of λ''.

Note 1. When the last article gives only two times for the phenomenon there proposed, the contact of the limbs of the sun and moon at the two places, found for a time prior to the earlier of these times, will be the beginning of the eclipse at the places; and for a time subsequent to the later time, it will be the end. For any intermediate time, between the times there found, the contact at one place will be the beginning and at the other the end.

When that article gives four times, the same observations apply to the earliest and latest of these times; observing that during the interval between the later of the two earliest times and the earlier of the two latest times, the phenomenon of beginning or end in the horizon cannot have place.

2. Generally the contact in the horizon takes place at sunrise for a time prior to the time T', and at sunset, for a time subsequent to the time T'. This does not, however, necessarily hold true for a time near to the time T'. When there is a doubt whether the contact takes place at sunrise or sunset, it may be removed by converting the given Greenwich mean time into apparent time at the place found. If this time is more than 12 hours, the contact takes place at sunrise; if it is less than 12 hours, the contact takes place at sunset. It may be observed that it is not necessary to change the mean time at the place into apparent, except the resulting mean time is within a few minutes of 12 hours.

Exam. Required the places that will have a contact of the limbs in the horizon, at 0 h. 30 m.

h. m. — m. m.

$T' = 2$ 1.4 — 60 : 91.4 :: 5.11 : fourth term = 7.78

Given time = 0 30.0

$t = 1$ 31.4 = 91.4 m.

log. G. 1.74439 — m' sq 22.31

$t = 91.4$ Ar. Co. log. 8.03905 — 4th term sq 60.53

$G = +31°\ 16'$ tan. 9.78344 — $m = 9.10$ — sq 82.84

$N = +\ 5\ \ 44$ — 10.00

$(G-N) = +25\ \ 32$ — Diff. 0.90

$K = +34\ \ 0$ — $k = 5.65$

$\lambda'' = +59\ \ 32$ — Sum. 6.55 log. 0.81624

Rem. 4.75 " 0.67669

also, $\lambda'' = -\ 8\ \ 28$ — 40 m. = 364 Ar. Co. " 7.43890

2)18.93183

$\frac{1}{2}K = 17°\ 0'$ sin. 9.46591

$K = 34\ \ 0$

With the above values of λ'', the longitudes and latitudes of the two places will be found to be,

1st place, Long. 116° 40′ W.; Lat. 42° 34′ N.
2d place, Long. 90 43 W.; Lat. 21 24 S.

Using the same value of K, and taking (G + N) instead of (G — N) we may find two other places, which at 1 h. 31.4 m. after the time T′, that is at 3 h. 32.8 m., will have a contact of the limbs in the horizon.

7. *To find the earliest and latest times, and the places at which the greatest phase will occur in the horizon.*

When m' is numerically equal to, or greater than, k, the times and places found by art. 5, will be the times and places required.

When m' is less than k, find an interval t, from the proportion, $n : 10 :: 1$ hour $: t$. Subtract t from T′ and add it to T′, and the remainder and sum will be the times required.

The arc N with its sign changed will be the value of λ'' at the earliest time, and without change of sign it will be the value of λ'' at the latest time. The values of λ'' being known, the latitudes and longitudes of the places may be found as before. The calculation need not however be repeated for the second time, as the value of F, will evidently be the same at each time, and the latitude of the second place will only differ from the first in having a contrary name.

In the present eclipse m' is less than k.

h. h. m. h. m.
5.11 : 10 :: 1 : t = 1 57.4 T′ = 2 1.4
t = 1 57.4
0 4.0, earliest time
3 58.8, latest do.

At earliest time, $\lambda'' = -5°\ 44'$
" latest " $\lambda'' = +5\ \ 44$

With these values of λ'', we find,

Long. 1st place 85° 12′ W.; Lat. 18° 49′ S.
Long. 2d " 35 56 E.; Lat. 18 49 N.

8. *To find, for a given time, the place or places that will have the greatest phase in the horizon.*

Let t be the interval between the time T′ and the given time. Then, as 1 hour $: t :: n :$ a fourth term. Subtract this fourth term from 10, and add the logarithm of the remainder to the constant logarithm 8.69897, increasing the index by 10, except when the remainder is less than a unit, and divide by 2. The quotient will be the sine of an arc ½ G. When m' is numerically equal to, or greater than, k, the arc ½ G must be taken with the same sign as m'. When m' is less than k, the arc ½ G may be taken either affirmative or negative for any time between the earliest time found by the last article, and the earliest found by art. 5, or between the latest times found by these two articles, and two places may be found that have the phenomenon at the same time; for any other time the arc $\frac{1}{2}$ G must have the same sign as m'; observing that between the later of the two earliest times found by art. 5, and the earlier of the two latest times, the phenomenon of greatest phase in the horizon cannot have place.

When the given time is earlier than T′, subtract N from G, for the value of λ''; but when it is later, add N to G.

Exam. Let it be required to find the places that will have the greatest phase in the horizon at 1 h. 30 m., and at 2 h. 32.8 m.; that is, at 31.4 m. before and after the time T′.

Here $t = 0° \; 31.4'$

		10.00
		fourth term 2.67
h. h. m.		
1 : 0 31.4 :: 5.11 : fourth term = 2.67		Remainder = 7.33

		8.69897
Rem. = 7.33	log.	0.86510
		2)19.56407
$\frac{1}{2}$G = + 37° 15′	sin.	9.78203
G = + 74 30		
N = 5 44		
λ'' = 68 46, at earlier time,		
λ'' = 80 14, " later "		

With these values of λ'', we find,

Long. of place at earlier time, 138° 5′ W.; Lat. 50° 42′ N.
" " " later " 153 45 E.; Lat. 70 30 N.

9. *To find the place in the northern or southern limit, which, at a given time, will have only a simple contact.*

Find the values of k sin. N and k cos. N. Then using the upper signs for a northern limit, but the lower ones for a southern limit, put $a' = a \mp k$ sin. N, and $b' = b \mp k$ cos. N. These values of a' and b', may be considered constant during the eclipse.

Let t, expressed in hours and decimals of an hour, be the interval between the time T′ and the given time, and find the products tp', tq' and tn. To the value of s or r, found by art. 5, add the square of tn, and subtract the sum from 100. The square root of the remainder will be a quantity g, which will have the same value for equal times before and after the time T′. When the given time is earlier than T′, attending to the signs, subtract the products tp' and tq' from a' and b', respectively, and call the remainders a'' and b''. When the given time is later than T′, add the products tp' and tq', to a' and b', for the values of a'' and b''.

To the logarithm of a'', add the arithmetical complement of the logarithm of g, and the sum will be the tangent of an arc D, less than 90°, which must have the same sign as a''. Attending to its sign, add the arc D to the sun's longitude and it will give L″, the longitude of the zenith of the required place. Add together the cosine of D, the logarithm of b'', and the arithmetical complement of the logarithm of g, and the sum will be the tangent of λ'', with the same sign as b''. With the longitude and latitude of the zenith, its right ascension and its declination, or the latitude of the place, may be found by prob. XVIII; observing that to have the geographic latitude, the constant logarithm 0.00290, should be added with the tangent of (B + E) and sine of the right ascension.

Note. A place in the annular or total limits may be found in the same manner, only taking k equal to the difference between 2.73 and R, instead of their sum.

Required the place that will have only a simple contact at 3 h. 0 m. in the southern limit, which, as appears by art. 4, is, in the present eclipse, the only limit of simple contact.

$k = 5.65$	log. 0.75205	k - -	log. 0.75205
$N = +5°\ 44'$	sin. 8.99956	N - -	cos. 9.99782
$k \sin N = +0.56$	9.75161	$k \cos N = +5.62$	0.74987
$a = -0.47$		$b = +4.70$	
$a' = +0.09$		$b' = -0.92$	

	h.	m.
Given time,	3	0
T′ =	2	1.4
t =	0	58.6 = 0.977 h.

$tp' = +4.96$; $tq' = +0.50$; $tn = +4.99$

$r =$	0.85	$a' = +0.09$	$b' = -0.92$
tn	sq. 24.90	$tp' = +4.96$	$tq' = +0.50$
	25.75	$a'' = +5.05$	$b'' = -0.42$
	100.00		
$g = 8.62$	sq. 74.25		

$a'' = +5.05$ log. 0.70329
$g = 8.62$ Ar. Co. " 9.06449

$D = +30° 22'$ tan. 9.76778
$L' = 54 42$

$L'' = 85 4$

$b'' = -0.42$ log. 9.62325 n
g Ar. Co. " 9.06449
D - - - cos. 9.93591

λ'' - - tan. 8.62365 n
L'' - Ar. Co. sin. 0.00161

$B = 177° 35'$ 8.62526 n
$E = 23 28$

$B + E = 201 3$

$B + E = 201° 3'$ cos. 9.97001 n
$L'' = 85 4$ tan. 11.06391
$B = 177 35$ Ar. Co. cos. 0.00039 n

Right asc. zen. = 84° 43' tan. 11.03431

$B + E$ tan. 9.58531
Right asc. zen. sin. 9.99815
0.00290

Lat. 21° 6' N. tan. 9.58636

h. h. m.
1 : 0 58.6 :: 15° 2.5' : 14° 41'

Z - - - - - - -	at time T'	83°	40'
Change of Z - - - - - -		14	41
Z - - - - - -	at given time	98	21
z - - - - - - -	" "	84	43
Long. of required place		13	38 W.

9. *To find the place that will have the sun centrally eclipsed, at a given time.*

Taking t equal to the interval between the given time and the time T', find, as in the last article, the products tp', tq' and tn. When the given time is earlier than T', subtract tp' and tq' from a and b, respectively, and call the remainders a'' and b''. When the given time is later than T', add tp' and tq' to a and b, for the values of a'' and b''. To the square of m', add the square of tn, and subtracting the sum from 100, call the square root of the remainder g. The value of g will be the same for equal times before and after the time T'. With the values of a'', b'', and g, proceed as in the last article, to find the latitude and longitude of the place.

Required the place that will have the sun centrally eclipsed at 1 h. 10 m. after the time T'; that is, at 3 h. 11.4 m.

$$tp' = +5.93 \; ; \; tq' = +0.59 \; ; \; tn = 5.96$$

$a = -0.47$	$b = +4.70$	m'	sq. 22.31
$tp' = +5.93$	$tq' = +0.59$	tn	sq. 35.52
$a'' = +5.46$	$b'' = +5.29$		57.83
			100.00
		$g = 6.49$	sq. 42.17

$a'' = +5.46$	log.	0.73719	$b'' = +5.29$	log.	0.72346
$g =$ 6.49	Ar. Co. "	9.18776	g	Ar. Co. "	9.18776
			D	cos.	9.88383
D $= +40°$ 4′	tan.	9.92495			
L′ $= +54$ 42			λ''	tan.	9.79505
			L″	Ar. Co. sin.	0.00150
L″ = 94 46					
			B = 32° 3′		9.79655
			E = 23 28		

B + E = 55 31

B + E = 55° 31′	cos.	9.75294	B + E	tan.	10.16314
L″ = 94 46	tan.	11.07890 *n*	R. asc. zen.	sin.	9.99664
B = 32 3	Ar. Co. cos.	0.07182			0.00290
R. asc. zen. = 97° 7′	tan.	10.90366 *n*	Lat. 55° 29′ N.	tan.	10.16268

h. h. m. h. m.
1 : 1 10 :: 15 2.5 : 17° 33′

Z	at time T′	83°	40′
Change of Z		17	33
Z	at given time	101	13
z	" "	97	7
Long. of required place,		4	6 W.

SCHOLIUM. By calculating for various times during the eclipse, the places that will have given phenomena, denoting these places on a map, and drawing lines through the corresponding ones, a representation of the limits &c. of the eclipse will be formed, as in *Fig.* 62, which exhibits them for the present eclipse.

The results that have been obtained are to be regarded as only approximative. If, in any case, greater accuracy is desired, it may be obtained by corrections and recalculations mentioned in the appendix to Part I., from the formula in which the rules in this problem and the preceding one have been deduced.

PROBLEM XXXVI.

The values of p or q, at three consecutive whole hours being given, to find their values at the intermediate half hours.

Let the times at the whole hours be $t - 1$, t, and $t + 1$. Attending to the signs, subtract the value of the quantity at the time $t - 1$, from its value at the time t, and its value at the time t, from its value at the time $t + 1$, and the remainders will be the *first differences*. Subtract the first of these from the second, and the remainder will be the *second difference*. Attending to the signs, add $\frac{1}{8}$ of the second difference to the value of the quantity at the time t; and from the sum, and to it, subtract and add $\frac{1}{4}$ of the sum of the first differences, and the results will be the values of the quantity at the times $t - \frac{1}{2}$ and $t + \frac{1}{2}$, respectively.

Exam. Given the values of p on any day, at 3 h. + 4.4913; at 4 h. + 9.5759; and at 5 h. + 14.6596; to find its values at 3 h. 30 m. and 4 h. 30 m.

h.	p.	1st diff.	8
3	+ 4.4913		
		+ 5.0846	
4	+ 9.5759		− 0.0009
		+ 5.0837	
5	+ 14.6596		
		4) + 10.1683	− 0.00011
		+ 2.54207	

+ 9.5759
− 0.00011

+ 9.57579
+ 2.54207

+ 7.0337, value of p at 3 h. 30 m.

+ 12.1179, " " 4 h. 30 m.

PROBLEM XXXVII.

To calculate with precision an Eclipse of the Sun for a given place, using the places, &c. of the moon and sun, as obtained from the Nautical Almanac, or other accurate Ephemeris.

1. *To find the values of certain quantities that are general and serve for any place.*

Let L and L′ represent the longitudes of the moon and sun, respectively, λ and λ', their latitudes, π and π', their equatorial horizontal parallaxes, d' the sun's semidiameter, and ϵ the apparent obliquity of the ecliptic.

Also let T be the Greenwich* mean time of new moon, taken to the nearest whole hour; a time an hour earlier being represented by T — 1, an hour later, by T + 1, &c.

For the times T — 3, T — 2, T — 1, T, T + 1, T + 2, and T + 3, find the values of L, L′, λ, λ', ϖ, and d'. And for the time T, find the values of ϖ' and s. These two values may be considered constant during the eclipse. Put,

$$p = \frac{10 \sin (L - L') \cos \lambda}{\sin (\varpi - \varpi')}$$

$$q = \frac{10 \sin (\lambda - \lambda')}{\sin (\varpi - \varpi')}$$

$$R = \frac{10 \cos (L - L') \cos (\lambda - \lambda') \tan d'}{\sin (\varpi - \varpi')}$$

Find the values of p, q, and R, for each of the hours mentioned above, extending them to four decimal figures, and by the last problem find the values of p and q, for the intermediate half hours. The value of R, at each half hour, may be taken equal to the half sum of its values at the preceding and following whole hours. Arrange the values found in a table, placing in columns respectively adjacent to those containing the values of p and q, the differences of their values for each half hour. Then the values of p' and q', for any whole hour, will be equal to the sum of the preceding and following differences. The values of p' and q' may be placed in columns adjacent to the columns of differences. From the values of p, q, &c. for the times contained in the table, their values for any intermediate time may be readily obtained by proportion.

Add to, or subtract from, the sidereal time at mean noon at Greenwich, taken from the Nautical Almanac and converted into degrees, the product of 15° 2′ 27.8″ by the number of hours the time T is after or before noon, and the result will be the Greenwich sidereal time, or, which is the same, the right ascension of the zenith of Greenwich, at the time T.

Taking the value of L′ at the time T, add together tan. $\frac{1}{2}$ s and cos. L′, rejecting ten from the index of the sum, and call the result log. C. Take the tangent of the same value of L′, and call it log. C″.

With the moon's equatorial horizontal parallax at the time T, take the value of r, from table LXXV.

2. Calculation for a given place.

Find by prob. XXXIII, the logarithms A, B, and G, to five decimal places, and the quantity f, to four decimal places. Taking the value of L′, at the time T, add cos. L′ to log. A, and call the natural number corresponding to the sum,* a. To the logarithm of a, add log. C$''$, and call the natural number corresponding to the sum, a''. And add together the logarithm of a'', the logarithm of the sun's hourly motion, in seconds, and the constant logarithm 4.68557, and call the natural number corresponding to the sum, a'. The values of a'' and a' may be regarded constant during the eclipse; but the value of a is only accurately true for the time T. To find its value for any other time, differing from T, by an interval t, say as 1 hour : t :: a' : a fourth term, with the same sign as a'. If the given time is after the time T, add this fourth term to the value of a, at the time T, but if it is earlier, subtract, and the result will be the required value.

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* The places, &c., of the sun and moon are here supposed to be taken from the Nautical Almanac, and consequently the time used is Greenwich time. When they are taken from an Ephemeris calculated for a different meridian, it will be more convenient to use the time at that meridian.

Let Z represent the right ascension of the zenith of the given place. To the right ascension of the zenith of Greenwich at the time T, or from it, *add* or *subtract* the longitude of the given place, according as it is *east* or *west*, and the sum or remainder will be the value of Z, at the time T. Its value at any other time during the eclipse may readily be found by table LXXI*.

Using logarithms to three decimal places, and natural numbers to two, proceed as in prob. XXXIV, to find near approximate values of the times of greatest approximation, &c. With these times repeat the calculation, using logarithms to five decimal places, and natural numbers to four, except for the values of a'', b', c', and g', which need not be extended beyond three. Observing, also, that in order to obtain the correction of R more accurately, we must add log. C$''$ to the logarithm of c, and taking the natural number corresponding to the sum, and calling it c'', we must take $E = a'' + b' + c''$.

Note. The general values found by the first part of the rule, serve, not only for calculating the times of beginning, &c., of an eclipse for any place at which it is visible, but also for calculations founded on the observed times of beginning or end, at any place, to determine its longitude. For simply determining the times, &c., for a single place, the calculation might be considerably shortened. It may further be observed, that if instead of finding near approximate values of the times of beginning and end, as in prob. XXXIV, the first approximate values are used, the difference in the results will seldom exceed a few tenths of a second.

Exam. Let it be required to calculate for Philadelphia, with data taken from the Nautical Almanac, the eclipse calculated in prob. XXXIV.

1. *For quantities independent of the place.*

By the Nautical Almanac, the time of new moon is May 15th, at 2 h. 6.9 m. Taking therefore T = 15 d. 2 h., we find the values of L, L$'$ &c. for the whole hours from 14 d. 23 h. to 15 d. 5 h. inclusive.

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* It is to be understood here and in other cases, except in that of the tangent of an arc, that ten is to be rejected from the index of the sum, whenever it is equal to, or exceeds, ten.

d.	h.	L	L′	λ	λ′	ϖ	d′
14	23	53° 8′ 54.63″	54°34′ 58.09″	+17′ 4.36″	+0.37″	54′ 26.30″	15 49.91
15	0	53 38 56.60	54 37 22.60	19 50.80	0.37	54 25.60	15 49.90
"	1	54 8 57.83	54 39 47.11	22 37.09	0.38	54 24.90	15 49.89
"	2	54 38 58.32	54 42 11.62	25 23.21	0.38	54 24.20	15 49.88
"	3	55 8 58.07	54 44 36.12	28 9.14	0.39	54 23.52	15 49.88
"	4	55 38 57.09	54 47 0.62	30 54.86	0.39	54 22.84	15 49.87
"	5	56 8 55.39	54 49 25.13	33 40.36	0.40	54 22.17	15 49.86

For time T, we find $\varpi' = 8.48''$
$\varepsilon = 23^\circ\ 27'\ 44''$

At time T — 3, or 14 d. 23 h.

$\varpi - \varpi' =$ 0° 54′ 17.82″, 1 + Ar. Co. sin. 2.8015161
$L - L' = -1$ 26 3.46 - - - sin. 8.3984703 *n*
$\lambda = +$ 17 4.36 - - cos. 9.9999947

$p = -15.8482$ - - - - - 1.1999811 *n*

$\varpi - \varpi'$ - - - - 1 + Ar. Co. sin. 2.8015161
$\lambda - \lambda' = +\ 17'\ 3.99''$ - - - - . sin. 7.6958688

$q = +\ 3.1433$ - - - - - - 0.4973849

$\varpi - \varpi'$ - - - - 1 + Ar. Co. sin. 2.8015161
$L - L'$ - - - - - - - cos. 9.9998639
$\lambda - \lambda'$ - - - - - - - cos. 9.9999947
$d' = 15'\ 49.91''$ - - - - - tan. 7.6632603

$R = 2.9150$ - - - - - - 0.4646350

Making similar calculations for the other hours, applying the last problem to find the quantities for the half hours, and proceeding as directed in the rule, we obtain the following values.

h.	m.	p	Diff.	p'	q	Diff.	q'	R
23	0	−15.8482	+2.5421		+3.1433	+0.2559		2.9150
	30	−13.3061	2.5422		3.3992	0.2558		2.9156
0	0	−10.7639	2.5424	+5.085	3.6550	0.2557	+0.511	2.9161
	30	− 8.2215	2·5426		3.9107	0.2557		2.9165
1	0	− 5.6789	2.5426	5.085	4.1664	0.2556	+0.511	2.9169
	30	− 3.1363	2.5426		4.4220	0.2555		2.9173
2	0	− 0.5937	2.5425	5.085	4.6775	0.2554	0.511	2.9176
	30	+ 1.9488	2.5425		4.9329	0.2554		2.9179
3	0	+ 4.4913	2.5424	5.085	5.1883	0.2553	0.511	2.9181
	30	+ 7.0337	2.5422		5.4435	0.2551		2.9183
4	0	+ 9.5759	2.5420	5.084	5.6986	0.2550	0.510	2.9185
	30	+12.1179	2.5417		5.9536	0.2548		2.9186
5	0	+14.6596			6.2084			2.9186

	h.	m.	sec.
Sidereal time at mean noon at Greenwich, tab. LV.	3	32	57.98
" " " " " in arc,	53°	14′	29.7″
Add, - - - - - - - - - -	30	4	55.6
Right ascen. of zen. of Greenwich at 2 h.	83	19	25.3

At time T = 2 h.

$\frac{1}{2}s = 11^\circ\ 43'\ 52''$ tan. 9.31727 Log. C'' = 10.14999
L' = 54 42 12 cos. 9.76178 $r =$ 2.7323

Log. C. 9.07905

2. *Calculation for Philadelphia.*

By prob. XXXIII, we find,

log. A. = 0.40535 log. G. = 0.48521 *n*
log. B. = 0.88517 $f = -5.8591$

At the time T = 2 h. 0 m.

log. A. 0.40535
L' - - - - cos. 9.76178

$a = +1.4694$ 0.16713
Log. C. 10.14999

$a'' = +2.075$ 0.31712
Sun's hor. mot. 144.5'', log. 2.15987
4.68557 *n*

$a' = -0.00145$ 7.16256 *n*

The approximate calculation, with the data obtained above, would give the times of greatest obscuration, beginning, and end, very nearly the same as obtained in prob. XXXIV, with the data there used. We may therefore take these times for the subsequent part of the calculation.

At 1 h. 14.2 m. = near approx. time of greatest obscur.

$p = -4.4754$ Z = 356° 40' 32''
$q = +4.2874$ L' = 54 40 21

Log. B. 0.88517
Z — L' = 302° 0' 11'' sin. 9.92841 *n*

$b = -6.5100$ 0.81358 *n*

Log. B. 0.88517
Z — L' cos. 9.72425

$b' = +4.068$ 0.60942

Log. G. 0.48521 *n*
Z - - sin. 8.76335 *n*

$g = +0.1772$ 9.24856
log. C. 9.07905

$c = +0.0213$ 8.32761
log. C'' 10.14999

$c'' = 0.030$ 8.47760

Log. G. 0.48521 *n*
Z - - cos. 9.99927

$g' = -3.051$ 0.48448 *n*
log. C. 9.07905

$c' = -0.366$ 9.56353 *n*

$u = -5.0182$		$u' = +0.969$	
$v = +6.0362$		$v' = -0.801$	
$x = +0.5428$		$x' = +4.116$	
$y = -1.7488$		$y' = +1.312$	
$xx' = +2.2342$		x'	sq. 16.94
$yy' = -2.2944$		y'	sq. 1.72
$xx' + yy' = -0.0602$		$n = 4.32$	sq. 18.66

$$18.66 : -0.0602 :: \overset{\text{m.}}{60} : \overset{\text{sec.}}{-11.6}$$

	h.	m.	sec.
Approx. time of greatest obscur.	1	14	12
Correction, subtract		—	11.6
True time,	1	14	23.6

$$n' = \frac{-0.0602}{4.32} = -0.014$$

x	sq. 0.2946	$E' = a'' + b' + c'' = 6.17$	
y	sq. 3.0583	$R' = 2.9171$	
	3.3529	$k = 5.6209$	
n'	sq. 0.0002		
$m = 1.8310$	sq. 3.3527		

$$\frac{6\,(k - m)}{R'} = 7.87 = \text{digits eclipsed on south limb.}$$

At 0 h. 3.8 m. = near approx. time of beg.

$p = -10.4419$	$Z = 339°$	1′	39″
$q = +\ \ 3.6874$	$L' = \ \ 54$	37	32

	Log. B	0.88517		Log. B	0.88517
$Z - L' = 284°\ 24'\ 7''$	sin.	9.98614 n	$Z - L'$	cos.	9.39572
$b = -7.4355$		0.87131 n	$b' = +1.909$		0.28089
	Log. G	0.48521 n		Log. G	0.48521 n
Z - -	sin.	9.55379 n	Z - -	cos.	9.97023
$g = +1.0939$		0.03900	$g' = -2.854$		0.45544 n
	log. C	9.07905		log. C	9.07905
$c = +0.1312$		9.11805	$c' = -0.342$		9.53449 n
	log. C″	10.14999			
$c'' = +0.185$		9.26804			

$u = -5.8321$	$u' = +0.410$	$E = 4.17$
$v = +6.9530$	$v' = -0.749$	$R' = 2.8969$
$x = -4.6098$	$x' = +4.675$	$k = 5.6292$
$y = -3.2656$	$y' = +1.261$	

x	sq. 21.2503	k	sq. 31.6887
y	sq. 10.6640	m	sq. 3.3527
	31.9143	$h' = 5.3232$	sq. 28.3360
m	sq. 3.3517	$h = 5.3443$	
$h = 5.3443$	sq. 28.5616	Diff. = 0.0211	

x'	sq. 21.85		m. sec.
		4.841 : 0.0211 : :	60 : 15.7
y'	sq. 1.59		
$n = 4.841$	sq. 23.44		

	h.	m.	sec.
Approx. time of beg.	0	3	48
Correction, add			15.7
True time of beg.	0	4	3.7

At 2 h. 33.2 m. = near approx. time of end.

$p = +2.2285$	$Z = 16° 30' 17''$
$q = +4.9610$	$L' = 54 \ 43 \ 31$

	Log. B 0.88517		Log. B 0.88517
$Z - L' = 321° 46' 46''$	sin. 9.79148 n	$Z - L'$	cos. 9.89522
$b = -4.7496$	0.67665 n	$b' = +6.031$	0.78039

	Log. G 0.48521 n		Log. G 0.48521 n
Z - -	sin. 9.45346	Z - -	cos. 9.98173
$g = -0.8683$	9.93867 n	$g' = -2.930$	0.46694 n
	log. C 9.07905		log. C 9.07905
$c = -0.1042$	9.01772 n	$c' = -0.352$	9.54599 n
	log. C'' 10.14999		
$c'' = -0.147$	9.16771 n		

$u = -3.3852$	$u' = +1.487$	$E = 7.96$
$v = +4.9908$	$v' = -0.769$	$R' = 2.8812$
$x = +5.6137$	$x' = +3.598$	$k = +5.6135$
$y + -0.0298$	$y' = +1.280$	

x	sq.	31.5136
y	sq.	0.0009
		31.5145
m	sq.	3.3527
$h = 5.3068$	sq.	28.1618

x'	sq.	12.95
y'	sq.	1.64
$n = 3.82$	sq.	14.59

k	sq.	31.5114
m	sq.	3.3527
$h' = 5.3065$	sq.	28.1587
$h = 5.3068$		
Diff. $= 0.0003$		

m. sec.
3.82 : 0.0003 :: 60 : 0.3.

	h.	m.	sec.
Approx. time of end,	2	33	18
Correction, subtract,		0	0.3
True time of end,	2	33	17.7

Expressing the times in mean time at Philadelphia, we have,

	h.	m.	sec.
Beginning, - - -	7	3	23.7
Greatest obscur. - -	8	13	43.6
End, - - - -	9	32	37.7

Digits eclipsed 7.87.

SCHOLIUM. In the preceding calculation, no allowance has been made for irradiation and inflexion. If we assume a correction on that account, amounting to 5″ in the sum of the semidiameters of the sun and moon, it will give the time of beginning about 11 seconds later than found above, and the time of end about 14 seconds earlier.

PROBLEM XXXVIII.

The observed time of beginning or end, or both, of an Eclipse of the Sun, at a place whose latitude is known, being given, to find the longitude of the place.

1. *When only the time of beginning is observed.*

Assume for the place, a longitude as nearly correct as existing knowledge of its position will permit.

With the assumed longitude find the sidereal time of mean noon at the place, by the note to the rule in prob. XXII. Take the interval between mean noon and the mean time of the observed beginning, and convert it into sidereal time, by the table of time equivalents at the end of the Nautical Almanac. When the observed time is *before* mean noon, *subtract* this sidereal time from the sidereal time of mean noon, but when it is *after* noon, *add* it, and converting the remainder or sum into arc, it will be the value of Z, the right ascension of the zenith of the place at the observed time of beginning.

Using the assumed longitude, find the Greenwich mean time corresponding to the observed time of beginning, and for this time find the values of L', p, q, R, p' and q', as in the last problem. To the logarithm of q', add the arithmetical complement of the logarithm of p', and the sum will be the tangent of an arc N, less than 90°, which must be marked with the same sign as q'. To the logarithm of p' add the arithmetical complement of the cosine of N, and the natural number corresponding to the sum will be the value of n.

Find for the place of observation, by prob. XXXIII, the logarithms A, B, and G, and the quantity f. To log. A, add cos. L', and the natural number corresponding to the sum will be the value of a. Also, to log. A, add sin. L', and the natural number corresponding to the sum will be the value of a''. Then proceed as in the last problem to find the values of b, b' g, c, c'', u, v, x, y, and k. The values of g', c', &c. need not be found.

To the logarithm of x, add sin. N, and call the natural number corresponding to the sum z; and to the logarithm of y, add cos. N, and call the natural number corresponding to the sum, z'. Take the difference between z and z' and call it z''. Add together the logarithms of $(k + z'')$ and $(k - z'')$ and taking *half* their sum, it will be the logarithm of an affirmative quantity h. To the logarithm of x, add cos. N, and the sum will be the logarithm of a quantity h'; and to the logarithm of y add sin. N, and the sum will be the logarithm of a quantity h''. Take the sum of h, h' and h''. Then, as n : the sum, taken with a *contrary* sign : : 60 minutes : an interval of time t. Attending to its sign, add t to the Greenwich time of observed beginning, as obtained with the assumed longitude, and it will give the corrected Greenwich time of observed beginning. The difference between the corrected Greenwich time and the mean time at the place, will be the longitude in time; to the *west* when the observed time is the *earlier* of the two, but to the *east* when it is the *later*.

2. *When only the end is observed.*

The proceeding is the same as for the beginning, except that in finding t, instead of the sum of h, h', and h'', with a contrary sign, we must take the result obtained by subtracting the sum of h' and h'', from h, and without change of sign.

3. *When the beginning and end are both observed.*

Find the values of t, for both the beginning and end, and taking half their sum call it t'. Apply t' to the Greenwich mean time, either of the beginning or end, as obtained with the assumed longitude, and it will give the corrected Greenwich time; whence the longitude is obtained as before. The longitude thus obtained is generally more accurate than when found from a single observation of the beginning or end. Considering p' and q' as constant during the eclipse, and they vary but little, the value of t' may be more concisely found as follows:

Find N and n, with the values of p' and q' at the time of middle, and regard them as constant. Find the values of k at the times both of beginning and end. Add together the values of x at the times of beginning and end, and to the logarithm of the sum add cos N, and call the natural number corresponding to the result, h'. Do the same with the two values of y, and sin. N, calling the quantity obtained, h''. Add together h', h'', and the value of k at the time of beginning, and subtract the sum from the value of k, at the time of end. Then, as n : *half* the remainder : : 60 minutes : t'. With t' proceed as above.*

Note 1. If the longitude found, differs greatly from the assumed longitude, and it is desired to give precision to the calculation, the operation may be repeated, taking the longitude found for the assumed longitude. In this case a slight correction in the mean time of observation may be necessary, as in determining that time the longitude of the place is requisite. A second calculation is sufficient, even supposing the first assumed longitude to differ several hours from the truth.

2. The longitude of a place found as above, from an accurately observed eclipse of the sun, when the eclipse is large and the times of both beginning and end have been obtained, differs but little from the truth. It is nevertheless liable to an error of some seconds, depending in part on unavoidable errors of observation, but mainly on errors in the places, &c. of the sun and moon, as obtained from the tables. The differences in the longitudes of places not very remote from one another, as obtained from

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* It is easily perceived that the values of u and v for a given time at any place are but slightly dependent on the longitude of the place, or at least are but slightly affected by a moderate error in the longitude. Let T be the Greenwich time corresponding to the observed time of the beginning or end of an eclipse of the sun at any place, as obtained with the assumed longitude, and let $T + t$ be the true time. Also let the values of p, q, and k, be those which they have at the time T. Then the values of p and q at the time $T + t$, will be $p + p't$ and $q + q't$. Hence taking for u and v their values found for the observed time of beginning or end, we must have,

$$k^2 = (p + p't - u)^2 + (q + q't - v)^2.$$

Or putting $x = p - u$ and $y = q - v$, we have,

$$k^2 = (x + p't)^2 + (y + q't)^2$$
$$= x^2 + y^2 + 2p'xt + 2q'yt + n^2t^2.$$

$$\text{Consequently } \frac{k^2}{n} = \frac{x^2 + y^2}{n} + 2xt.\frac{p'}{n} + 2yt\,\frac{q'}{n} + nt^2$$

$$= \frac{x^2 + y^2}{n} + 2xt \cos. \mathrm{N} + 2yt \sin. \mathrm{N} + nt^2.$$

Thence we readily obtain,

$$t + \frac{x \cos. \mathrm{N} + y \sin. \mathrm{N}}{n} = \pm \sqrt{\frac{[k^2 + (x \cos. \mathrm{N} + y \sin. \mathrm{N})^2 - x^2 - y^2]}{n}}$$

$$= \pm \sqrt{\frac{[k^2 - (x \sin \mathrm{N} \backsim y \cos. \mathrm{N})^2]}{n}}$$

From this formula the above rule is easily deduced.

accurate observations of the same eclipse, are however almost entirely independent of this last source of error.

3. When a table of the values of p, p', &c. has not been previously computed, calculate the values of p and q, not only for the Greenwich time of observed beginning or end, but also for a time an hour later or earlier. Subtract the values of p and q at the earlier of the two times repectively from their values at the later, and the remainders will be the values of p' and q'. If both the beginning and end are observed, subtract the value of p at the beginning, from its value at the end. Then, as the duration of the eclipse : 1 hour :: the remainder : p'. In like manner q' may be found.

Exam. The observed beginning of the eclipse of May 15th, 1836, at Haverford school, latitude 40° 1′ 12″ N., and assumed longitude 5 h. 1 m. 25 sec. W., was at 7 h. 3 m. 24.5 sec. A. M., mean time, and the end at 9 h. 31 m. 47 sec. Required the corrected longitude.

	h.	m.	sec
	12	0	0
Beginning, - - - - -	7	3	24.5
Interval in mean time, - -	4	56	35.5
" " sidereal time, - -	4	57	24.22

	h.	m.	sec.
Sidereal time at mean noon, Greenwich, tab. LV,	3	32	57.98
Add for assumed diff. long. - - - - - - -	0	0	49.52
Sid. time at mean noon at Haverford school,	3	33	47.50
Interval in sid. time, subtract - - - - -	4	57	24.22
Remainder, - - - - - - - - - -	22	36	23.28
Z - - - - - - - - - - - - -	339°	5′	49″

	d.	h.	m.	sec.
Observed beginning, Haverford time,	14	19	3	24.5
Assumed diff. of long., add - - - -		5	1	25
Greenwich time	15	0	4	49.5

For this time we have, from the values found in prob. XXXVII, $L' = 54° 37' 34''$, $p = -10.3550$, $q = +3.6961$, and $R = 2.9162$. We have also for the time of middle $p' = +5.085$ and $q' = +0.511$.

q'	log.	9.70842	p'	log.	0.70629
p'	Ar. Co. "	9.29371	N	Ar. Co. cos.	0.00218
$N = +5° 44' 18''$	tan.	9.00213	$n = 5.111$		0.70847

For the lat. 40° 1′ 12″ and $\varepsilon = 23° 27' 44''$, we find,
Log. A = 0.40599, Log. B = 0.88473, Log. G = 0.48477 *n*, and $f = + 5.8677$.

	Log. A	0.40599		Log. A	0.40599
L'	cos.	9.76261	L'	sin.	9.91137
$a = + 1.4743$		0.16860	$a' = + 2.077$		0.31736

	Log. B	0.88473		Log. B	0.88473
$Z - L' = 284° 28' 15''$	sin.	9.98600 *n*	$Z - L'$	cos.	9.39774
$b = - 7.4256$		0.87073 *n*	$b' = + 1.916$		0.28247

	Log. G	0.48477 *n*	$u = - 5.8206$
Z	sin.	9.55241 *n*	$v = + 6.9571$
$g = + 1.0894$		0.03718	
	log. C	9.07905	$x = - 4.5344$
$c = + 0.1307$		9.11623	$y = - 3.2610$
	log. C″	10.14999	$E = 4.18$
$c'' = + 0.185$		9.26622	$k = 5.6292$

x	log.	0.65652 *n*	y	log.	0.51335 *n*
N	sin.	8.99994	N	cos.	9.99782
$z = - 0.4534$		9.65646 *n*	$z' = - 3.2447$		0.51117 *n*

$z = - 0.4534$
$z' = - 3.2447$

$z'' = + 2.7913$

$k + z'' = 8.4205$	log.	0.92533
$k - z'' = 2.8379$	"	0.45300
		2)1.37833
$h = + 4.8883$		0.68916

Z, at time of beginning	339°	5′	49″
Change during 2 h. 28 m. 22.5 sec.	37	11 .	43
Z, at time of end	16	17	32

	h.	m.	sec.
Greenwich time of beginning	0	4	49.5
Duration	2	28	22.5
Greenwich time of end	2	33	12

For this time we have,

$L' = 54° 43' 32''$, $p = +2.2200$, $q = +4.9601$, $R = 2.9179$.

Log. A 0.40599 — Log. A 0.40599
L' - - cos. 9.76154 — L' - - sin. 9.91190

$a = +1.4707$ 0.16753 — $a'' = +2.079$ 0.31789

Log. B 0.88473 — Log. B 0.88473
$Z - L' = 321° 34' 0''$ sin. 9.79351 *n* — $Z - L'$ cos. 9.89395

$b = -4.7669$ 0.67824 *n* — $b' = +6.007$ 0.77868

Log. G 0.48477 *n* — $u = -3.3990$
Z - - - sin. 9.44799 — $v = +5.0111$

$g = -0.8566$ 9.93276 *n*
log. C 9.07905 — $x = +5.6190$

$c = -0.1028$ 9.01181 *n* — $y = -0.0510$
log. C'' 10.14999 — E = 7.94

$c'' = -0.145$ 9.16180 *n* — $k = 5.6136$

x - - log. 0.74966 — y - - log. 8.70757 *n*
N - sin. 8.99994 — N - cos. 9.99782

$z = +0.5618$ 9.74960 — $z' = -0.0507$ 8.70539 *n*
$z' = -0.0507$

$z' = +0.6125$ — $k + z'' = 6.2261$ log. 0.79422
$k - z'' = 5.0011$ " 0.69907

2)1.49329

$h = +5.5801$ 0.74664

x, at beg. $= -4.5344$ — y, at beg. $= -3.2610$
x, " end $= +5.6190$ — y, " end $= -0.0510$

Sum $= +1.0846$ log. 0.03527 — Sum $= -3.3120$ log. 0.52009 *n*
N cos. 9.99782 — N - - sin. 8.99994

$h' = +1.0792$ 0.03309 — $h'' = -0.3312$ 9.52003 *n*
$h'' = -0.3312$
h, at beg. $= +4.8883$

Sum $= +5.6363$
h, at end $= +5.5801$

Rem. $= -0.0562$

½ Rem. $= -0.0281$

	m.	sec.
5.111 : —0.0281 ::	60 :	— 19.8

	d.	h.	m.	sec.
Greenwich time of beg. - - -	15	0	4	49.5
Correction - - - - -			—	19.8
Greenwich time, corrected - - -	15	0	4	29.7
Observed time - - - - -	14	19	3	24.5
Long. Haverford school* - -		5	1	5.2 W.

PROBLEM XXXIX.

To calculate an Occultation of a fixed star by the Moon, for a given place.

Let A = moon's right ascension, A′ = star's apparent right ascension, D = moon's declination, D′ = star's apparent declination, π = moon's equatorial horizontal parallax, Z = right ascension of the zenith of the place, and put,

$$p = \frac{10\,(A - A')\cos D}{\pi}, \text{ and } q = \frac{10\,(D - D')}{\pi}.$$

Also let T be the mean time of conjunction of the moon and star in right ascension, taken to the nearest whole hour. Calculate the values of p and q, for the time T, and also for a time an hour *later* or *earlier*, according as the time T, is *before* or *after*, the time of conjunction. Subtract the value of p at the earlier of the two times, from its value at the later, and the remainder will be the value of p'. Do the same with the values of q, to obtain q'.

Find the values of log. B and log. F, for the given place, by prob. XXXIII. To log. F, add cos. D′, and the sum will be the logarithm of a constant quantity f.

Find the value of Z for the time T, as in prob. XXXIV. Put u = B sin. (Z — A′), b'' = B cos. (Z — A′), $g' = u$ sin D′, and $g = b''$ sin. D′, and find the values of u, b'', g' and g. Subtract g from f and the remainder will be the value of v. With b'' and g', respectively as arguments, find from table LXXIII, the values of u' and v', taking both from the column for v'. Then, taking the values of p and q at the time T, and putting $x = p - u$, $y = q - v$, $x' = p' - u'$, and $y' = q' - v'$, find $n = \sqrt{x'^2 + y'^2}$, $h = \frac{xx' + yy'}{n}$, $m = \sqrt{x^2 + y^2 - h}$, and $h' = \sqrt{7.43 - m^2}$. The quantities m and h', may be considered constant. Take $n : h :: 1$ hour : an interval t. Then will $T - t$ be the approximate time of the middle of the occultation. Call this time T'. Take $n : h' :: 1$ hour : an interval t'. Then will $T' - t'$ and $T' + t'$, be the approximate times of beginning and end.

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* From an article by S. C. Walker, in the journal of the Franklin Institute for August 1836, in which he has given the results of a number of calculations, founded on observations of this eclipse in Philadelphia, it appears, that assuming 5 h. 0 m. 40 sec. W., to be the correct longitude of Philadelphia, the longitudes of places in its vicinity, obtained from observations of the beginning and end of the eclipse, require to be increased by 11.04 sec., on account of errors in the tabular places, &c., of the sun and moon. Applying this correction to the longitude obtained in the text, it will give for the longitude of Haverford school, according to the given observations, 5 h. 1 m. 16.2 sec. W.

Taking now T' to stand for the approximate time of beginning, find for this time the values of p', q', and Z, as in prob. XXXIV, and the values of x, y, x', y', and n, as above. Find, also, $h'' = \sqrt{x^2 + y^2 - m^2}$. Take $n : h'' - h' :: 1$ hour : an interval t''. Then will $T' + t''$ be the corrected time of beginning.

Taking T' to stand for the approximate time of end, proceed in the same manner to find the corrected time, except that $(h' - h'')$ is to be used instead of $(h'' - h')$, in finding t''.

With the values of u, v, x, and y, at the approximate time of beginning, or with their values found by proportion for the corrected time of beginning, find the arc V, as in prob. XXXIV. The arc V will designate the place of the first contact, in reference to the moon's vertex, as seen through a telescope that inverts. A similar process will give the apparent place of contact at the end.

Note. If in any case it is required to give great precision to the calculation, the values of p and q should be computed by the formula given in the next problem.

Exam. Let it be required to calculate for Greenwich, latitude 51° 28′ 39″ N., the occultation of ι, Leonis, of Jan. 7th, 1836, having given the following data taken from the Nautical Almanac.

	h.	m.	sec.
Mean time of conjunc. in right ascen.	12	12	17
Star's apparent right ascen.	10	23	26.39
Moon's right ascen. at 12 h.	10	23	1.27
“ “ “ at 13 h.	10	25	3.92
Star's apparent declination,	14°	58′	38.8″
Moon's declin. at 12 h.	15	35	27.1
“ “ “ 13 h.	15	23	38.4
“ equat. hor. par. at 12 h.		56	3.5
“ “ “ “ 13 h.		56	4.8

Sidereal time at mean noon, 19 h. 4 m. 22.41 sec.

Taking T = 12 h. we have, at time T,

A — A′ = — 25.12 sec. = (in arc), — 377″, D — D′ = 36′ 48″ = 2208″.

$\pi = 3363.5''$	1 + Ar. Co. log.	7.47321	π	1 + Ar. Co. log.	7.47321
A—A′	log.	2.57634 n	D — D′	log.	3.34400
D	cos.	9.98372			
			$q = +6.565$		0.81721
$p = -1.080$		0.03327 n			

At time T + 1,

A — A′ = (in arc), + 1463″, D — D′ = 1500″.

$\pi = 3365''$	1 + Ar. Co. log.	7.47301	π	1 + Ar. Co. log.	7.47301
A—A′	log.	3.16524	D — D′	log.	3.17609
D	cos.	9.98413			
			$q = +4.458$		0.64910
$p = +4.192$		0.62238	$q = +6.565$		
$p = -1.080$					
$p' = +5.27$			$q' = -2.11$		

By prob. XXXIII, we find log. B = 0.795, and log. F = 0.891.

	Log. F	0.891
D′	cos.	9.985
$f = +7.52$		0.876

At time T = 12 h.

Z — A′ = 310° 44′

	Log. B	0.795		Log. B	0.795
Z — A′	sin.	9.880 n	Z — A′	cos.	9.815
$u = -4.73$		0.675 n	$b'' = +4.07$		0.610
D′	sin.	9.413	D′	sin.	9.413
$g' = -1.22$		0.088 n	$g = +1.05$		0.023

$v = +6.47$, $u' = +1.06$ $x = +3.65$ $x' = +4.21$

$v' = -0.32$ $y = +0.09$ $y' = -1.79$

x'	sq.	17.72			
y'	sq.	3.20			
$n = 4.57$	sq.	20.92			
x	sq.	13.32			7.43
y	sq.	0.01	m	sq.	2.24
		13.33	$h' = 2.28$	sq.	5.19
h	sq.	11.09			
m	sq.	2.24			

$$h = \frac{xx' + yy'}{n} = \frac{+15.21}{4.57} = +3.33.$$

4.57 : + 3.33 : : 60 m. : t = + 44 m.

4.57 : 2.28 : : 60 : t' = 30

	h.	m.	
$T =$	12	0	
$t =$	0	44	
$T' =$	11	16	= approx. time ot middle.
$t' =$	0	30	
$T' - t' =$	10	46	= approx. time of beginning.
$T' + t' =$	11	46	= " " end.

Taking now $T' = 10$ h. 46 m., we have,

$p = -7.58$, $q = +9.16$ and $Z - A' = 292° 10'$.

		Log. B	0.795			Log. B	0.795
$Z - A'$		sin.	9.967 *n*	$Z - A$		cos.	9.577
u	$= -5.78$		0.762 *n*	b''	$= +2.36$		0.372
D'		sin.	9.413	D'		sin.	9.413
g'	$= -1.50$		0.175 *n*	g	$= +0.61$		9.785

$v = +6.91$, $u' = +0.62$ $x = -1.80$ $x' = +4.65$

$v' = -0.39$ $y = +2.25$ $y' = -1.72$

x'		*sq.*	21.62	x		*sq.*	3.24
y'		*sq.*	2.96	y		*sq.*	5.06
n	$= 4.96$	*sq.*	24.58				8.30
				m		*sq.*	2.24
				h''	$= 2.46$	*sq.*	6.06
				h'	$= 2.28$		
				$h'' - h'$	$= +0.18$		

4.96 : +0.18 :: 60 m. : $t'' = +2.2$ m.

$T' + t'' = 10$ h. 48.2 m. = corrected time of beginning

Taking $T' = 11$ h. 46 m., we have,

$p = -2.31$, $q = +7.05$, $Z - A' = 307° 13'$

		Log. B	0.795			Log. B	0.795
$Z - A'$		sin.	9.901 *n*	$Z - A'$		cos.	9.781
u	$= -4.97$		0.696 *n*	b''	$= +3.77$		0.576
D'		sin.	9.413	D'		sin.	9.413
g'	$= -1.28$		0.109 *n*	g	$= -0.97$		9.989

$v = +6.55,\quad u' = +0.98 \quad x = +2.66 \quad x' = +4.29$

$v' = -0.33 \quad y = +0.50 \quad y' = -1.78$

x' - - - - sq. 18.40 $\qquad x$ - - - - sq. 7.08

y' - - - sq. 3.17 $\qquad y$ - - - sq. 0.25

$n = 4.64$ sq. 21.57 $\qquad$ 7.33

m - - - - sq. 2.24

$h'' = 2.26$ sq. 5.09

$h' = 2.28$

$h' - h'' = +0.02$

$4.64 : +0.02 :: 60 : t'' = +0.3$

h. m.

$T' + t'' = 11\ 46.3$ = corrected time of end.

At 10 h. 48.2 m.

$x = -1.63 \quad ; \quad u = -5.76 \quad ; \quad P = -36.7°$

$y = +2.19 \quad ; \quad y = +6.90 \quad ; \quad Q = -39.8$

Hence, V = 3.1°, to the east.

At 11 h. 46.3 m.

$x = +2.68 \quad ; \quad u = -4.97 \quad ; \quad P = +79.6°$

$y = +0.49 \quad ; \quad v = +6.55 \quad ; \quad Q = -37.1$

Hence, V = 116.7,° to the east.

PROBLEM XL.

To find the longitude of a place from an observed occultation of a fixed star by the moon.

Let A, A′, D, D′, π, Z, u, v, x and y represent the same quantities as in the last problem, $k = 2.725$, and put,

$$p = \frac{10 \sin (A - A') \cos D}{\sin \pi}$$

$$q = \frac{10 \sin (D - D')}{\sin \pi} + \tfrac{1}{2} p \sin (A - A') \sin D'.$$

Then finding the values of p and q from these expressions and the values of f, u, b'', g' g, v, x, and y, as in the last problem, proceed as in prob. XXXVIII, except that p' should be corrected by subtracting from it the product of 0.0007 by b'', and q', by subtracting from it the product of 0.0007 by g'.

When both the beginning and end are observed and the calculation is made by the second method, take for the correction of p', the product of 0.0007 by *half* the sum of the values of b'' at the beginning and end; and in like manner for the correction of q'.

Note. When there is reason to believe the estimated longitude is nearly correct, the corrections of p' and q' may be omitted. In this case the values of b'' and g' need not be found, and the value of g will be at once obtained from the expression $g = B \cos. (A - A') \sin. D'$.

EXAM. Suppose the observed immersion of ι, Leonis, on Jan. 7th, 1836, at a place in latitude 52° 8′ 28″ N., estimated longitude 0 h. 1 m. W., was at 10 h. 45 m. 53.3 sec., mean time; required the longitude, having given the following data, obtained from the Nautical Almanac.

$$D = 14^\circ\ 58'\ 39''\ \text{N.}$$

	h.	m.	sec.
At Greenwich time	10	46	53.3 = T,

$A - A' = -\ 43'\ 41.5''$; $D - D' = +\ 51'\ 5.4''$; $\pi = 56'\ 1.9''$
$D' = -\ 15^\circ\ 49'\ 44''$ N. ; $Z - A' = 292^\circ\ 8'\ 54''$

	h.	m.	sec.
And at Greenwich time	11	46	53.3 = T + 1,

$A - A' = -\ 12'\ 59.1''$; $D - D' = +\ 39'\ 22.6''$
$\pi = 56\ \ 3.2$; $D = 15^\circ\ 38'\ 1''$ N.

At time T.

π	1 + Ar. Co. sin.	2.78786	π	1 + Ar. Co. sin.	2.78786
$A - A'$	sin.	8.10412 *n*	$D - D'$	sin.	8.17205
D	cos.	9.98321			———
		———	+ 9.1182		0.95991
$p = -\ 7.5022$		0.87519 *n*	+ 0.0123		
$A - A'$	sin.	8.10412 *n*	———		
D'	sin.	9.41236	$q = +\ 9.1305$		
		———			
+ 0.0246		8.39167			
———					
+ 0.0123					

At time T + 1.

π	1 + Ar. Co. sin.	2.78769	π	1 + Ar. Co. sin.	2.78769
$A - A'$	sin.	7.57717 *n*	$D - D'$	sin.	8.05895
D	cos.	9.98363			———
		———	+ 7.0249		0.84664
$p = -\ 2.2310$		0.34849 *n*	+ 0.0011		
$A - A'$	sin.	7.57717 *n*	———		
D'	sin.	9.41236	$q = +\ 7.0260$		
		———	$q = +\ 9.1305$		
+ 0.0022		7.33802	———		
———			$q' = -\ 2.104$		
+ 0.0011					

$p = -2.2310$, at time T + 1,
$p = -7.5022$, " T

$p' = +5.271$

By prob. XXXIII, we find log. B = 0.78887 and log. F = 0.89537, and by the last problem, we find $f = +7.5920$.

	Log. B 0.78887		Log. B 0.78887
Z — A′	sin. 9.96671 *n*	Z — A′	cos. 9.57635
$u = -5.6961$	0.75558 *n*	$b'' = +2.32$	0.36522
D′	sin. 9.41236	D′	sin. 9.41236
$g' = -1.47$	0.16794 *n*	$g = +0.5992$	9.77758
		$f = +7.5920$	
		$v = +6.9928$	

$+2.32 \times 0.0007 = +0.002$
$-1.47 \times 0.0007 = -0.001$

p', corrected	$= +5.269$	$x = -1.8061$	
q' "	$= -2.103$	$y = +2.1377$	

q' - - -	log. 0.32284 *n*	p' - -	log. 0.72173
p' Ar. Co.	" 9.27827	N Ar. Co.	cos. 0.03209
N = — 21° 45′ 29″	tan. 9.60111 *n*	$n = 5.675$	0.75382
x - - -	log. 0.25673 *n*	y - - -	log. 0.32995
N - - -	sin. 9.56901 *n*	N - -	cos. 9.96791
$z = +0.6695$	9.82574	$z' = +1.9855$	0.29786

$$z'' = z' - z = +1.3160.$$

$k + z'' = 4.041$	log. 0.60649
$k - z'' = 1.409$	" 0.14891
	2)0.75540
$h = +2.3862$	0.37770

x - - -	log. 0.25673 *n*	y - -	log. 0.32995
N - -	cos. 9.96791	N - -	sin. 9.56901 *n*
$h' = -1.6774$	0.22464 *n*	$h'' = -0.7924$	9.89896

$$h + h' + h'' = -0.0836.$$

m. sec.
5.675 : + 0.0836 :: 60 : + 53.0

	h.	m.	sec.
Greenwich time of immer.	10	46	53.3
Correction - - - -		+	53.0
Greenwich time, corrected	10	47	46.3
Observed time - - -	10	45	53.3
Longitude of place - -		1	53 W.

PROBLEM XLI.

To Calculate a Transit of Mercury.

For Greenwich mean noon of the day on which the transit occurs, find the sun's longitude and hourly motion, and Mercury's heliocentric longitude. To Mercury's mean anomaly, add 10′ 14″, the mean hourly motion in anomaly, and the sum will be the mean anomaly, an hour after noon. With this anomaly take out again the equation of the centre, and adding 10′ 14″ to it, subtract from the sum the equation of the centre at noon. The remainder will be Mercury's hourly motion in longitude, nearly. To the sun's longitude, add 180° 0′ 20″, and the sum will be the earth's longitude. Then, as the difference of the hourly motions of Mercury and the Sun : the difference of their longitudes :: 1 hour : an interval of time. When the earth's longitude is *greater* than Mercury's, *add* this interval to the mean noon at Greenwich, but when it is *less*, *subtract* the interval, and the sum or remainder will be the approximate time of conjunction in longitude.

Let T be the approximate time of conjunction, taken to the nearest whole hour, and t an interval of two or three hours. For the times $T - t$ and $T + t$, find the sun's longitude and radius vector, Mercury's heliocentric longitude, latitude, and the radius vector, and thence the apparent geocentric longitude and latitude. Take half the sum of the sun's mean anomalies at the times $T - t$ and $T + t$, and it will be the mean anomaly at the time T. In like manner find the radius vectors of the earth and Mercury at the time T. With these find the sun's semidiameter and Mercury's equatorial horizontal parallax and the semidiameter. Add together the semidiameters of the sun and Mercury, and expressing the sum in seconds, call it k. To the constant logarithm 7.95071, add the logarithm of the sun's semidiameter, in seconds, and the sum, rejecting ten from the index, will be the logarithm of the sun's horizontal parallax.

Subtract the sun's longitude at the time $T - t$, from Mercury's apparent geocentric longitude at the same time, and reducing the remainder to seconds, call it p. Reducing Mercury's apparent geocentric latitude at the same time to seconds, and marking it *affirmative* or *negative* according as it is *north* or *south*, call it q[*]. To the difference between the apparent geocentric longitude of Mercury, at the times $T - t$ and $T + t$, add the difference between the sun's longitudes at the same times, and reducing the sum to seconds, and marking it *negative*, call it p'. Subtract the value of the apparent geocentric latitude at the time $T - t$, from its value at the time $T + t$, and reducing the remainder to seconds, call it q'.

To the logarithm of q', add the arithmetical complement of the logarithm of p', and the sum will be the tangent of an arc N, between 90° and 180°, which must be marked with the same sign as q'. Add together the logarithm of $2t$, reduced to seconds, the cosine of N, and the arithmetical complement of the logarithm of p', and the sum, rejecting the tens from the index, will be the logarithm of a quantity n.

Put $z = p \sin N$ and $z' = q \cos.\ N$, and finding the values of z and z', take their difference and call it z''. Add together the logarithms of $(k + z'')$ and $(k - z'')$, and *half* the sum will be the logarithm of an affirmative quantity h. Put $h' = p \cos.\ N$, and $h'' = q \sin.\ N$, and find h' and h''. To the logarithm of the sum of h, h' and h'', add the logarithm of n, and the sum will be the logarithm of an interval, in seconds of time, which being *subtracted* from the time $T - t$, will give the time of the first external contact as seen from the earth's centre. Call this time T'. Subtract the sum of h' and h'', from h, and to the logarithm of the remainder add the logarithm of n, and the sum will be the logarithm of an interval in seconds, which being *added* to the time $T - t$, will give the time of last external contact as seen from the earth's centre. Call this time T''.

Using p' and q', which are the changes of the values of p and q in the time $2t$, find by proportion the values of p and q, at the times T' and T''.

Let $d = \frac{1}{2}(T'' - T')$, expressed in seconds, ϖ = Mercury's parallax, and ϖ' = sun's parallax. Add together twice the logarithm of n, the logarithm of $\frac{1}{10}(\varpi - \varpi')$, and the arithmetical complement of the logarithm of d, and call the sum, with 10 rejected from the index, log. n'. Using the values of p and q, at the time T', find the values of e and e', from the expressions, log. e = log. p + log. n' — 10, and log. e' = log. q + log. n' — 10. With the values of p and q, at the time T'', find in like manner the values of e and e', for that time. The values of e and e' will be expressed in seconds of time.

Let Z' be the right ascension of the zenith of Greenwich, Z the right ascension of the zenith of a given place, for which it is required to calculate the effects of parallax in changing the times of contact, l, the longitude of the place from Greenwich, *affirmative* when *east*, but *negative* when *west*, t'', a correction in seconds, to be *subtracted* from the time T', or *added* to the time T'', to obtain the time of contact at the given place, and let D, L′, u, and v, represent the same quantities as in prob. XXXIV. Then $D = Z' - Z' + l$, and $Z = L' + l$.

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[*] When the sun's place is found from complete tables in which its latitude is given, q should be taken equal to the difference of the latitudes of Mercury and the sun.

With the values of Z′, and L′ found for each of the times T′ and T″, find the values of D and Z at these times, and thence by prob. XXXIV, the values of u and v. Then using the values of the quantities corresponding to either of the times T′ or T″, the value of t', the correction for that time, will be equal to the sum of the products of e by u, and e' by v.

Taking p and q, instead of x and y, the place of contact with reference to the sun's vertex may be found in the same manner as for an eclipse of the sun.

Note. 1. To find the times of internal contact take $k =$ to the difference of the semidiameters of the Sun and Mercury, and with z'' and this value of k, find h, as in the rule. To the logarithm of the difference between the true values of h, add the logarithm of n, and the sum will be the logarithm of an interval of time, in seconds, which being added to the time of the first external contact and subtracted from the time of the second, will give the times of the internal contacts.

2. A transit of Venus may be calculated in a similar manner. Table LXVII, contains the heliocentric longitudes and latitudes and the radius vector of Venus at the times of the next two transits.

Exam. It is required to calculate the transit of mercury that will occur on the 8th of May, 1845, and to find the effect of parallax in changing the times of ingress and egress, for Philadelphia.

The sun's long. at Greenwich mean noon is	47°	42′	47″
" earth's " " "	227	43	7
Mercury's helio. long. - - - - -	227	5	7
Sun's hourly motion in long. - - - -		2	25
Mercury's " "		7	19

h. h. m.
4′ 54″ : 38′ 0″ :: 1 : 7 45

Hence the time of conjunction in long. is 7 h. 45 m. P. M., nearly. Taking T to denote the 8th day at 8 h., let $t = 3$ h.

For the time $T - t$, we find,

Sun's longitude - - - - - - - -		47°	54′	51″
Mercury's app. geoc. longitude - - - -		48	6	35
" " " latitude - -			7	11 S.
Earth's or sun's radius vector	1.01012			
Mercury's " "	0.45357			

For the time $T + t$,

Sun's longitude, - - - - - - - -		48°	9′	20″
Mercury's app. geoc. longitude - - - -		47	57	29
" " " latitude . - - -			11	33 S.
Earth's radius vector	1.01018			
Mercury's " "	0.45423			

We find for the time T, or without sensible error, during the transit,

Sun's semidiameter	951.8″
" hor. parallax	8.5
Mercury's semidiameter	5.8
" hor. parallax	15.4

We also readily obtain, $p' = -1415''$ and $q' = -262''$.

q'	log.	2.41830 n	$2t = 21600''$	log.	4.33445
p' Ar. Co.	"	6.84924 n	N	cos.	9.99268 n
			p' Ar. Co.	log.	6.84924 n
N = — 169° 30′ 36″	tan.	9.26754			
			n	log.	1.17637

At time T — t.

$p = +704''$ and $q = -431''$

p	log.	2.84757	q	log.	2.63448 n
N	sin.	9.26022 n	N	cos.	9.99268 n
$z = -128.2$		2.10779 n	$z' = +423.8$		2.62716

$k = 957.6 \quad z'' = +552.0.$

$k + z'' = 1509.6$	log.	3.17886
$k - z' = 405.6$	"	2.60810
		2)5.78696
$h = +782.5$		2.89348

p	log.	2.84757	q	log.	2.63448 n
N	cos.	9.99268 n	N	sin.	9.26022 n
$h' = -692.2$		2.84025 n	$h'' = +78.5$		1.89470

$h+h'+h''=+168.8$	log.	2.22737	$h-(h'+h'')=+1396.2$	log.	3.14495
n	log.	1.17637	n	log.	1.17637
+2534 sec.		3.40374	+20957 sec.		4.32132
42 m. 14 sec.			5 h. 49 m. 17 sec.		
T — t' = 5 h. 0 0			T — t = 5 0 0		
T′ = 4 17 46			T″ = 10 49 17		

$d = \frac{1}{2}(T'' - T') = 11745$ sec.

At time T′.

$p = +870, \quad q = -400, \quad Z' = 110° 47', \quad L' = 47° 53',$

$D = 62° 54' + l,$ and $Z = 110° 47' + l.$

At time T″

$p = -669, \quad q = -685, \quad Z' = 208° 56', \quad L' = 48° 9',$

$D = 160° 47' + l,$ and $Z = 208° 56' + l.$

n	log.	1.17637
		2
		2.35274
$\frac{1}{10}(\pi - \pi') = 0.69''$	log.	9.83885
$d = 11745$	Ar. Co. "	5.93015
	log. n'	8.12174

At time T'

p	log.	2.93952	q	log.	2.60206 n
	log. n'	8.12174		log. n'	8.12174
$e = + 11.51$ sec.		1.06126	$e' = - 5.29$ sec.		0.72380 n

At time T''.

p	log.	2.82543 n	q	log.	2.83569 n
	log. n'	8.12174		log. n'	8.12174
$e = - 8.85$ sec.		0.94717 n	$e' = - 9.07$ sec.		0.95743 n

To find a and log. C, by prob. XXXIV, take L' = 48° 1', the mean of its values at T' and T''.

For Philadelphia.

	Log. A	0.405		C log.	9.317
L'	cos.	9.826	L'	cos.	9.826
$a = + 2.04$		0.231		Log. C	9.143

At time T'.

	Log. B	0.885		Log. G	0.485 n
D = 347° 44'	sin.	9327 n	Z = 35° 37'	sin.	9.765
$b = - 1.63$		0.212	$g = - 1.78$		0.250 n
				log. C	9.143
			$c = - 0.25$		9.393 n

$$u = a + b + c = + 0.16 \qquad v = f + g = + 4.08$$

$$t'' = eu + e'v = - 35.2 \text{ sec.}$$

Time of first contact at Philadelphia, 4 h. 18 m. 21 sec.

At time T''.

	Log. B	0.885		Log. G	0.485 n
D = 85° 37'	sin.	9.999	Z = 133° 46'	sin.	9.859
$b = + 7.66$		0.884	$g = - 2.21$		0.344 n
				log. C	9.143
			$c = - 0.31$		9.487 n

$$u = +9.39 \qquad v = -3.65$$

$$t'' = eu + e'v = -116.2 \text{ sec.} = 1 \text{ m. } 56.2 \text{ sec.}$$

Time of last contact at Philadelphia, 10 h. 47 m. 21 sec.

At time T'.

p	- - -	log.	2.940	u	- - -	log.	9.204
q	Ar. Co.	"	7.398 n	v	Ar. Co.	"	9.389
$P = +114.7°$		tan.	10.338 n	$Q = +2.3°$		tan.	8.593

$$V = Q - P = -112.4°$$

In like manner we find at time T'',

$$V = +113.1°$$

Reducing the times to Philadelphia time we have,

	h.	m.	sec.	
First contact, at	11	17	41	A. M. mean time
Last " "	5	46	41	P. M. " "

First contact, 112.4°, from vertex to the east,
Last " 113.1 " " " west,

SCHOLIUM. The *corrections* of the times of contact on account of parallax, obtained as above, may be regarded as very nearly true. But the times of contact obtained for the earth's centre, and consequently those for a given place, cannot be depended on as equally correct; as an error of three or four seconds in the longitude of the sun or Mercury, may produce an error of a minute in time.

THE END.

ASTRONOMICAL TABLES.

Latitudes, and Longitudes from the Meridian of Greenwich, of some cities, and other conspicuous places.

Names of Places.		Latitude.	Longitude in Degrees.	Longitude in Time.
		° ′ ″	° ′ ″	h. m. sec.
Albany, *Capitol,*	New York,	42 39 3N.	73 44 45W.	4 54 59
Altona, *Obs.,*	Denmark,	53 32 45N.	9 56 45E.	0 39 47
Amsterdam,	Holland,	52 22 30N.	4 53 16E.	0 19 33
Baltimore, *B. Mon't.,*	Maryland,	39 17 13N.	76 37 50W.	5 6 31
Berlin, *Obs.,*	Germany,	52 31 13N.	13 23 52E.	0 53 35.5
Boston, *State House,*	Mass'ts,	42 21 15N.	71 4 9W.	4 44 16.6
Brest, *Obs.,*	France,	48 23 32N.	4 29 25W.	0 17 58
Canton,	China,	23 8 9N.	113 16 54E.	7 33 8
Cape G. Hope, *Obs.,*	Africa,	33 56 3S.	18 28 45E.	1 13 55
Charleston, *Coll.,*	S. Carolina,	32 47 0N.	80 0 52W.	5 20 3
Charlottesville, *Univers.,*	Virginia,	38 2 3N.	78 31 29W.	5 14 6
Cincinnati,	Ohio,	39 5 54N.	84 24 0W.	5 37 36
Copenhagen, *Obs.,*	Denmark,	55 40 53N.	12 34 57E.	0 50 19.8
Dublin, *Obs.,*	Ireland,	53 23 13N.	6 20 30W.	0 25 22
Edinburgh, *Obs.,*	Scotland,	55 57 20N.	3 10 54W.	0 12 43.6
Gotha, *Seeberg Obs.,*	Germany,	50 56 5N.	10 44 6E.	0 42 56.4
Göttingen, *Obs.,*	Germany,	51 31 48N.	9 56 37E.	0 39 46.5
Greenwich, *Obs.,*	England,	51 28 39N.	0 0 0	0 0 0
Königsberg, *Obs.,*	Prussia,	54 42 50N.	20 30 7E.	1 22 0.5
Lancaster,	Penn.,	40 2 36N.	76 20 33W.	5 5 22
London, *St. Paul's Ch.,*	England,	51 30 49N.	0 5 47W.	0 0 23
Marseilles, *Obs.,*	France,	43 17 50N.	5 22 15E.	0 21 29
Milan, *Obs.,*	Italy,	45 28 1N.	9 11 48E.	0 36 47
Naples, *Obs.,*	Italy,	40 51 47N.	14 15 4E.	0 57 0
New Haven, *Coll.,*	Connecticut,	41 17 58N.	72 57 46W.	4 51 51
New Orleans, *C. Hall,*	Louisiana,	29 57 45N.	90 6 49W.	6 0 27
New York, *C. Hall,*	New York,	40 42 40N.	74 1 8W.	4 56 4
Palermo, *Obs.,*	Italy,	38 6 44N.	13 21 24E.	0 53 25.6
Paramatta, *Obs.*	New Hol.,	33 48 50S.	151 1 34E.	10 4 6
Paris, *Obs.,*	France,	48 50 13N.	2 20 24E.	0 9 21.6
Petersburgh, *Obs.,*	Russia,	59 56 31N.	30 19 0E.	2 1 16
Philadelphia, *Ind'ce H.*	Penn.,	39 56 59N.	75 10 59W.	5 0 40
Pittsburgh,	Penn.,	40 26 15N.	79 58 6W.	5 19 52
Point Venus,	Otaheite,	17 29 21S.	149 28 55W.	9 57 56
Princeton, *Coll.,*	New Jersey,	40 22 N.	74 35 W.	4 58 20
Providence, *Univers.,*	Rhode Isl.,	41 49 25N.	71 25 56W.	4 45 44
Quebec, *Castle,*	L. Canada,	46 49 12N.	71 16 0W.	4 45 4
Richmond, *Capitol,*	Virginia,	37 32 17N.	77 26 28W.	5 9 46
Rome, *St. Peter's Ch.,*	Italy,	41 54 8N.	12 27 5E.	0 49 48
Savannah, *Exch.,*	Georgia,	32 4 56N.	81 7 9W.	5 24 29
Stockholm, *Obs.,*	Sweden,	59 20 31N.	18 3 34E.	1 12 14
Turin, *Obs.,*	Italy,	45 4 6N.	7 42 6E.	0 30 48.4
Vienna, *Obs.,*	Austria,	48 12 35N.	16 23 0E.	1 5 32
Wardhus,	Lapland,	70 22 36N.	31 7 54E.	2 4 32
Washington, *Capitol,*	Dist. Colum.,	38 52 54N.	77 1 48W.	5 8 7

TABLE II.

Mean Astronomical Refractions.

Ap. Alt.	Refr.	Ap. Alt.	Refr.	Ap. Alt.	Refr.	Ap. Alt.	Refr.
0° 0'	33' 0"	4° 0	11' 51"	12 20'	4' 16"	45 0'	0' 57"
0 5	32 10	4 10	11 29	12 40	4 9	46 0	0 55
0 10	31 22	4 20	11 8	13 0	4 3	47 0	0 53
0 15	30 35	4 30	10 48	13 20	3 57	48 0	0 51
0 20	29 50	4 40	10 29	13 40	3 51	49 0	0 49
0 25	29 6	4 50	10 11	14 0	3 45	50 0	0 48
0 30	28 22	5 0	9 54	14 20	3 40	51 0	0 46
0 35	27 41	5 10	9 38	14 40	3 35	52 0	0 44
0 40	27 0	5 20	9 23	15 0	3 30	53 0	0 43
0 45	26 20	5 30	9 8	15 30	3 24	54 0	0 41
0 50	25 42	5 40	8 54	16 0	3 17	55 0	0 40
0 55	25 5	5 50	8 41	16 30	3 10	56 0	0 38
1 0	24 29	6 0	8 28	17 0	3 4	57 0	0 37
1 5	23 54	6 10	8 15	17 30	2 59	58 0	0 35
1 10	23 20	6 20	8 3	18 0	2 54	59 0	0 34
1 15	22 47	6 30	7 51	18 30	2 49	60 0	0 33
1 20	22 15	6 40	7 40	19 0	2 45	61 0	0 31
1 25	21 44	6 50	7 30	19 30	2 39	62 0	0 30
1 30	21 15	7 0	7 20	20 0	2 35	63 0	0 29
1 35	20 46	7 10	7 11	20 30	2 31	64 0	0 28
1 40	20 18	7 20	7 2	21 0	2 27	65 0	0 26
1 45	19 51	7 30	6 53	21 30	2 24	66 0	0 25
1 50	19 25	7 40	6 45	22 0	2 20	67 0	0 24
1 55	19 0	7 50	6 37	23 0	2 14	68 0	0 23
2 0	18 35	8 0	6 29	24 0	2 7	69 0	0 22
2 5	18 11	8 10	6 22	25 0	2 2	70 0	0 21
2 10	17 48	8 20	6 15	26 0	1 56	71 0	0 19
2 15	17 26	8 30	6 8	27 0	1 51	72 0	0 18
2 20	17 4	8 40	6 1	28 0	1 47	73 0	0 17
2 25	16 44	8 50	5 55	29 0	1 42	74 0	0 16
2 30	16 24	9 0	5 48	30 0	1 38	75 0	0 15
2 35	16 4	9 10	5 42	31 0	1 35	76 0	0 14
2 40	15 45	9 20	5 36	32 0	1 31	77 0	0 13
2 45	15 27	9 30	5 31	33 0	1 28	78 0	0 12
2 50	15 9	9 40	5 25	34 0	1 24	79 0	0 11
2 55	14 52	9 50	5 20	35 0	1 21	80 0	0 10
3 0	14 36	10 0	5 15	36 0	1 18	81 0	0 9
3 5	14 20	10 15	5 7	37 0	1 16	82 0	0 8
3 10	14 4	10 30	5 0	38 0	1 13	83 0	0 7
3 15	13 49	10 45	4 53	39 0	1 10	84 0	0 6
3 20	13 34	11 0	4 47	40 0	1 8	85 0	0 5
3 25	13 20	11 15	4 40	41 0	1 5	86 0	0 4
3 30	13 6	11 30	4 34	42 0	1 3	87 0	0 3
3 40	12 40	11 45	4 29	43 0	1 1	88 0	0 2
3 50	12 15	12 0	4 23	44 0	0 59	89 0	0 1

Mean New Moons and Arguments in January.

	Mean New Moon in January.			I.	II.	III.	IV.	N.
A. D.	D.	H.	M.					
1821	2	17	59	0092	7859	80	78	823
1822	21	15	32	0602	7182	78	66	930
1823	11	0	20	0304	5787	61	55	953
1824 B.	29	21	53	0814	5110	59	43	060
1825	18	6	41	0516	3716	42	32	083
1826	7	15	30	0218	2321	25	21	105
1827	26	13	3	0728	1644	24	09	213
1828 B.	15	21	51	0430	0250	07	98	235
1829	4	6	40	0131	8855	90	87	257
1830	23	4	12	0642	8178	88	75	365
1831	12	13	1	0343	6784	71	64	387
1832 B.	1	21	50	0045	5389	54	53	409
1833	19	19	22	0555	4712	53	42	517
1834	9	4	11	0257	3318	36	31	539
1835	28	1	43	0768	2641	34	19	647
1836 B.	17	10	32	0469	1246	17	08	669
1837	5	19	20	0171	9852	00	97	692
1838	24	16	53	0681	9175	99	85	799
1839	14	1	42	0383	7780	82	74	822
1840 B.	3	10	30	0085	6386	65	63	844
1841	21	8	3	0595	5709	63	51	951
1842	10	16	51	0297	4314	46	40	974
1843	29	14	24	0807	3637	44	28	081
1844 B.	18	23	13	0509	2243	28	17	104
1845	7	8	1	0211	0848	11	06	126
1846	26	5	34	0721	0171	09	94	234
1847	15	14	22	0423	8777	92	84	256
1848 B.	4	23	11	0125	7382	75	73	278
1849	22	20	43	0635	6705	73	61	386
1850	12	5	32	0337	5311	56	50	408
1851	1	14	21	0038	3916	40	39	431
1852 B.	20	11	53	0549	3239	38	27	538
1853	8	20	42	0251	1845	21	16	560
1854	27	18	14	0761	1168	19	04	668
1855	17	3	3	0463	9773	02	93	690
1856 B.	6	11	51	0164	8379	85	82	713
1857	24	9	24	0675	7702	84	70	820
1858	13	18	13	0376	6307	67	59	843
1859	3	3	1	0078	4913	50	48	865
1860 B.	22	0	34	0588	4236	48	36	972
1861	10	9	22	0290	2840	31	25	995
1862	29	6	55	0800	2163	30	14	102
1863	18	15	44	0504	0769	13	03	125
1864 B.	8	0	32	0204	9374	96	92	147
1865	25	22	5	0714	8698	94	80	256
1866	15	6	53	0416	7303	77	69	277
1867	4	15	42	0118	5909	60	58	299
1868 B.	23	13	14	0628	5231	59	46	407
1869	11	22	3	0330	3837	42	35	429
1870	1	6	51	0032	2442	25	24	451

TABLE IV.

Mean Lunations and Change in Arguments.

Num.	Lunations.			I.	II.	III.	IV.	N.
	D.	H.	M.					
½	14	18	22	404	5359	58	50	43
1	29	12	44	808	717	15	99	85
2	59	1	28	1617	1434	31	98	170
3	88	14	12	2425	2151	46	97	256
4	118	2	56	3234	2869	61	96	341
5	147	15	40	4042	3586	76	95	426
6	177	4	24	4851	4303	92	95	511
7	206	17	8	5659	5020	7	94	566
8	236	5	52	6468	5737	22	93	622
9	265	18	36	7276	6454	37	92	767
10	295	7	20	8085	7171	53	91	852
11	324	20	5	8893	7889	68	90	937
12	354	8	49	9702	8606	83	89	22
13	383	21	33	510	9323	98	88	108

TABLE V.

Number of Days from the commencement of the year to the first of each month.

Months,	Com.	Bis.
	Days.	Days.
January, -	0	0
February, -	31	31
March, - -	59	60
April, - -	90	91
May, - -	120	121
June, - -	151	152
July, - -	181	182
August, -	212	213
September,	243	244
October, -	273	274
November,	304	305
December,	334	335

Equations for New and Full Moon.

Arg.	I.	II.	Arg.	I.	II.	Arg.	III.	IV.	Arg.
	H. M.	H. M.		H. M	H. M.		M.	M.	
000	4 20	10 10	5000	4 20	10 10	25	3	31	25
100	4 36	9 36	5100	4 5	10 50	26	3	31	24
200	4 52	9 2	5200	3 49	11 30	27	3	30	23
300	5 8	8 28	5300	3 34	12 9	28	3	30	22
400	5 24	7 55	5400	3 19	12 48	29	3	30	21
500	5 40	7 22	5500	3 4	13 26	30	3	30	20
600	5 55	6 49	5600	2 49	14 3	31	3	30	19
700	6 10	6 17	5700	2 35	14 39	32	4	30	18
800	6 24	5 46	5800	2 21	15 13	33	4	29	17
900	6 38	5 15	5900	2 8	15 46	34	4	29	16
1000	6 51	4 46	6000	1 55	16 18	35	4	29	15
1100	7 4	4 17	6100	1 42	16 48	36	5	28	14
1200	7 15	3 50	6200	1 31	17 16	37	5	28	13
1300	7 27	3 24	6300	1 19	17 42	38	5	27	12
1400	7 37	2 59	6400	1 9	18 6	39	5	27	11
1500	7 47	2 35	6500	0 59	18 28	40	6	26	10
1600	7 55	2 14	6600	0 50	18 48	41	6	26	9
1700	8 3	1 53	6700	0 42	19 6	42	7	25	8
1800	8 10	1 35	6800	0 34	19 21	43	7	25	7
1900	8 16	1 18	6900	0 28	19 33	44	7	24	6
2000	8 21	1 3	7000	0 22	19 44	45	8	23	5
2100	8 25	0 51	7100	0 17	19 52	46	8	23	4
2200	8 29	0 40	7200	0 14	19 57	47	9	22	3
2300	8 31	0 32	7300	0 11	20 0	48	9	21	2
2400	8 32	0 25	7400	0 9	20 1	49	10	21	1
2500	8 32	0 21	7500	0 8	19 59	50	10	20	0
2600	8 31	0 19	7600	0 8	19 55	51	10	19	99
2700	8 29	0 20	7700	0 9	19 48	52	11	19	98
2800	8 26	0 23	7800	0 11	19 40	53	11	18	97
2900	8 23	0 28	7900	0 15	19 29	54	12	17	96
3000	8 18	0 36	8000	0 19	19 17	55	12	17	95
3100	8 12	0 47	8100	0 24	19 2	56	13	16	94
3200	8 6	0 59	8200	0 30	18 45	57	13	15	93
3300	7 58	1 14	8300	0 37	18 27	58	13	15	92
3400	7 50	1 32	8400	0 45	18 6	59	14	14	91
3500	7 41	1 52	8500	0 53	17 45	60	14	14	90
3600	7 31	2 14	8600	1 3	17 21	61	15	13	89
3700	7 21	2 38	8700	1 13	16 56	62	15	13	88
3800	7 9	3 4	8800	1 25	16 30	63	15	12	87
3900	6 58	3 32	8900	1 36	16 3	64	15	12	86
4000	6 45	4 2	9000	1 49	15 34	65	16	11	85
4100	6 32	4 34	9100	2 2	15 5	66	16	11	84
4200	6 19	5 7	9200	2 16	14 34	67	16	11	83
4300	6 5	5 41	9300	2 30	14 3	68	16	10	82
4400	5 51	6 17	9400	2 45	13 31	69	17	10	81
4500	5 36	6 54	9500	3 0	12 58	70	17	10	80
4600	5 21	7 32	9600	3 16	12 25	71	17	10	79
4700	5 6	8 11	9700	3 32	11 52	72	17	10	78
4800	4 51	8 50	9800	3 48	11 18	73	17	10	77
4900	4 35	9 30	9900	4 4	10 44	74	17	9	76
5000	4 20	10 10	10000	4 20	10 10	75	17	9	75

Sun's Epochs.

Years.	M. Long.	Long. Perig.	I.	II.	III.	N.
1821	9s 8° 48′ 19″	9s 7° 50′ 43″	920	782	260	036
1822	9 8 34 0	9 7 51 45	280	697	886	090
1823	9 8 19 40	9 7 52 47	640	612	511	143
1824 B.	9 9 4 29	9 7 53 49	034	530	138	197
1825	9 8 50 9	9 7 54 51	394	445	763	251
1826	9 8 35 49	9 7 55 52	754	360	388	304
1827	9 8 21 30	9 7 56 54	114	275	013	358
1828 B.	9 9 6 18	9 7 57 56	508	192	640	412
1829	9 8 51 59	9 7 58 58	868	107	265	466
1830	9 8 37 39	9 8 0 0	228	022	890	519
1831	9 8 23 19	9 8 1 2	588	937	515	573
1832 B.	9 9 8 8	9 8 2 4	982	855	142	627
1833	9 8 53 49	9 8 3 6	342	770	767	681
1834	9 8 39 29	9 8 4 8	702	684	392	734
1835	9 8 25 9	9 8 5 10	062	600	017	788
1836 B.	9 9 10 6	9 8 7 2	456	517	644	842
1837	9 8 55 46	9 8 8 3	816	432	269	895
1838	9 8 41 57	9 8 9 4	176	347	894	949
1839	9 8 27 7	9 8 10 6	536	262	519	003
1840 B.	9 9 11 56	9 8 11 8	930	180	146	056
1841	9 8 57 37	9 8 12 9	290	095	771	110
1842	9 8 43 17	9 8 13 11	650	009	397	164
1843	9 8 28 9	9 8 14 12	010	925	021	218
1844 B.	9 9 13 47	9 8 15 14	404	843	648	272
1845	9 8 59 27	9 8 16 16	764	757	273	325
1846	9 8 45 8	9 8 17 17	124	673	897	379
1847	9 8 30 48	9 8 18 19	484	588	623	433
1848 B.	9 9 15 37	9 8 19 20	878	505	151	487
1849	9 9 1 17	9 8 20 22	238	420	775	540
1850	9 8 46 58	9 8 21 23	598	336	400	594
1851	9 8 32 39	9 8 22 24	958	250	025	648
1852 B.	9 9 17 27	9 8 23 26	353	168	653	701
1853	9 9 3 8	9 8 24 27	713	083	277	755
1854	9 8 48 48	9 8 25 29	073	998	902	809
1855	9 8 34 29	9 8 26 30	433	913	527	863
1856 B.	9 9 19 18	9 8 27 32	827	832	153	916
1857	9 9 4 58	9 8 28 34	187	746	779	970
1858	9 8 50 39	9 8 29 35	547	661	404	024
1859	9 8 36 19	9 8 30 37	907	576	029	078
1860 B.	9 9 21 8	9 8 31 38	301	494	656	131
1861	9 9 6 49	9 8 32 39	661	409	281	185
1862	9 8 52 29	9 8 33 41	021	324	906	239
1863	9 8 38 10	9 8 34 42	381	239	530	292
1864 B.	9 9 22 58	9 8 35 44	775	157	157	346
1865	9 9 8 39	9 8 36 45	135	072	783	400
1866	9 8 54 20	9 8 37 47	495	986	408	453
1867	9 8 40 0	9 8 38 49	855	902	033	507
1868 B.	9 9 24 49	9 8 39 50	249	820	659	561
1869	9 9 10 30	9 8 40 52	609	734	285	615
1870	9 8 56 10	9 8 41 53	969	649	910	668
1882	9 9 1 41	9 8 54 10	391	638	416	313

TABLE VIII.

Sun's Motions for Months.

Months.		Longitude.	Per.	I.	II.	III.	N.
Jan.	Com.	0^s 0° 0′ 0″	0″	0	0	0	0
	Bis.	11 29 0 52	0	966	997	998	0
Feb.	Com.	1 0 33 18	5	47	78	53	4
	Bis.	0 29 34 10	5	13	75	51	4
March,		1 28 9 11	10	993	148	101	9
April, - - -		2 28 42 30	15	42	226	154	13
May, - - - - -		3 28 16 40	20	59	301	206	18
June, - - -		4 28 49 58	26	110	379	259	22
July, - - - - -		5 28 24 8	31	129	454	310	27
August, - - -		6 28 57 26	36	182	531	363	31
September, - -		7 29 30 44	41	233	609	416	36
October, - -		8 29 4 54	46	250	684	468	40
November, - -		9 29 38 12	52	300	762	521	45
December, - -		10 29 12 22	57	313	837	572	49

TABLE IX.

Sun's Horary Motion.

ARGUMENT. Sun's Mean Anomaly.

	0^s	I^s	II^s	III^s	IV^s	V^s	
0°	2′ 33″	2′ 32″	2′ 30″	2′ 28″	2′ 25″	2′ 24″	30°
10	2 33	2 32	2 29	2 27	2 25	2 23	20
20	2 33	2 31	2 29	2 26	2 24	2 23	10
30	2 32	2 30	2 28	2 25	2 24	2 23	0
	XI^s	X^s	IX^s	VIII^s	VII^s	VI^s	

TABLE X.

Sun's Semidiameter.

ARGUMENT. Sun's Mean Anomaly.

	0^s	I^s	II^s	III^s	IV^s	V^s	
0°	16′ 18″	16′ 15″	16′ 9″	16′ 1″	15′ 53″	15′ 48″	30°
10	16 18	16 14	16 7	15 58	15 51	15 46	20
20	16 17	16 12	16 4	15 56	15 49	15 46	10
30	16 15	16 9	16 1	15 53	15 48	15 45	0
	XI^s	X^s	IX^s	VIII^s	VII^s	VI^s	

Sun's Motions for Days and Hours.

Days.	Longitude.	Per	I.	II.	III.	N.
1	0° 0′ 0″	0″	0	0	0	0
2	0 59 8	0	34	3	2	0
3	1 58 17	0	68	5	3	0
4	2 57 25	0	101	8	5	0
5	3 56 33	1	135	10	7	1
6	4 55 42	1	169	13	9	1
7	5 54 50	1	203	15	10	1
8	6 53 58	1	236	18	12	1
9	7 53 7	1	270	20	14	1
10	8 52 15	1	304	23	15	1
11	9 51 23	2	338	25	17	1
12	10 50 32	2	371	28	19	2
13	11 49 40	2	405	30	21	2
14	12 48 48	2	439	33	22	2
15	13 47 57	2	473	35	24	2
16	14 47 5	3	506	38	26	2
17	15 46 13	3	540	40	27	2
18	16 45 22	3	574	43	29	2
19	17 44 30	3	608	45	31	3
20	18 43 38	3	641	48	33	3
21	19 42 47	3	675	50	34	3
22	20 41 55	4	709	53	36	3
23	21 41 3	4	743	55	38	3
24	22 40 12	4	777	58	39	3
25	23 39 20	4	810	60	41	4
26	24 38 28	4	844	63	43	4
27	25 37 37	4	878	65	45	4
28	26 36 45	5	912	68	46	4
29	27 35 53	5	945	70	48	4
30	28 35 2	5	979	73	50	4
31	29 34 10	5	13	75	51	4

Hours.	Long.	I.
1	2′ 28″	1
2	4 56	3
3	7 23	4
4	9 51	6
5	12 19	7
6	14 47	8
7	17 15	10
8	19 43	11
9	22 11	13
10	24 38	14
11	27 6	16
12	29 34	17
13	32 2	18
14	34 30	20
15	36 58	21
16	39 26	23
17	41 53	24
18	44 21	25
19	46 49	27
20	49 17	28
21	51 45	30
22	54 13	31
23	56 40	32
24	59 8	34

Sun's Motions for Minutes and Seconds.

Min.	Long.	Min.	Long.	Sec.	Long.	Sec.	Long.
1	0′ 2″	31	1′ 16″	1	0″	31	1″
2	0 5	32	1 19	2	0	32	1
3	0 7	33	1 21	3	0	33	1
4	0 10	34	1 24	4	0	34	1
5	0 12	35	1 26	5	0	35	1
6	0 15	36	1 29	6	0	36	1
7	0 17	37	1 31	7	0	37	2
8	0 20	38	1 34	8	0	38	2
9	0 22	39	1 36	9	0	39	2
10	0 25	40	1 39	10	0	40	2
11	0 27	41	1 41	11	0	41	2
12	0 30	42	1 43	12	0	42	2
13	0 32	43	1 46	13	1	43	2
14	0 34	44	1 48	14	1	44	2
15	0 37	45	1 51	15	1	45	2
16	0 39	46	1 53	16	1	46	2
17	0 42	47	1 56	17	1	47	2
18	0 44	48	1 58	18	1	48	2
19	0 47	49	2 1	19	1	49	2
20	0 49	50	2 3	20	1	60	2
21	0 52	51	2 6	21	1	51	2
22	0 54	52	2 8	22	1	52	2
23	0 57	53	2 11	23	1	53	2
24	0 59	54	2 13	24	1	54	2
25	1 2	55	2 16	25	1	55	2
26	1 4	56	2 18	26	1	56	2
27	1 7	57	2 20	27	1	57	2
28	1 9	58	2 23	28	1	58	2
29	1 11	59	2 25	29	1	59	2
30	1 14	60	2 28	30	1	60	2

Equation of the Sun's Centre.

ARGUMENT. Sun's Mean Anomaly.

	0^s	I^s	II^s	III^s	IV^s	V_s
0°	1°59′ 30″	2°58′ 15″	3°40′ 27″	3°54′ 50″	3°38′ 21″	2°56′ 9″
1	2 1 33	3 0 0	3 41 25	3 54 47	3 37 18	2 54 25
2	2 3 37	3 1 44	3 42 21	3 54 41	3 36 14	2 52 40
3	2 5 40	3 3 27	3 43 15	3 54 33	3 35 8	2 50 54
4	2 7 43	3 5 9	3 44 8	3 54 23	3 34 1	2 49 8
5	2 9 46	3 6 49	3 44 58	3 54 11	3 32 51	2 47 20
6	2 11 49	3 8 28	3 45 47	3 53 57	3 31 41	2 45 32
7	2 13 51	3 10 6	3 46 33	3 53 41	3 30 28	2 43 43
8	2 15 54	3 11 43	3 47 17	3 53 23	3 29 14	2 41 53
9	2 17 56	3 13 18	3 48 0	3 53 3	3 27 58	2 40 3
10	2 19 57	3 14 51	3 48 40	3 52 40	3 26 41	2 38 11
11	2 21 58	3 16 24	3 49 18	3 52 16	3 25 22	2 36 19
12	2 23 59	3 17 54	3 49 55	3 51 50	3 24 2	2 34 27
13	2 25 59	2 19 24	3 50 29	3 51 21	3 22 40	2 32 34
14	2 27 59	3 20 51	3 51 1	3 50 51	3 21 17	2 30 40
15	2 29 58	3 22 18	3 51 31	3 50 18	3 19 52	2 28 46
16	2 31 57	3 23 42	3 51 59	3 49 44	3 18 26	2 26 52
17	2 33 55	3 25 5	3 52 25	3 49 7	3 16 58	2 24 56
18	2 35 52	3 26 26	3 52 49	3 48 29	3 15 30	2 23 0
19	2 37 49	3 27 46	3 53 10	3 47 49	3 14 0	2 21 4
20	2 39 45	3 29 4	3 53 30	3 47 7	3 12 28	2 19 8
21	2 41 40	3 30 24	3 53 47	3 46 22	3 10 55	2 17 11
22	2 43 34	3 31 35	3 54 3	3 45 36	3 9 22	2 15 14
23	2 45 28	3 32 48	3 54 16	3 44 48	3 7 46	2 13 16
24	2 47 20	3 33 59	3 54 27	3 43 58	3 6 10	2 11 19
25	2 49 12	3 35 8	3 54 36	3 43 7	3 4 33	2 9 21
26	2 51 2	3 36 16	3 54 43	3 42 13	3 2 54	2 7 23
27	2 52 52	3 37 21	3 54 48	3 41 18	3 1 14	2 5 25
28	2 54 41	3 38 25	3 54 51	3 40 21	2 59 33	2 3 27
29	2 56 28	3 39 27	3 54 52	3 39 22	2 57 52	2 1 28
30	2 58 15	3 40 27	3 54 50	3 38 21	2 56 9	1 59 30

Equations of the Sun's Centre.

ARGUMENT. Sun's Mean Anomaly.

	VI^s	VII^s	VIII^s	IX^s	X^s	XI^s
0°	1°59′ 30″	1° 2′ 51′	0°20′ 39″	0° 4′ 10″	0°18′ 33″	1° 0′ 45″
1	1 57 32	1 1 8	0 19 38	0 4 8	0 19 33	1 2 32
2	1 55 33	0 59 27	0 18 39	0 4 9	0 20 35	1 4 19
3	1 53 35	0 57 46	0 17 42	0 4 12	0 21 39	1 6 8
4	1 51 37	0 56 6	0 16 47	0 4 17	0 22 44	1 7 58
5	1 49 39	0 54 27	0 15 53	0 4 24	0 23 52	1 9 48
6	1 47 41	0 52 47	0 15 2	0 4 33	0 25 1	1 11 40
7	1 45 44	0 51 14	0 14 12	0 4 44	0 26 12	1 13 32
8	1 43 46	0 49 38	0 13 24	0 4 57	0 27 25	1 15 26
9	1 41 49	0 48 5	0 12 38	0 5 13	0 28 40	1 17 20
10	1 39 52	0 46 32	0 11 53	0 5 30	0 29 56	1 19 15
11	1 37 56	0 45 0	0 11 11	0 5 50	0 31 14	1 21 11
12	1 36 0	0 43 30	0 10 31	0 6 11	0 32 34	1 23 8
13	1 34 4	0 42 1	0 9 53	0 6 35	0 33 55	1 25 5
14	1 32 9	0 40 34	0 9 16	0 7 1	0 35 18	1 27 3
15	1 30 14	0 39 8	0 8 42	0 7 29	0 36 42	1 29 2
16	1 28 20	0 37 43	0 8 9	0 7 59	0 38 9	1 31 1
17	1 26 26	0 36 20	0 7 39	0 8 31	0 39 36	1 33 1
18	1 24 33	0 34 58	0 7 10	0 9 5	0 41 6	1 35 1
19	1 22 41	0 33 38	0 6 44	0 9 42	0 42 36	1 37 1
20	1 20 49	0 32 19	0 6 20	0 10 20	0 44 9	1 39 3
21	1 18 57	0 31 2	0 5 57	0 11 0	0 45 42	1 41 4
22	1 17 7	0 29 46	0 5 37	0 11 43	0 47 17	1 43 6
23	1 15 17	0 28 32	0 5 19	0 12 27	0 48 54	1 45 9
24	1 13 28	0 27 19	0 5 3	0 13 13	0 50 32	1 47 11
25	1 11 40	0 26 9	0 4 49	0 14 2	0 52 11	1 49 14
26	1 9 52	0 24 59	0 4 37	0 14 52	0 53 51	1 51 17
27	1 8 6	0 23 52	0 4 27	0 15 45	0 55 33	1 53 20
28	1 6 20	0 22 46	0 4 19	0 16 39	0 57 16	1 55 23
29	1 4 35	0 21 41	0 4 13	0 17 35	0 59 0	1 57 27
30	1 2 51	0 20 39	0 4 10	0 18 33	1 0 45	1 59 30

TABLE XIV.

Small Equations of Sun's Longitude.

Arg.	I.	II.	III.	Arg.	I.	II.	III.
0	10″	10″	10″	500	10″	10″	10″
10	10	11	9	510	10	10	9
20	11	11	9	520	9	10	8
30	11	12	8	530	9	10	7
40	11	13	8	540	9	10	7
50	12	14	7	550	8	10	6
60	12	14	7	560	8	9	5
70	12	15	7	570	8	9	4
80	13	15	7	580	7	9	3
90	13	16	7	590	7	9	3
100	13	16	7	600	7	9	2
110	14	17	7	610	6	8	1
120	14	17	7	620	6	8	1
130	14	18	8	630	6	8	1
140	15	18	8	640	5	7	0
150	15	18	9	650	5	7	0
160	15	18	9	660	5	6	0
170	15	18	10	670	5	6	1
180	15	18	10	680	5	6	1
190	16	18	11	690	4	5	2
200	16	18	11	700	4	5	2
210	16	18	12	710	4	4	3
220	16	18	12	720	4	4	3
230	16	18	13	730	4	4	4
240	16	17	14	740	4	3	5
250	16	17	14	750	4	3	6
260	16	17	15	760	4	3	6
270	16	16	16	770	4	2	7
280	16	16	17	780	4	2	8
290	16	16	17	790	4	2	8
300	16	15	18	800	4	2	9
310	16	15	18	810	4	2	9
320	15	14	19	820	5	2	10
330	15	14	19	830	5	2	10
340	15	14	20	840	5	2	11
350	15	13	20	850	5	2	11
360	15	13	20	860	5	2	12
370	14	12	19	870	6	2	12
380	14	12	19	880	6	3	13
390	14	12	19	890	6	3	13
400	13	11	18	900	7	4	13
410	13	11	17	910	7	4	13
420	13	11	17	920	7	5	13
430	12	11	16	930	8	5	13
440	12	11	15	940	8	6	13
450	12	10	14	950	8	6	13
460	11	10	13	960	9	7	12
470	11	10	13	970	9	8	12
480	11	10	12	980	9	9	11
490	10	10	11	990	10	9	11
500	10	10	10	1000	10	10	10

TABLE XV.

Mean Obliquity of the Ecliptic.

Years.	M. Obliqu.
1821	23° 27′ 46″
1822	23 27 46
1823	23 27 45
1824	23 27 44
1825	23 27 44
1826	23 27 43
1827	23 27 43
1828	23 27 42
1829	23 27 42
1830	23 27 41
1831	23 27 41
1832	23 27 40
1833	23 27 40
1834	23 27 39
1835	23 27 39
1836	23 27 38
1837	23 27 38
1838	23 27 37
1839	23 27 37
1840	23 27 37
1841	23 27 36
1842	23 27 36
1843	23 27 35
1844	23 27 35
1845	23 27 34
1846	23 27 34
1847	23 27 33
1848	23 27 33
1849	23 27 32
1850	23 27 32
1851	23 27 31
1852	23 27 31
1853	23 27 31
1854	23 27 30
1855	23 27 30
1856	23 27 29
1857	23 27 29
1858	23 27 28
1859	23 27 28
1860	23 27 27
1861	23 27 27
1862	23 27 26
1863	23 27 26
1864	23 27 26
1865	23 27 25
1866	23 27 25
1867	23 27 24
1868	23 27 24
1869	23 27 23
1870	23 27 23
1882	23 27 17

TABLE XVI.

Nutations.

Argument. Supplement of the Node, or N.

N.	Long.	R. Asc.	Obliq.	N.	Long.	R. Asc.	Obliq.
0	+ 0″	+ 0″	+ 10″	500	— 0″	— 0″	— 10″
10	1	1	10	510	1	1	10
20	2	2	10	520	2	2	9
30	3	3	9	530	3	3	9
40	4	4	9	540	4	4	9
50	+ 6	+ 5	+ 9	550	— 6	— 5	— 9
60	7	6	9	560	7	6	9
70	8	7	9	570	8	7	8
80	9	8	8	580	9	8	8
90	10	9	8	590	10	9	8
100	+ 11	+ 10	+ 8	600	— 11	— 10	— 8
110	11	10	7	610	11	10	7
120	12	11	7	620	12	11	7
130	13	12	7	630	13	12	7
140	14	13	6	640	14	13	6
150	+ 15	+ 13	+ 6	650	— 15	— 13	— 6
160	15	14	5	660	15	14	5
170	16	14	5	670	16	14	5
180	16	15	4	680	16	15	4
190	17	15	3	690	17	15	3
200	+ 17	+ 16	+ 3	700	— 17	— 16	— 3
210	17	16	2	710	17	16	2
220	18	16	2	720	18	16	2
230	18	16	1	730	18	16	1
240	18	16	1	740	18	16	1
250	+ 18	+ 16	+ 0	750	— 18	— 16	— 0
260	18	16	— 1	760	18	16	+ 1
270	18	16	1	770	18	16	1
280	18	16	2	780	18	16	2
290	17	16	2	790	17	16	2
300	+ 17	+ 16	— 3	800	— 17	— 16	+ 3
310	17	15	3	810	17	15	3
320	16	15	4	820	16	15	4
330	16	14	5	830	16	14	5
340	15	14	5	840	15	14	5
350	+ 15	+ 13	— 6	850	— 15	— 13	+ 6
360	14	13	6	860	14	13	6
370	13	12	7	870	13	12	7
380	12	11	7	880	12	11	7
390	11	10	7	890	11	10	7
400	+ 11	+ 10	— 8	900	— 11	— 10	+ 8
410	10	9	8	910	10	9	8
420	9	8	8	920	9	8	8
430	8	7	9	930	8	7	9
440	7	6	9	940	7	6	9
450	+ 6	+ 5	— 9	950	— 6	— 5	+ 9
460	4	4	9	960	4	4	9
470	3	3	9	970	3	3	9
480	2	2	10	980	2	2	10
490	1	1	10	990	1	1	10
500	+ 0	+ 0	— 10	1000	— 0	— 0	+ 10

Equation of Time, to convert Apparent *into* Mean.

Argument. Sun's Mean Longitude.

	0^s	I^s	II^s	III^s	IV^s	V^s
°	m. sec.	m. sec.	m. sec.	m. sec.	m. sec.	m. sec.
0	+ 6 21	— 1 55	— 3 32	+ 1 50	+ 6 6	+ 2 17
1	6 2	2 6	3 26	2 3	6 7	2 1
2	5 43	2 17	3 20	2 16	6 7	1 45
3	5 25	2 27	3 13	2 29	6 7	1 28
4	5 6	2 37	3 6	2 42	6 6	1 11
5	4 47	2 46	2 59	2 54	6 5	0 53
6	4 28	2 55	2 51	3 6	6 3	0 35
7	4 9	3 3	2 42	3 18	6 0	+ 0 17
8	3 51	3 11	2 33	3 30	5 57	— 0 2
9	3 33	3 18	2 24	3 41	5 53	0 21
10	3 14	3 24	2 14	3 52	5 49	0 40
11	2 56	3 30	2 4	4 3	5 44	1 0
12	2 38	3 36	1 54	4 14	5 38	1 19
13	2 20	3 40	1 43	4 24	5 32	1 39
14	2 2	3 45	1 32	4 33	5 25	2 0
15	1 45	3 48	1 21	4 43	5 17	2 20
16	1 28	3 51	1 9	4 52	5 9	2 41
17	1 11	3 53	0 57	5 0	5 0	3 2
18	0 55	3 55	0 45	5 8	4 51	3 23
19	0 38	3 56	0 33	5 16	4 41	3 44
20	0 22	3 57	0 20	5 23	4 31	4 5
21	+ 0 7	3 57	— 0 7	5 30	4 20	4 26
22	— 0 8	3 57	+ 0 5	5 36	4 8	4 47
23	0 23	3 56	0 18	5 42	3 56	5 9
24	0 37	3 54	0 32	5 47	3 43	5 30
25	0 51	3 51	0 45	5 52	3 30	5 51
26	1 4	3 49	0 58	5 56	3 17	6 13
27	1 17	3 45	1 11	5 59	3 3	6 34
28	1 30	3 41	1 24	6 2	2 48	6 56
29	1 43	3 37	1 37	6 4	2 33	7 17
30	— 1 55	— 3 32	+ 1 50	+ 6 6	+ 2 17	— 7 38

Equation of Time, to convert Apparent *into* Mean.

ARGUMENT. Sun's Mean Longitude.

	VI^s	VII^s	VIII^s	IX^s	X^s	XI^s
°	m. sec.	m. sec.	m. sec.	m. sec.	m. sec.	m. sec.
0	— 7 38	— 15 35	— 13 24	— 0 27	+ 12 7	+ 13 50
1	7 59	15 43	13 7	+ 0 4	12 22	13 42
2	8 19	15 49	12 48	0 34	12 36	13 33
3	8 40	15 56	12 29	1 4	12 50	13 23
4	9 1	16 1	12 10	1 35	13 3	13 13
5	9 21	16 5	11 49	2 4	13 15	13 2
6	9 41	16 9	11 28	2 34	13 27	12 51
7	10 0	16 12	11 6	3 4	13 37	12 39
8	10 20	16 14	10 43	3 33	13 47	12 26
9	10 39	16 15	10 20	4 2	13 55	12 13
10	10 58	16 16	9 56	4 30	14 3	12 0
11	11 16	16 15	9 31	4 59	14 10	11 46
12	11 34	16 14	9 6	5 26	14 16	11 32
13	11 52	16 12	8 40	5 54	14 22	11 17
14	12 9	16 9	8 14	6 21	14 26	11 2
15	12 26	16 5	7 47	6 47	14 30	10 46
16	12 43	16 1	7 20	7 13	14 33	10 30
17	12 59	15 55	6 52	7 38	14 35	10 14
18	13 14	15 49	6 24	8 3	14 36	9 58
19	13 29	15 42	5 56	8 27	14 36	9 41
20	13 44	15 33	5 27	8 50	14 35	9 23
21	13 58	15 24	4 58	9 13	14 34	9 6
22	14 11	15 14	4 28	9 35	14 32	8 48
23	14 24	15 3	3 59	9 57	14 30	8 31
24	14 36	14 52	3 29	10 18	14 26	8 13
25	14 48	14 39	2 59	10 38	14 22	7 54
26	14 58	14 26	2 28	10 57	14 17	7 36
27	15 9	14 12	1 58	11 16	14 11	7 17
28	15 18	13 57	1 28	11 33	14 5	6 59
29	15 27	13 41	0 57	11 50	13 58	6 40
30	— 15 35	— 13 24	— 0 27	+ 12 7	+ 13 50	+ 6 21

Earth's Radius Vector.

ARGUMENT. Sun's Mean Anomaly.

	0^s	I^s	II^s	III^s	IV^s	V^s	
0°	0.98313	0.98545	0.99173	1.00018	1.00850	1.01450	30°
1	0.98313	0.98560	0.99199	1.00047	1.00874	1.01464	29
2	0.98314	0.98576	0.99225	1.00077	1.00899	1.01477	28
3	0.98315	0.98592	0.99251	1.00106	1.00923	1.01490	27
4	0.98317	0.98608	0.99278	1.00135	1.00947	1.01503	26
5	0.98319	0.98625	0.99304	1.00164	1.00971	1.01515	25
6	0.98322	0.98643	0.99331	1.00193	1.00994	1.01527	24
7	0.98326	0.98661	0.99359	1.00222	1.01017	1.01538	23
8	0.98330	0.98679	0.99386	1.00251	1.01040	1.01549	22
9	0.98334	0.98698	0.99414	1.00280	1.01062	1.01560	21
10	0.98339	0.98717	0.99441	1.00308	1.01084	1.01569	20
11	0.98344	0.98736	0.99469	1.00337	1.01106	1.01579	19
12	0.98350	0.98756	0.99497	1.00366	1.01128	1.01588	18
13	0.98357	0.98777	0.99526	1.00394	1.01149	1.01596	17
14	0.98364	0.98797	0.99554	1.00422	1.01170	1.01604	16
15	0.98372	0.98818	0.99582	1.00450	1.01190	1.01612	15
16	0.98380	0.98840	0.99611	1.00478	1.01210	1.01619	14
17	0.98388	0.98861	0.99640	1.00506	1.01230	1.01626	13
18	0.98397	0.98883	0.99668	1.00534	1.01249	1.01632	12
19	0.98407	0.98906	0.99697	1.00561	1.01268	1.01638	11
20	0.98417	0.98929	0.99726	1.00588	1.01286	1.01643	10
21	0.98428	0.98952	0.99755	1.00615	1.01304	1.01647	9
22	0.98439	0.98975	0.99784	1.00642	1.01322	1.01652	8
23	0.98450	0.98999	0.99813	1.00669	1.01340	1.01655	7
24	0.98462	0.99023	0.99843	1.00695	1.01357	1.01659	6
25	0.98475	0.99047	0.99872	1.00722	1.01373	1.01661	5
26	0.98488	0.99072	0.99901	1.00748	1.01389	1.01663	4
27	0.98501	0.99096	0.99930	1.00774	1.01405	1.01665	3
28	0.98515	0.99122	0.99960	1.00799	1.01420	1.01666	2
29	0.98530	0.99147	0.99989	1.00825	1.01435	1.01667	1
30	0.98545	0.99173	1.00018	1.00850	1.01450	1.01667	0
	XI^s	X^s	IX^s	VIII^s	VII^s	VI^s	

Perturbations of Earth's Radius Vector.

Arg.	I.	II.	III.	Arg.	I.	II.	III.
0	8	4	3	500	2	0	4
50	8	4	3	550	2	1	4
100	7	4	2	600	3	1	3
150	7	4	1	650	3	2	2
200	6	4	0	700	4	3	1
250	5	4	0	750	5	4	0
300	4	3	1	800	6	4	0
350	3	2	2	850	7	4	1
400	3	1	3	900	7	4	2
450	2	1	4	950	8	4	3
500	2	0	4	1000	8	4	3

Mean Right Ascensions and Declinations of some of the Fixed Stars, for the beginning of 1820, *with their Annual Variations.*

Names and Magnitude.		Right Asc.	An. Var.	Declination.	An. Var.
	Mag.	° ′ ″	″	° ′ ″	′
γ Pegasi, - -	2	0 59 35	+46.1	14 10 56N.	+20.0
α Polaris, -	2.3	14 13 7	216.4	88 20 55N.	+19.4
α Arietis, -	3	29 15 38	50.3	22 36 23N.	+17.3
α Ceti, - - -	2	43 13 8	46.7	3 22 39N.	+14.5
γ Tauri, - -	2	62 23 23	51.0	15 11 3N.	+ 9.2
Aldebaran, -	1	66 24 0	51.4	16 8 19N.	+ 7.8
Capella, -	1	75 51 7	66.3	45 48 8N.	+ 4.5
Rigel, - -	1	76 28 21	51.8	8 25 2S.	— 4.7
β Tauri, - -	2	78 43 47	56.7	28 26 42N.	+ 3.8
ζ Tauri, - -	3	81 43 14	53.6	21 1 23N.	+ 2.8
η Geminorum, -	2.3	91 0 6	54.3	22 32 56N.	— 0.4
μ Geminorum, -	3	93 1 0	54.5	22 35 48N.	— 1.1
γ Geminorum,	2.3	96 49 37	52.0	16 32 36N.	— 2.5
Sirius, - -	1	99 18 18	39.8	16 28 33S.	+ 4.4
δ Geminorum, -	3	107 20 25	53.8	22 18 15N.	— 6.0
Procyon, -	1.2	112 28 2	47.1	5 40 46N.	— 8.6
Pollux, - -	2.3	113 34 16	55.3	28 27 7N.	— 8.0
ε Leonis, - -	3	149 22 33	49.2	17 38 10N.	—17.3
Regulus, -	1	149 41 39	48.1	12 50 36N.	—17.3
β Virginis, - -	3	175 19 45	46.9	2 46 44N.	—20.3
γ Virginis, -	3	188 8 4	45.3	0 27 38S.	+20.0
α Virginis, - -	1	198 55 30	47.2	10 13 5S.	+19.0
Arcturus, -	1	211 51 45	40.9	20 7 28N.	—19.0
α 2 Libræ, -	2.3	220 14 2	49.5	15 17 13S.	+15.4
δ Scorpii, - -	3	237 25 37	52.9	22 5 59S.	+10.9
β Scorpii, - -	2	238 44 48	52.0	19 18 13S.	+10.5
Antares, -	1	244 35 49	54.9	26 1 21S.	+ 8.7
α Lyræ, - -	1	277 42 37	30.4	38 37 19N.	+ 2.9
σ Sagittarii, -	2.3	281 1 28	55.8	26 30 34S.	— 3.7
π Sagittarii, - -	3	284 45 47	53.5	21 18 0S.	— 5.0
α 1 Capricorni, -	3.4	301 54 56	50.0	13 3 22S.	—1 05
α 2 Capricorni, -	3	302 0 53	50.0	13 5 33S.	—10.8
β Capricorni, -	3	302 43 18	50.6	15 20 29S.	—10.8
γ Capricorni,	3.4	322 31 32	50.0	17 28 10S.	—15.8
δ Capricorni, -	3	324 16 24	49.8	16 56 16S.	—16.0
Fomalhaut, -	1	341 55 14	50.1	30 34 25S.	—18.8

Moon's Epochs.

Years.	1	2	3	4	5	6	7	8	9
1821	0027	8365	5389	1368	6970	7714	6319	7024	7800
1822	0020	5573	5054	6112	9441	3512	7380	9481	6664
1823	0012	2782	4720	0856	1913	9309	8440	1938	5528
1824 B.	0033	0640	5426	5887	4720	5478	9559	4787	4417
1825	0026	7849	5092	0631	7192	1276	0619	7243	3280
1826	0018	5057	4758	5375	9663	7073	1680	9701	2144
1827	0011	2265	4424	0119	2135	2871	2740	2158	1008
1828 B.	0032	0124	5129	5150	4942	9040	3859	5007	9896
1829	0024	7332	4795	9894	7414	4837	4919	7463	8760
1830	0017	4541	4461	4638	9885	0635	5979	9921	7623
1831	0010	1749	4127	9381	2357	6432	7040	2378	6487
1832 B.	0030	9607	4833	4412	5164	2601	8158	5226	5376
1833	0023	6816	4499	9156	7636	8399	9219	7683	4239
1834	0016	4024	4164	3900	0107	4196	0279	0140	3103
1835	0009	1232	3830	8644	2579	9993	1340	2598	1967
1836 B.	0029	9091	4536	3675	5386	6163	2458	5446	0856
1837	0022	6299	4202	8419	7858	1960	3518	7903	9719
1838	0015	3508	3868	3163	0329	7757	4579	0360	8583
1839	0008	0716	3534	7907	2801	3555	5639	2818	7447
1840 B.	0028	8575	4239	2938	5608	9724	6758	5666	6335
1841	0021	5783	3906	7682	8080	5522	7818	8123	5199
1842	0014	2991	3571	2425	0551	1319	8879	0580	4062
1843	0007	0200	3237	7169	3023	7116	9939	3038	2926
1844 B.	0027	8058	3943	2200	5830	3286	1058	5886	1815
1845	0020	5266	3609	6944	8302	9083	2118	8343	0678
1846	0013	2475	3275	1688	0773	4880	3179	0800	9542
1847	0006	9683	2941	6432	3245	0678	4239	3257	8406
1848 B.	0026	7542	3646	1463	6052	6847	5358	6106	7295
1849	0019	4750	3312	6207	8524	2644	6418	8563	6158
1850	0012	1958	2978	0951	0995	8442	7479	1020	5022
1851	0005	9167	2644	5695	3467	4239	8539	3477	3885
1852 B.	0025	7025	3350	0726	6274	0408	9658	6326	2774
1853	0018	4233	3016	5469	8746	6206	0718	8782	1637
1854	0011	1442	2681	0213	1217	2003	1778	1240	0501
1855	0004	8650	2347	4957	3689	7801	2839	3697	9365
1856 B.	0024	6509	3053	9988	6496	3970	3957	6546	8254
1857	0017	3717	2719	4732	8968	9767	5018	9002	7117
1858	0010	0925	2385	9476	1439	5565	6078	1460	5981
1859	0003	8134	2051	4220	3911	1362	7139	3917	4845
1860 B.	0023	5992	2756	9251	6718	7531	8257	6765	3734
1861	0016	3200	2423	3995	9190	3329	9317	9222	2597
1862	0009	0409	2088	8739	1661	9126	0378	1679	1461
1863	0002	7617	1754	3483	4133	4923	1438	4137	0324
1864 B.	0022	5476	2460	8514	6941	1093	2557	6984	9212
1865	0015	2684	2126	3257	9412	6890	3617	9442	8076
1866	0008	9893	1792	8001	1883	2687	4678	1899	6940
1867	0001	7101	1457	2745	4355	8485	5738	4357	5804
1868 B.	0021	4959	2163	7776	7163	4654	6857	7204	4692
1869	0014	2168	1829	2520	9634	0452	7917	9662	3556
1870	0007	9376	1495	7264	2105	6249	8978	2119	2420

TABLE XX.

Moon's Epochs.

Years.	10	11	12	13	14	15	16	17	18	19	20
1821	620	917	842	142	979	067	923	331	134	036	036
1822	226	278	562	615	172	208	282	684	609	090	202
1823	833	639	281	088	366	348	641	036	084	143	369
1824 B.	509	030	070	595	659	519	037	431	585	197	537
1825	116	391	790	068	853	659	397	783	060	251	703
1826	722	752	510	541	047	800	756	136	536	304	869
1827	329	113	229	014	241	940	115	488	011	358	036
1828 B.	005	505	019	521	533	111	511	883	512	412	204
1829	612	866	738	994	727	251	871	235	987	466	370
1830	219	226	458	468	921	392	230	588	462	519	536
1831	825	587	177	940	115	532	589	940	937	573	703
1832 B.	502	979	967	447	408	704	985	335	438	627	871
1833	108	340	687	920	602	844	345	688	913	681	037
1834	715	701	406	393	796	984	704	040	388	734	203
1835	321	061	125	866	989	124	063	393	863	788	370
1836 B.	998	453	915	373	282	296	459	787	364	842	538
1837	605	814	635	846	476	436	819	140	840	895	704
1838	211	175	354	319	670	576	178	492	315	949	870
1839	818	536	074	792	864	716	537	845	790	003	037
1840 B.	494	927	863	299	157	888	933	239	291	056	205
1841	101	288	583	772	351	028	293	592	766	110	371
1842	707	649	302	245	544	168	652	944	241	164	537
1843	314	010	022	718	738	308	012	297	716	218	704
1844 B.	990	402	811	225	031	480	407	691	217	272	872
1845	597	763	531	698	225	620	767	044	692	325	038
1846	203	123	250	171	419	760	126	396	167	379	204
1847	810	484	970	644	613	901	486	749	643	433	371
1848 B.	486	876	759	151	905	072	881	143	144	487	539
1849	093	237	479	624	099	212	241	496	619	540	705
1850	700	597	199	097	293	352	600	848	094	594	871
1851	306	958	918	570	487	493	960	201	569	648	038
1852 B.	983	350	707	077	780	664	355	595	070	701	206
1853	589	711	427	550	974	804	715	948	545	755	372
1854	196	072	147	023	168	944	074	300	020	809	539
1855	802	432	866	496	361	085	434	653	495	863	705
1856 B.	479	824	656	003	654	256	829	047	996	916	873
1857	086	185	375	476	848	396	189	400	471	970	039
1858	692	546	095	949	042	537	548	752	947	024	206
1859	299	907	814	422	236	677	908	105	422	078	372
1860 B.	975	298	604	929	529	848	303	499	923	131	540
1861	581	659	323	402	723	988	662	852	398	185	706
1862	187	020	042	875	916	129	021	204	873	239	873
1863	794	381	761	348	110	269	381	557	348	292	039
1864 B.	470	773	551	855	403	440	777	951	849	346	207
1865	077	134	271	328	597	580	136	304	324	400	373
1866	684	494	990	801	791	721	495	657	799	458	540
1867	290	855	710	274	985	861	855	009	274	507	707
1868 B.	967	247	500	781	277	032	251	404	775	561	874
1869	573	608	219	254	471	172	610	756	251	615	040
1870	180	968	939	727	665	313	969	109	726	668	207

Moon's Epochs.

Years.	Evection.	Anomaly.	Variation.	Longitude.
1821	1s 19° 43′ 47″	8s 9° 54′ 17″	10s 22° 4′ 2″	8s 2° 7′ 41″
1822	7 10 15 16	11 8 37 37	3 1 41 27	0 11 30 46
1823	1 0 46 45	2 7 20 57	7 11 18 51	4 20 53 51
1824 B.	7 2 37 15	5 19 8 11	0 3 7 43	9 13 27 31
1825	0 23 8 44	8 17 51 31	4 12 45 7	1 22 50 37
1826	6 13 40 14	11 16 34 50	8 22 22 32	6 2 13 42
1827	0 4 11 44	2 15 18 10	1 1 59 56	10 11 36 47
1828 B.	6 6 2 13	5 27 5 24	5 23 48 49	3 4 10 27
1829	11 26 33 43	8 25 48 44	10 3 26 13	7 13 33 32
1830	5 17 5 12	11 24 32 4	2 13 3 38	11 22 56 37
1831	11 7 36 41	2 23 16 24	6 22 41 3	4 2 19 42
1832 B.	5 9 27 11	6 5 2 38	11 14 29 54	8 24 53 23
1833	10 29 58 40	9 3 45 58	3 24 7 20	1 4 16 28
1834	4 20 30 11	0 2 29 18	8 3 44 44	5 13 39 33
1835	10 11 1 40	3 1 12 38	0 13 22 9	9 23 2 38
1836 B.	4 12 52 9	6 12 59 52	5 5 11 0	2 15 36 19
1837	10 3 23 39	9 11 43 12	9 14 48 26	6 24 59 24
1838	3 23 55 9	0 10 26 32	1 24 25 50	11 4 22 29
1839	9 14 26 38	3 9 9 53	6 4 3 15	3 13 45 35
1840 B.	3 16 17 8	6 20 57 7	10 25 52 8	8 6 19 15
1841	9 6 48 37	9 19 40 27	3 5 29 32	0 15 42 21
1842	2 27 20 7	0 18 23 47	7 15 6 57	4 25 5 26
1843	8 17 51 37	3 17 7 7	11 24 44 22	9 4 28 31
1844 B.	2 19 42 7	6 28 54 22	4 16 33 14	1 27 2 12
1845	8 10 13 36	9 27 37 42	8 26 10 39	6 6 25 17
1846	2 0 45 6	0 26 21 2	1 5 48 4	10 15 48 23
1847	7 21 16 35	3 25 4 23	5 15 25 29	2 25 11 28
1848 B.	1 23 7 5	7 6 51 37	10 7 14 21	7 17 45 8
1849	7 13 38 35	10 5 34 57	2 16 51 46	11 27 8 14
1850	1 4 10 4	1 4 18 18	6 26 29 11	4 6 31 20
1851	6 24 41 35	4 3 1 38	11 6 6 36	8 15 54 25
1852 B.	0 26 32 5	7 14 48 53	3 27 55 29	1 8 28 6
1853	6 17 3 34	10 13 32 13	8 7 32 53	5 17 51 11
1854	0 7 35 4	1 12 15 34	0 17 10 19	9 27 14 17
1855	5 28 6 33	4 10 58 54	4 26 47 43	2 6 37 22
1856 B.	11 29 57 3	7 22 46 9	9 18 36 36	6 29 11 3
1857	5 20 28 33	10 21 29 29	1 28 14 1	11 8 34 9
1858	11 11 0 2	1 20 12 50	6 7 51 26	3 17 57 14
1859	5 1 31 33	4 18 56 10	10 17 28 52	7 27 20 20
1860 B.	11 3 22 3	8 0 43 25	3 9 17 44	0 19 54 0
1861	4 23 53 33	10 29 26 45	7 18 55 9	4 29 17 6
1862	10 14 25 3	1 28 10 6	11 28 32 34	9 8 40 12
1863	4 4 56 33	4 26 53 27	4 8 10 0	1 18 3 18
1864 B.	10 6 47 2	8 8 40 41	8 29 58 51	6 10 36 58
1865	3 27 18 32	11 7 24 2	1 9 36 17	10 20 0 4
1866	9 17 50 2	2 6 7 23	5 19 13 42	2 29 23 10
1867	3 8 21 32	5 4 50 43	9 28 51 8	7 8 46 15
1868 B.	9 10 12 2	8 16 37 58	2 20 40 0	0 1 19 56
1869	3 0 43 33	11 15 21 19	7 0 17 25	4 10 43 2
1870	8 21 15 2	2 14 4 40	11 9 54 50	8 20 6 8

Moon's Epochs.

Years.	Supp. of Node.	II.	V.	VI.	VII.	VIII.	IX.	X.
1821	0^s 13° 3′ 29″	0^s 27° 41′	706	711	074	079	637	596
1822	1 2 23 11	4 18 13	120	124	382	386	717	536
1823	1 21 42 56	8 8 45	533	536	689	692	796	475
1824 B.	2 11 5 47	0 10 26	981	988	026	032	912	420
1825	3 0 25 29	4 0 58	395	401	333	338	992	359
1826	3 19 45 11	7 21 30	809	813	641	645	072	299
1827	4 9 4 53	11 12 2	223	225	949	951	151	238
1828 B.	4 28 27 46	3 13 43	670	677	285	291	267	182
1829	5 17 47 29	7 4 15	084	090	592	597	347	122
1830	6 7 7 11	10 24 47	498	502	900	904	427	062
1831	6 26 26 53	2 15 19	912	914	208	210	506	001
1832 B.	7 15 49 46	6 17 0	360	366	545	550	622	945
1833	8 5 9 28	10 7 32	774	779	852	856	702	885
1834	8 24 29 11	1 28 4	187	191	159	163	782	825
1835	9 13 48 53	5 18 36	601	603	467	469	861	764
1836 B.	10 3 11 46	9 20 18	048	055	804	809	977	708
1837	10 22 31 28	1 10 50	463	468	111	116	057	648
1838	11 11 51 10	5 1 22	876	880	419	423	137	588
1839	0 1 10 52	8 21 54	290	292	726	729	217	527
1840 B.	0 20 33 45	0 23 35	738	744	063	069	332	471
1841	1 9 53 28	4 14 7	152	157	370	375	412	411
1842	1 29 13 10	8 4 39	566	569	678	682	492	350
1843	2 18 32 52	11 25 11	980	980	986	988	572	290
1844 B.	3 7 55 45	3 26 52	427	433	322	328	687	234
1845	3 27 15 27	7 17 24	840	846	629	634	767	174
1846	4 16 35 9	11 7 56	254	258	937	941	847	113
1847	5 5 54 52	2 28 38	668	670	245	247	927	053
1848 B.	2 25 17 45	7 0 9	116	122	582	587	042	997
1849	6 14 37 27	10 20 41	531	535	889	893	122	937
1850	7 3 57 9	2 11 13	944	947	196	200	202	876
1851	7 23 16 51	6 1 45	358	359	504	506	282	816
1852 B.	8 12 39 44	10 3 27	806	811	841	846	398	760
1853	9 1 59 26	1 23 59	220	223	148	152	477	700
1854	9 21 19 9	5 14 31	634	636	456	459	557	639
1855	10 10 38 51	9 5 3	047	048	763	765	637	579
1856 B.	11 0 1 44	1 6 44	495	500	100	105	753	523
1857	11 19 21 26	4 27 16	909	912	407	411	832	463
1858	0 8 41 8	8 17 48	323	325	715	718	912	402
1859	0 28 0 51	0 8 20	736	737	023	024	992	342
1860 B.	1 17 23 43	4 10 1	184	189	359	364	108	286
1861	2 6 43 27	8 0 33	598	601	666	670	187	226
1862	2 26 3 9	11 21 5	012	014	974	977	267	165
1863	3 15 23 11	3 11 37	426	426	282	283	347	105
1864 B.	4 4 45 44	7 13 18	873	878	618	623	463	049
1865	4 24 5 46	11 3 50	287	291	926	929	542	989
1866	5 13 25 28	2 24 22	701	703	233	236	622	928
1867	6 2 45 10	6 14 54	115	115	541	542	702	868
1868 B.	6 22 7 43	10 16 36	563	567	877	882	818	812
1869	7 11 27 46	2 7 8	977	980	185	188	897	752
1870	8 0 47 28	5 27 40	390	392	493	495	977	691

Moon's Motions for Months.

Months.		1	2	3	4	5	6	7	8	9
Jan.	Com.	0000	0000	0000	0000	0000	0000	0000	0000	0000
	Bis.	9973	9350	8960	9713	9664	9628	9942	9610	9976
Feb.	Com.	849	146	2246	8896	402	1533	1789	2099	753
	Bis.	821	9497	1205	8609	66	1161	1731	1709	729
March, -		1615	8343	1371	6931	9797	1951	3404	3027	1433
April, -		2464	8490	3616	5827	199	3484	5193	5126	2186
May, -		3285	7986	4822	4436	265	4646	6924	6835	2914
June, -		4134	8133	7067	3332	666	6179	8713	8934	3667
July, -		4955	7629	8273	1942	732	7341	444	643	4396
August, -		5804	7776	518	838	1134	8874	2233	2742	5148
September,		6653	7922	2764	9734	1536	408	4021	4842	5901
October, -		7474	7419	3969	8343	1602	1569	5752	6550	6630
November,		8323	7565	6215	7239	2004	3102	7541	8649	7382
December,		9144	7062	7420	5848	2070	4264	9272	358	8111

TABLE XXI.

Moon's Motions for Months.

Months.		Evection.	Anomaly.	Variation.	M. Long.
Jan.	Com.	0s 0° 0′ 0″	0s 0° 0′ 0″	0s 0° 0′ 0″	0s 0° 0′ 0″
	Bis.	11 18 41 1	11 16 56 6	11 17 48 33	11 16 49 25
Feb.	Com.	11 20 48 42	1 15 0 53	0 17 54 48	1 18 28 6
	Bis.	11 9 29 43	1 1 56 59	0 5 43 21	1 5 17 31
March, -		10 7 40 26	1 20 50 4	11 29 15 15	1 27 24 27
April, -		9 28 29 8	3 5 50 57	0 17 10 3	3 15 52 32
May, -		9 7 58 51	4 7 47 56	0 22 53 24	4 21 10 3
June, -		8 28 47 33	5 22 48 49	1 10 48 11	6 9 38 9
July, -		8 8 17 16	6 24 45 48	1 16 31 32	7 14 55 40
August,		7 29 5 59	8 9 46 42	2 4 26 20	9 3 23 46
September,		7 19 54 41	9 24 47 35	2 22 21 7	10 21 51 52
October,		6 29 24 24	10 26 44 34	2 28 4 28	11 27 9 22
November,		6 20 13 6	0 11 45 27	3 15 59 16	1 15 37 28
December,		5 29 42 49	1 13 42 26	3 21 42 37	2 20 54 59

Moon's Motions for Months.

Months.		10	11	12	13	14	15	16	17	18	19	20
Jan.	Com.	000	000	000	000	000	000	000	000	000	000	000
	Bis.	930	969	930	966	901	969	963	958	974	000	000
Feb.	Com.	175	965	184	59	74	946	135	304	805	5	14
	Bis.	105	934	114	25	975	916	98	262	779	5	14
March, - - -		139	836	157	16	851	801	159	482	532	9	27
April, - - -		314	801	342	76	925	747	294	786	336	13	41
May, - - - -		419	735	456	101	899	663	392	47	115	18	55
June, - - -		593	700	640	160	973	609	527	351	920	22	69
July, - - - -		698	634	754	185	948	525	625	613	699	27	83
August, - - -		873	599	938	245	22	471	759	917	503	31	97
September, - -		48	563	123	304	96	417	894	221	308	36	111
October, - -		152	497	237	329	71	333	992	483	87	40	125
November, - -		327	462	421	388	145	279	127	787	892	45	139
December, - -		432	396	535	414	120	194	225	49	670	49	153

TABLE XXI.

Moon's Motions for Months.

Months.		Supp. of Node.				II.			V.	VI.	VII.	VIII.	IX.	X.
Jan.	Com.	0^s	0°	0′	0″	0^s	0°	0′	000	000	000	000	000	000
	Bis.	11	29	56	49	11	18	51	966	961	972	966	964	995
Feb.	Com.	0	1	38	30	11	15	43	54	224	875	45	111	165
	Bis.	0	1	35	19	11	4	34	20	185	847	11	75	159
March, - -		0	3	7	27	9	27	59	7	330	666	989	114	313
April, - -		0	4	45	57	9	13	42	61	554	542	34	225	478
May, - -		0	6	21	16	8	18	15	81	738	389	46	300	638
June, - -		0	7	59	46	8	3	58	136	962	264	91	411	802
July, - - -		0	9	35	5	7	8	32	156	147	112	103	486	962
August, -		0	11	13	35	6	24	15	210	371	987	147	497	126
September, -		0	12	52	5	6	9	58	265	595	862	193	708	291
October, -		0	14	27	24	5	14	32	285	780	710	204	783	451
November, -		0	16	5	53	5	0	15	339	4	585	250	894	615
December,		0	17	41	13	4	4	49	359	188	432	261	969	775

Moon's Motions for Days.

Days	1	2	3	4	5	6	7	8	9
1	0000	0000	0000	0000	0000	0000	0000	0000	0000
2	27	650	1040	287	336	372	58	390	24
3	55	1300	2080	574	671	744	115	781	49
4	82	1950	3121	861	1007	1116	173	1171	73
5	109	2600	4161	1148	1342	1488	231	1561	97
6	137	3249	5201	1435	1678	1860	289	1952	121
7	164	3899	6241	1722	2013	2232	346	2342	146
8	192	4549	7281	2009	2349	2604	404	2732	170
9	219	5199	8321	2296	2684	2976	462	3122	194
10	246	5849	9362	2583	3020	3348	519	3513	219
11	274	6499	402	2870	3355	3720	577	3903	243
12	301	7149	1442	3157	3691	4093	635	4293	267
13	328	7799	2482	3444	4026	4465	692	4684	291
14	356	8449	3522	3731	4362	4837	750	5074	316
15	383	9098	4563	4018	4698	5209	808	5464	340
16	411	9748	5603	4305	5033	5581	866	5854	364
17	438	398	6643	4592	5369	5953	923	6245	389
18	465	1048	7683	4878	5704	6325	981	6635	413
19	493	1698	8723	5165	6040	6697	1039	7025	437
20	520	2348	9763	5452	6375	7069	1096	7416	461
21	548	2998	804	5739	6711	7441	1154	7806	486
22	575	3648	1844	6026	7046	7813	1212	8196	510
23	602	4298	2884	6313	7382	8185	1269	8586	534
24	630	4947	3924	6600	7717	8557	1327	8977	559
25	657	5597	4964	6887	8053	8929	1385	9367	583
26	684	6247	6005	7174	8389	9301	1443	9757	607
27	712	6897	7045	7461	8724	9673	1500	148	631
28	739	7547	8085	7748	9060	45	1558	538	656
29	767	8197	9125	8035	9395	417	1616	928	680
30	794	8847	165	8322	9731	789	1673	1319	704
31	821	9497	1205	8609	66	1161	1731	1709	729

Moon's Motions for Days.

Days.	10	11	12	13	14	15	16	17	18	19	20
1	000	000	000	000	000	000	000	000	000	000	000
2	70	31	70	34	99	31	37	42	26	0	0
3	140	62	141	68	198	61	73	84	52	0	1
4	210	93	211	103	297	92	110	126	78	0	1
5	281	125	282	137	397	122	146	168	104	1	2
6	351	156	352	171	496	153	183	210	130	1	2
7	421	187	423	205	595	183	220	252	156	1	3
8	491	218	493	239	694	214	256	294	182	1	3
9	561	249	564	273	793	244	293	336	208	1	4
10	631	280	634	308	892	275	329	379	234	1	4
11	702	311	705	342	992	305	366	421	260	1	5
12	772	342	775	376	91	336	403	463	286	2	5
13	842	374	845	410	190	366	439	505	312	2	5
14	912	405	916	444	289	397	476	547	337	2	6
15	982	436	986	478	388	427	512	589	363	2	6
16	52	467	57	513	487	458	549	631	389	2	7
17	122	498	127	547	587	488	586	673	415	2	7
18	193	529	198	581	686	519	622	715	441	2	8
19	263	560	268	615	785	549	659	757	467	3	8
20	333	591	339	649	884	580	695	799	493	3	9
21	403	623	409	683	983	611	732	841	519	3	9
22	473	654	480	718	82	641	769	883	545	3	10
23	543	685	550	752	182	672	805	925	571	3	10
24	614	716	621	786	281	702	842	967	597	3	11
25	684	747	691	820	380	733	878	9	623	4	11
26	754	778	762	854	479	763	915	52	649	4	11
27	824	809	832	888	578	794	952	94	675	4	12
28	894	840	903	923	677	824	988	136	701	4	12
29	964	872	973	957	777	855	25	178	727	4	13
30	34	903	43	991	876	885	61	220	753	4	13
31	105	934	114	25	975	916	98	262	779	4	14

Moon's Motions for Days.

Days	Evection.	Anomaly.	Variation.	M. Long.
1	0s 0° 0′ 0″	0s 0° 0′ 0″	0s 0° 0′ 0″	0s 0° 0′ 0″
2	0 11 18 59	0 13 3 54	0 12 11 27	0 13 10 35
3	0 22 37 59	0 26 7 48	0 24 22 53	0 26 21 10
4	1 3 56 58	1 9 11 42	1 6 34 20	1 9 31 45
5	1 15 15 58	1 22 15 36	1 18 45 47	1 22 42 20
6	1 26 34 57	2 5 19 30	2 0 57 13	2 5 52 55
7	2 7 53 57	2 18 23 24	2 13 8 40	2 19 3 30
8	2 19 12 56	3 1 27 18	2 25 20 7	3 2 14 5
9	3 0 31 55	3 14 31 12	3 7 31 34	3 15 24 40
10	3 11 50 55	3 27 35 6	3 19 43 0	3 28 35 15
11	3 23 9 54	4 10 39 0	4 1 54 27	4 11 45 50
12	4 4 28 54	4 23 42 54	4 14 5 54	4 24 56 25
13	4 15 47 53	5 6 46 48	4 26 17 20	5 8 7 0
14	4 27 6 53	5 19 50 42	5 8 28 47	5 21 17 35
15	5 8 25 52	6 2 54 36	5 20 40 14	6 4 28 10
16	5 19 44 51	6 15 58 29	6 2 51 40	6 17 38 45
17	6 1 3 51	6 29 2 23	6 15 3 7	7 0 49 20
18	6 12 22 50	7 12 6 17	6 27 14 34	7 13 59 55
19	6 23 41 50	7 25 10 11	7 9 26 1	7 27 10 30
20	7 5 0 49	8 8 14 5	7 21 37 27	8 10 21 5
21	7 16 19 49	8 21 17 59	8 3 48 54	8 23 31 40
22	7 27 38 48	9 4 21 53	8 16 0 21	9 6 42 16
23	8 8 57 47	9 17 25 47	8 28 11 47	9 19 52 51
24	8 20 16 47	10 0 29 41	9 10 23 14	10 3 3 26
25	9 1 35 46	10 13 33 35	9 22 34 41	10 16 14 1
26	9 12 54 46	10 26 37 29	10 4 46 7	10 29 24 36
27	9 24 13 45	11 9 41 23	10 16 57 34	11 12 35 11
28	10 5 32 45	11 22 45 17	10 29 9 1	11 25 45 46
29	10 16 51 44	0 5 49 11	11 11 20 28	0 8 56 21
30	10 28 10 43	0 18 53 5	11 23 31 54	0 22 6 56
31	11 9 29 43	1 1 56 59	0 5 43 21	1 5 17 31

Moon's Motions for Days.

Days	Sup. of Node.	II.	V.	VI.	VII.	VIII.	IX.	X.
1	0° 0′ 0″	0s 0° 0′	000	000	000	000	000	000
2	0 3 11	11 9	34	39	28	34	36	5
3	0 6 21	22 18	68	79	56	67	72	11
4	0 9 32	1 3 27	102	118	85	101	108	16
5	0 12 52	1 14 37	136	158	113	135	143	21
6	0 15 53	1 25 46	170	197	141	169	179	27
7	0 19 4	2 6 55	204	237	169	202	215	32
8	0 22 14	2 18 4	238	276	198	236	251	37
9	0 25 25	2 29 13	272	316	226	270	287	43
10	0 28 36	3 10 22	306	355	254	303	323	48
11	0 31 46	3 21 31	340	395	282	337	358	53
12	0 34 57	4 2 40	374	434	311	371	394	58
13	0 38 7	4 13 50	408	474	339	405	430	64
14	0 41 18	4 24 59	442	513	367	438	466	69
15	0 44 29	5 6 8	476	553	395	472	502	74
16	0 47 39	5 17 17	510	592	424	506	538	80
17	0 50 50	5 28 26	544	632	452	539	573	85
18	0 54 1	6 9 35	578	671	480	573	609	90
19	0 57 11	6 20 44	612	711	508	607	645	96
20	1 0 22	7 1 53	646	750	537	641	681	101
21	1 3 33	7 13 3	680	790	565	674	717	106
22	1 6 43	7 24 12	714	829	593	708	753	112
23	1 9 54	8 5 21	748	869	621	742	788	117
24	1 13 5	8 16 30	782	908	650	775	824	122
25	1 16 15	8 27 39	816	948	678	809	860	128
26	1 19 26	9 8 48	850	987	706	843	896	133
27	1 22 36	9 19 57	884	027	734	877	932	138
28	1 25 47	10 1 6	918	066	762	910	968	143
29	1 28 58	10 12 16	952	106	791	944	003	149
30	1 32 8	10 23 25	986	145	819	978	039	154
31	1 35 19	11 4 34	020	185	847	011	075	159

TABLÆ XXIII.

Moon's Motions for Hours.

Hours.	1	2	3	4	5	6	7	8	9
1	1	27	43	12	14	16	2	16	1
2	2	54	87	24	28	31	5	33	2
3	3	81	130	36	42	47	7	49	3
4	5	108	173	48	56	62	10	65	4
5	6	135	217	60	70	78	12	81	5
6	7	162	260	72	84	93	14	98	6
7	8	190	303	84	98	109	17	114	7
8	9	217	347	96	112	124	19	130	8
9	10	244	390	108	126	140	22	146	9
10	11	271	433	120	140	155	24	163	10
11	12	298	477	131	154	171	26	179	11
12	14	325	520	143	168	186	29	195	12
13	15	352	563	155	182	202	31	211	13
14	16	379	607	167	196	217	34	228	14
15	17	406	650	179	210	233	36	244	15
16	18	433	693	191	224	248	38	260	16
17	19	460	737	203	238	264	41	276	17
18	20	487	780	215	252	279	43	293	18
19	22	515	823	227	266	295	46	309	19
20	23	542	867	239	280	310	48	325	20
21	24	569	910	251	294	326	50	341	21
22	25	596	953	263	308	341	53	358	22
23	26	623	997	275	322	357	55	374	23
24	27	650	1040	287	336	372	58	390	24

Hours.	Evection.	Anomaly.	Variation.	Longitude.
1	0°28′ 17″	0°32′ 40″	0°30′ 29″	0°32′ 56″
2	0 56 35	1 5 19	1 0 57	1 5 53
3	1 24 52	1 37 59	1 31 26	1 38 49
4	1 53 10	2 10 39	2 1 54	2 11 46
5	2 21 27	2 43 19	2 32 23	2 44 42
6	2 49 45	3 15 58	3 2 52	3 17 39
7	3 18 2	3 48 38	3 33 20	3 50 35
8	3 46 20	4 21 18	4 3 49	4 23 32
9	4 14 37	4 53 58	4 34 17	4 56 28
10	4 42 55	5 26 37	5 4 46	5 29 25
11	5 11 12	5 59 17	5 35 15	6 2 21
12	5 39 30	6 31 57	6 5 43	6 35 17
13	6 7 47	7 4 37	6 36 12	7 8 14
14	6 36 5	7 37 16	7 6 40	7 41 10
15	7 4 22	8 9 56	7 37 9	8 14 7
16	7 32 40	8 42 36	8 7 38	8 47 3
17	8 0 57	9 15 16	8 38 6	9 20 0
18	8 29 15	9 47 55	9 8 35	9 52 56
19	8 57 32	10 20 35	9 39 3	10 25 53
20	9 25 50	10 53 15	10 9 32	10 58 49
21	9 54 7	11 25 55	10 40 1	11 31 46
22	10 22 24	11 58 34	11 10 29	12 4 42
23	10 50 42	12 31 14	11 40 58	12 37 39
24	11 18 59	13 3 54	12 11 27	13 10 35

Moon's Motions for Hours.

Hours.	10	11	12	13	14	15	16	17	18
1	3	1	3	1	4	1	2	2	1
2	6	3	6	3	8	3	3	4	2
3	9	4	9	4	12	4	5	5	3
4	12	5	12	6	16	5	6	7	4
5	15	6	15	7	21	6	8	9	5
6	18	8	18	9	25	8	9	11	6
7	20	9	20	10	29	9	11	12	8
8	23	10	23	11	33	10	12	14	9
9	26	12	26	13	37	11	14	16	10
10	29	13	29	14	41	13	15	18	11
11	32	14	32	16	45	14	17	19	12
12	35	16	35	17	49	15	18	21	13
13	38	17	38	18	54	16	20	23	14
14	41	18	41	20	58	18	21	25	15
15	44	19	44	21	62	19	23	26	16
16	47	21	47	23	66	20	25	28	17
17	50	22	50	24	70	21	26	30	18
18	53	23	53	25	74	23	28	32	19
19	56	25	56	27	78	24	29	33	21
20	58	26	58	28	83	25	31	35	22
21	61	27	61	30	87	26	32	37	23
22	64	28	64	31	91	28	34	39	24
23	67	30	67	33	95	29	35	40	25
24	70	31	70	34	99	31	37	42	26

Hours.	Sup. of Node.	II.	V.	VI.	VII.	VIII	IX.	X.
1	0′ 8″	0°28′	1	2	1	1	1	0
2	0 16	0 56	3	3	2	3	3	0
3	0 24	1 24	4	5	4	4	4	1
4	0 32	1 52	6	7	5	6	6	1
5	0 40	2 19	7	8	6	7	7	1
6	0 48	2 47	9	10	7	9	9	1
7	0 56	3 15	10	12	8	10	10	2
8	1 4	3 43	11	13	9	11	12	2
9	1 11	4 11	13	15	11	13	13	2
10	1 19	4 39	14	16	12	14	15	2
11	1 27	5 7	16	18	13	15	16	2
12	1 35	5 35	17	20	14	17	18	3
13	1 43	6 2	18	21	15	18	19	3
14	1 51	6 30	20	23	16	19	21	3
15	1 59	6 58	21	25	18	21	22	3
16	2 7	7 26	23	26	19	22	24	4
17	2 15	7 54	24	28	20	24	25	4
18	2 23	8 22	26	29	21	25	27	4
19	2 31	8 50	27	31	22	27	28	4
20	2 39	9 18	28	32	24	28	30	4
21	2 47	9 45	30	34	25	29	31	5
22	2 55	10 13	31	36	26	31	33	5
23	3 3	10 41	33	38	27	32	34	5
24	3 11	11 9	34	39	28	34	36	5

Moon's Motions for Minutes and Seconds.

Min.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2	0	1	1	0	0	1	0	1	0	0	0	0	0	0
3	0	1	2	1	1	1	0	1	0	0	0	0	0	0
4	0	2	3	1	1	1	0	1	0	0	0	0	0	0
5	0	2	4	1	1	1	0	1	0	0	0	0	0	0
6	0	3	4	1	1	2	0	2	0	0	0	0	0	0
7	0	3	5	1	2	2	0	2	0	0	0	0	0	0
8	0	4	6	2	2	2	0	2	0	0	0	0	0	1
9	0	4	6	2	2	2	0	2	0	0	0	0	0	1
10	0	5	7	2	2	3	0	3	0	0	0	0	0	1
11	0	5	8	2	3	3	0	3	0	1	0	1	0	1
12	0	5	9	2	3	3	0	3	0	1	0	1	0	1
13	0	6	9	3	3	3	1	4	0	1	0	1	0	1
14	0	6	10	3	3	4	1	4	0	1	0	1	0	1
15	0	7	11	3	3	4	1	4	0	1	0	1	0	1
16	0	7	12	3	4	4	1	4	0	1	0	1	0	1
17	0	8	12	3	4	4	1	5	0	1	0	1	0	1
18	0	8	13	4	4	5	1	5	0	1	0	1	0	1
19	0	9	14	4	4	5	1	5	0	1	0	1	0	1
20	0	9	14	4	5	5	1	5	0	1	0	1	0	1
21	0	10	15	4	5	5	1	6	0	1	0	1	0	1
22	0	10	16	4	5	6	1	6	0	1	0	1	1	2
23	0	10	17	5	5	6	1	6	0	1	0	1	1	2
24	0	11	17	5	6	6	1	7	0	1	1	1	1	2
25	0	11	18	5	6	6	1	7	0	1	1	1	1	2
26	0	12	19	5	6	7	1	7	0	1	1	1	1	2
27	1	12	19	5	6	7	1	7	0	1	1	1	1	2
28	1	13	20	6	7	7	1	8	0	1	1	1	1	2
29	1	13	21	6	7	7	1	8	0	1	1	1	1	2
30	1	14	22	6	7	8	1	8	0	1	1	1	1	2

Moon's Motions for Minutes and Seconds.

Min.	Evect.	Anom.	Variat.	Long.	Sup. nod.	II.
31	14′ 37″	16′ 52″	15′ 45″	17′ 1″	4″	14′
32	15 5	17 25	16 15	17 34	4	15
33	15 34	17 58	16 46	18 7	4	15
34	16 2	18 30	17 16	18 40	4	16
35	16 30	19 3	17 47	19 13	5	16
36	16 58	19 36	18 17	19 46	5	17
37	17 27	20 8	18 48	20 19	5	17
38	17 55	20 41	19 18	20 52	5	18
39	18 23	21 14	19 49	21 25	5	18
40	18 52	21 46	20 19	21 58	5	19
41	19 20	22 19	20 50	22 31	5	19
42	19 48	22 52	21 20	23 3	6	20
43	20 16	23 24	21 51	23 36	6	20
44	20 45	23 57	22 21	24 9	6	21
45	21 13	24 30	22 52	24 42	6	21
46	21 41	25 2	23 22	25 15	6	21
47	22 10	25 35	23 53	25 48	6	22
48	22 38	26 8	24 23	26 21	6	22
49	23 6	26 40	24 54	26 54	6	23
50	23 34	27 13	25 24	27 27	7	23
51	24 3	27 46	25 55	28 0	7	24
52	24 31	28 18	26 25	28 33	7	24
53	24 59	28 51	26 56	29 6	7	25
54	25 28	29 24	27 26	29 39	7	25
55	25 56	29 56	27 56	30 12	7	26
56	26 24	30 29	28 27	30 45	7	26
57	26 52	31 2	28 57	31 18	7	27
58	27 21	31 34	29 28	31 51	8	27
59	27 49	32 7	29 58	32 23	8	28
60	28 17	32 40	30 29	32 56	8	28

Sec.	Ev.	Au.	Var.	Lon.
31	15″	17″	16″	17″
32	15	17	16	18
33	16	18	17	18
34	16	18	17	19
35	17	19	18	19
36	17	20	18	20
37	18	20	19	20
38	18	21	19	21
39	18	21	20	21
40	19	22	20	22
41	19	22	21	22
42	20	23	21	23
43	20	23	22	24
44	21	24	22	24
45	21	24	23	25
46	22	25	23	25
47	22	26	24	26
48	23	26	24	26
49	23	27	25	27
50	24	27	25	27
51	24	28	26	28
52	25	28	26	28
53	25	29	27	29
54	26	29	27	30
55	26	30	28	30
56	26	30	28	31
57	27	31	29	31
58	27	32	29	32
59	28	32	30	32
60	28	33	30	33

First Equation of Moon's Longitude. ARGUMENT 1.

Arg.	1	Diff.	Arg.	1	Diff.
0	12′ 40″	42″	5000	12′ 40″	40″
100	11 58	42	5100	13 20	41
200	11 16	42	5200	14 1	40
300	10 34	41	5300	14 41	39
400	9 53	41	5400	15 20	40
500	9 12	40	5500	16 0	38
600	8 32	38	5600	16 38	37
700	7 54	38	5700	17 15	37
800	7 16	36	5800	17 52	35
900	6 40	34	5900	18 27	34
1000	6 6	33	6000	19 1	32
1100	5 33	31	6100	19 33	31
1200	5 2	30	6200	20 4	29
1300	4 32	27	6300	20 33	28
1400	4 5	25	6400	21 1	26
1500	3 40	23	6500	21 27	23
1600	3 17	21	6600	21 50	22
1700	2 56	18	6700	22 12	19
1800	2 38	16	6800	22 31	17
1900	2 22	13	6900	22 48	15
2000	2 9	11	7000	23 3	12
2100	1 58	8	7100	23 15	10
2200	1 50	6	7200	23 25	7
2300	1 44	3	7300	23 32	5
2400	1 41	0	7400	23 37	2
2500	1 41	2	7500	23 39	0
2600	1 43	5	7600	23 39	3
2700	1 48	7	7700	23 36	6
2800	1 55	10	7800	23 30	8
2900	2 5	12	7900	23 22	11
3000	2 17	15	8000	23 11	13
3100	2 32	17	8100	22 58	16
3200	2 49	19	8200	22 42	18
3300	3 8	22	8300	22 24	21
3400	3 30	23	8400	22 3	23
3500	3 53	26	8500	21 40	25
3600	4 19	27	8600	21 15	27
3700	4 46	30	8700	20 48	30
3800	5 16	31	8800	20 18	31
3900	5 47	32	8900	19 47	33
4000	6 19	34	9000	19 14	34
4100	6 53	35	9100	18 40	36
4200	7 28	37	9200	18 4	38
4300	8 5	37	9300	17 26	38
4400	8 42	38	9400	16 48	40
4500	9 20	39	9500	16 8	41
4600	9 59	40	9600	15 27	41
4700	10 39	40	8700	14 46	42
4800	11 19	40	9800	14 4	42
4900	11 59	41	9900	13 22	42
5000	12 40		10000	12 40	

Equations 2 to 7 of Moon's Longitude. ARGUMENTS 2 to 7.

Arg.	2	3	4	5	6	7	Arg.
2500	4′ 57″	0′ 2″	6′ 30″	3′ 39″	0′ 6″	0′ 1″	2500
2600	4 57	0 2	6 30	3 39	0 6	0 1	2400
2700	4 56	0 3	6 29	3 38	0 7	0 1	2300
2800	4 55	0 3	6 27	3 37	0 8	0 2	2200
2900	4 53	0 4	6 24	3 36	0 9	0 3	2100
3000	4 50	0 5	6 21	3 34	0 10	0 4	2000
3100	4 47	0 6	6 17	3 32	0 12	0 5	1900
3200	4 43	0 8	6 12	3 29	0 14	0 6	1800
3300	4 39	0 9	6 7	3 26	0 17	0 8	1700
3400	4 34	0 11	6 1	3 22	0 19	0 10	1600
3500	4 29	0 13	5 54	3 18	0 22	0 12	1500
3600	4 23	0 15	5 47	3 14	0 25	0 14	1400
3700	4 17	0 18	5 39	3 10	0 29	0 17	1300
3800	4 11	0 20	5 30	3 5	0 33	0 19	1200
3900	4 4	0 23	5 21	3 0	0 37	0 22	1100
4000	3 57	0 26	5 12	2 54	0 41	0 25	1000
4100	3 49	0 29	5 2	2 49	0 45	0 28	900
4200	3 41	0 32	4 52	2 43	0 50	0 31	800
4300	3 33	0 35	4 41	2 37	0 54	0 35	700
4400	3 24	0 39	4 30	2 30	0 59	0 38	600
4500	3 15	0 42	4 19	2 24	1 4	0 42	500
4600	3 7	0 46	4 7	2 17	1 9	0 45	400
4700	2 58	0 49	3 56	2 10	1 14	0 49	300
4800	2 48	0 53	3 44	2 4	1 19	0 53	200
4900	2 39	0 56	3 32	1 57	1 25	0 56	100
5000	2 30	1 0	3 20	1 50	1 30	1 0	0000
5100	2 21	1 4	3 8	1 43	1 35	1 4	9900
5200	2 11	1 7	2 56	1 36	1 40	1 7	9800
5300	2 2	1 11	2 44	1 29	1 46	1 11	9700
5400	1 53	1 14	2 33	1 23	1 51	1 15	9600
5500	1 44	1 18	2 21	1 16	1 56	1 18	9500
5600	1 36	1 21	2 10	1 10	2 1	1 22	9400
5700	1 27	1 25	1 59	1 3	2 6	1 25	9300
5800	1 19	1 28	1 48	0 57	2 10	1 28	9200
5900	1 11	1 31	1 38	0 51	2 15	1 32	9100
6000	1 3	1 34	1 28	0 46	2 19	1 35	9000
6100	0 56	1 37	1 19	0 40	2 23	1 38	8900
6200	0 49	1 39	1 10	0 35	2 27	1 40	8800
6300	0 43	1 42	1 1	0 30	2 31	1 43	8700
6400	0 36	1 44	0 53	0 26	2 35	1 46	8600
6500	0 31	1 47	0 46	0 21	2 38	1 48	8500
6600	0 26	1 49	0 39	0 18	2 41	1 50	8400
6700	0 21	1 51	0 33	0 14	2 43	1 52	8300
6800	0 17	1 52	0 28	0 11	2 46	1 54	8200
6900	0 13	1 54	0 23	0 8	2 48	1 55	8100
7000	0 10	1 55	0 19	0 6	2 50	1 56	8000
7100	0 7	1 56	0 16	0 4	2 51	1 57	7900
7200	0 5	1 57	0 13	0 2	2 52	1 58	7800
7300	0 4	1 57	0 11	0 1	2 53	1 59	7700
7400	0 3	1 58	0 10	0 1	2 54	1 59	7600
7500	0 3	1 58	0 10	0 1	2 54	1 59	7500

TABLE XXVII.

Equations 8 *and* 9.

Arg.	8	9	Arg.	8	9
0	1′ 20″	1′ 20″	5000	1′ 20″	1′ 20″
100	1 15	1 29	5100	1 24	1 26
200	1 11	1 37	5200	1 29	1 31
300	1 7	1 46	5300	1 33	1 37
400	1 2	1 54	5400	1 37	1 42
500	0 58	2 1	5500	1 42	1 47
600	0 54	2 8	5600	1 46	1 51
700	0 50	2 15	5700	1 50	1 55
800	0 46	2 20	5800	1 54	1 58
900	0 42	2 25	5900	1 58	2 0
1000	0 38	2 29	6000	2 1	2 1
1100	0 35	2 32	6100	2 5	2 2
1200	0 31	2 34	6200	2 8	2 2
1300	0 28	2 35	6300	2 11	2 1
1400	0 25	2 35	6400	2 14	1 59
1500	0 23	2 34	6500	2 17	1 56
1600	0 20	2 32	6600	2 19	1 52
1700	0 18	2 29	6700	2 22	1 48
1800	0 16	2 26	6800	2 24	1 43
1900	0 14	2 21	6900	2 25	1 38
2000	0 13	2 16	7000	2 27	1 32
2100	0 11	2 11	7100	2 28	1 25
2200	0 10	2 4	7200	2 29	1 18
2300	0 10	1 58	7300	2 30	1 11
2400	0 9	1 51	7400	2 30	1 4
2500	0 9	1 43	7500	2 31	0 56
2600	0 10	1 36	7600	2 30	0 49
2700	0 10	1 29	7700	2 30	0 42
2800	0 11	1 22	7800	2 29	0 36
2900	0 12	1 15	7900	2 28	0 29
3000	0 13	1 8	8000	2 27	0 24
3100	0 15	1 2	8100	2 26	0 18
3200	0 16	0 57	8200	2 24	0 14
3300	0 18	0 52	8300	2 22	0 10
3400	0 21	0 47	8400	2 20	0 8
3500	0 23	0 44	8500	2 17	0 6
3600	0 26	0 41	8600	2 15	0 5
3700	0 29	0 39	8700	2 12	0 5
3800	0 32	0 38	8800	2 9	0 6
3900	0 35	0 38	8900	2 5	0 8
4000	0 39	0 39	9000	2 2	0 11
4100	0 42	0 40	9100	1 58	0 15
4200	0 46	0 42	9200	1 54	0 20
4300	0 50	0 45	9300	1 50	0 25
4400	0 54	0 49	9400	1 46	0 32
4500	0 58	0 53	9500	1 42	0 39
4600	1 3	0 58	9600	1 38	0 46
4700	1 7	1 3	9700	1 33	0 54
4800	1 11	1 9	9800	1 29	1 3
4900	1 16	1 14	9900	1 24	1 11
5000	1 20	1 20	10000	1 20	1 20

TABLE XXVIII.

Equations 10 *and* 11.

Arg.	10	11	Arg.	10	11
0	10″	10″	500	10″	10″
10	9	11	510	10	11
20	9	12	520	9	11
30	8	13	530	9	12
40	7	14	540	8	13
50	7	15	550	8	14
60	6	16	560	8	14
70	6	17	570	8	15
80	5	17	580	7	15
90	5	18	590	7	15
100	5	18	600	7	16
110	4	19	610	7	16
120	4	19	620	7	16
130	4	19	630	7	16
140	4	19	640	7	15
150	4	19	650	8	15
160	4	19	660	8	15
170	4	18	670	8	14
180	5	18	680	9	13
190	5	17	690	9	13
200	5	16	700	10	12
210	6	16	710	10	11
220	6	15	720	11	10
230	7	14	730	11	9
240	7	13	740	12	9
250	8	12	750	12	8
260	8	11	760	13	7
270	9	10	770	13	6
280	9	10	780	14	5
290	10	9	790	14	4
300	10	8	800	15	3
310	11	7	810	15	3
320	11	6	820	15	2
330	12	6	830	16	2
340	12	5	840	16	1
350	12	5	850	16	1
360	12	5	860	16	1
370	13	4	870	16	1
380	13	4	880	16	1
390	13	4	890	16	1
400	13	4	900	15	2
410	13	5	910	15	2
420	12	5	920	15	3
430	12	5	930	14	3
440	12	6	940	14	4
450	12	6	950	13	5
460	11	7	960	13	6
470	11	8	970	12	7
480	11	8	980	11	8
490	10	9	990	11	9
500	10	10	1000	10	10

TABLE XXIX.

Equations 12 *to* 19.

Arg.	12	13	14	15	16	17	18	19	Arg.
250	2″	2″	8″	0″	34″	3″	17″	3″	250
260	2	2	8	0	34	3	17	3	240
270	2	2	8	0	34	3	17	3	230
280	3	2	8	0	33	3	17	3	220
290	3	2	8	0	33	4	16	3	210
300	3	2	8	0	33	4	16	3	200
310	3	3	9	1	33	4	16	3	190
320	4	3	9	1	32	4	16	4	180
330	4	4	9	1	32	4	16	4	170
340	5	4	10	2	32	4	16	4	160
350	6	5	10	2	31	5	15	4	150
360	6	6	11	2	31	5	15	5	140
370	7	7	11	3	30	5	15	5	130
380	8	7	12	3	29	5	15	5	120
390	9	8	12	4	29	6	14	6	110
400	10	9	13	4	28	6	14	6	100
410	10	10	13	5	27	6	14	6	90
420	11	11	14	5	27	7	13	7	80
430	12	12	15	6	26	7	13	7	70
440	13	13	15	6	25	8	12	7	60
450	14	14	16	7	24	8	12	8	50
460	16	15	17	7	23	8	12	8	40
470	17	16	18	8	23	9	11	9	30
480	18	18	18	9	22	9	11	9	20
490	19	19	19	9	21	10	10	10	10
500	20	20	20	10	20	10	10	10	000
510	21	21	21	11	19	10	10	10	990
520	22	22	21	11	18	11	9	11	980
530	23	23	22	12	17	11	9	11	970
540	24	25	23	12	17	12	8	12	960
550	25	26	24	13	16	12	8	12	950
560	26	27	24	14	15	12	7	13	940
570	27	28	25	14	14	13	7	13	930
580	28	29	26	15	13	13	7	13	920
590	29	30	26	15	13	13	6	14	910
600	30	31	27	16	12	14	6	14	900
610	31	32	28	16	11	14	6	14	890
620	32	33	28	17	11	14	5	15	880
630	33	33	29	17	10	15	5	15	870
640	34	34	29	18	9	15	5	15	860
650	34	35	30	18	9	15	5	16	850
660	35	36	30	18	8	16	4	16	840
670	35	36	31	19	8	16	4	16	830
680	36	37	31	19	8	16	4	16	820
690	36	37	31	19	7	16	4	17	810
700	37	37	32	19	7	16	4	17	800
710	37	38	32	20	7	16	3	17	790
720	37	38	32	20	6	16	3	17	780
730	38	38	32	20	6	16	3	17	770
740	38	38	32	20	6	17	3	17	760
750	38	38	32	20	6	17	3	17	750

TABLE XXX.

Equation 20.

Arg.	20	Arg.
0	10″	500
10	11	510
20	12	520
30	13	530
40	13	540
50	14	550
60	15	560
70	16	570
80	16	580
90	17	590
100	17	600
110	17	610
120	17	620
130	17	630
140	17	640
150	17	650
160	17	660
170	16	670
180	16	680
190	15	690
200	14	700
210	13	710
220	13	720
230	12	730
240	11	740
250	10	750
260	9	760
270	8	770
280	7	780
290	6	790
300	6	800
310	5	810
320	4	820
330	4	830
340	3	840
350	3	850
360	3	860
370	3	870
380	3	880
390	3	890
400	3	900
410	3	910
420	4	920
430	4	930
440	5	940
450	6	950
460	6	960
470	7	970
480	8	980
490	9	990
500	10	1000

Evection.

Argument. Evection, corrected.

	0^s	I^s	II^s	III^s	IV^s	V^s
0°	1°30′ 0″	2°10′ 43″	2°40′ 10″	2°50′ 25″	2°39′ 8″	2° 9′ 42″
1	1 31 25	2 11 57	2 40 51	2 50 23	2 38 25	2 8 29
2	1 32 51	2 13 9	2 41 30	2 50 20	2 37 40	2 7 16
3	1 34 16	2 14 21	2 42 8	2 50 15	2 36 55	2 6 2
4	1 35 42	2 15 31	2 42 45	2 50 9	2 36 8	2 4 47
5	1 37 7	2 16 41	2 43 21	2 50 1	2 35 19	2 3 32
6	1 38 32	2 17 50	2 43 55	2 49 52	2 34 30	2 2 16
7	1 39 57	2 18 58	2 44 27	2 49 41	2 33 40	2 1 0
8	1 41 21	2 20 5	2 44 59	2 49 29	2 32 48	1 59 43
9	1 42 46	2 21 11	2 45 29	2 49 15	2 31 55	1 58 26
10	1 44 10	2 22 17	2 45 57	2 49 0	2 31 2	1 57 8
11	1 45 34	2 23 21	2 46 24	2 48 43	2 30 7	1 55 49
12	1 46 58	2 24 24	2 46 50	2 48 26	2 29 11	1 54 30
13	1 48 21	2 25 26	2 47 14	2 48 6	2 28 14	1 53 11
14	1 49 44	2 26 28	2 47 37	2 47 45	2 27 16	1 51 51
15	1 51 7	2 27 28	2 47 59	2 47 23	2 26 17	1 50 31
16	1 52 29	2 28 27	2 48 19	2 47 0	2 25 17	1 49 11
17	1 53 51	2 29 25	2 48 37	2 46 35	2 24 16	1 47 50
18	1 55 12	2 30 21	2 48 54	2 46 8	2 23 14	1 46 29
19	1 56 33	2 31 17	2 49 10	2 45 41	2 22 11	1 45 7
20	1 57 53	2 32 11	2 49 24	2 45 12	2 21 7	1 43 46
21	1 59 13	2 33 5	2 49 37	2 44 41	2 20 2	1 42 24
22	2 0 32	2 33 57	2 49 48	2 44 9	2 18 56	1 41 2
23	2 1 51	2 34 48	2 49 58	2 43 36	2 17 50	1 39 39
24	2 3 9	2 35 38	2 50 6	2 43 2	2 16 43	1 38 17
25	2 4 26	2 36 26	2 50 13	2 42 26	2 15 34	1 36 54
26	2 5 43	2 37 13	2 50 19	2 41 49	2 14 25	1 35 32
27	2 6 59	2 37 59	2 50 23	2 41 11	2 13 16	1 34 9
28	2 8 15	2 38 44	2 50 25	2 40 31	2 12 5	1 32 46
29	2 9 30	2 39 28	2 50 26	2 39 50	2 10 54	1 31 23
30	2 10 43	2 40 10	2 50 25	2 39 8	2 9 42	1 30 0

Evection.

Argument. Evection, corrected.

	VI^{s}	VII^{s}	VIII^{s}	IX^{s}	X^{s}	XI^{s}
0°	1°30′ 0″	0°50′18″	0°20′52″	0° 9′34″	0°19′50″	0°49′16″
1	1 28 37	0 49 6	0 20 10	0 9 34	0 20 32	0 50 30
2	1 27 14	0 47 55	0 19 29	0 9 35	0 21 16	0 51 45
3	1 25 51	0 46 44	0 18 49	0 9 37	0 22 1	0 53 1
4	1 24 28	0 45 34	0 18 11	0 9 41	0 22 47	0 54 17
5	1 23 6	0 44 26	0 17 34	0 9 47	0 23 34	0 55 33
6	1 21 43	0 43 17	0 16 58	0 9 54	0 24 22	0 56 51
7	1 20 20	0 42 10	0 16 24	0 10 2	0 25 12	0 58 9
8	1 18 58	0 41 4	0 15 50	0 10 12	0 26 3	0 59 28
9	1 17 36	0 39 58	0 15 19	0 10 23	0 26 55	1 0 47
10	1 16 14	0 38 53	0 14 48	0 10 36	0 27 48	1 2 7
11	1 14 52	0 37 49	0 14 19	0 10 50	0 28 43	1 3 27
12	1 13 31	0 36 46	0 13 51	0 11 5	0 29 39	1 4 48
13	1 12 10	0 35 44	0 13 25	0 11 23	0 30 35	1 6 9
14	1 10 49	0 34 43	0 13 0	0 11 41	0 31 33	1 7 31
15	1 9 29	0 33 43	0 12 37	0 12 1	0 32 32	1 8 53
16	1 8 9	0 32 44	0 12 14	0 12 23	0 33 32	1 10 16
17	1 6 49	0 31 46	0 11 54	0 12 45	0 34 34	1 11 39
18	1 5 30	0 30 49	0 11 34	0 13 10	0 35 36	1 13 2
19	1 4 11	0 29 53	0 11 16	0 13 35	0 36 39	1 14 26
20	1 2 52	0 28 58	0 11 0	0 14 3	0 37 43	1 15 50
21	1 1 34	0 28 5	0 10 45	0 14 31	0 38 48	1 17 14
22	1 0 17	0 27 12	0 10 31	0 15 1	0 39 55	1 18 39
23	0 59 0	0 26 20	0 10 19	0 15 33	0 41 2	1 20 3
24	0 57 44	0 25 30	0 10 8	0 16 5	0 42 10	1 21 28
25	0 56 28	0 24 40	0 9 59	0 16 39	0 43 19	1 22 53
26	0 55 13	0 23 52	0 9 51	0 17 15	0 44 29	1 24 18
27	0 53 58	0 23 5	0 9 45	0 17 52	0 45 39	1 25 44
28	0 52 44	0 22 20	0 9 40	0 18 30	0 46 51	1 27 9
29	0 51 31	0 21 35	0 9 36	0 19 9	0 48 3	1 28 34
30	0 50 18	0 20 52	0 9 34	0 19 50	0 49 16	1 30 0

Equation of Moon's Centre.

Argument. Anomaly, corrected.

	0ˢ	Iˢ	IIˢ	IIIˢ	IVˢ	Vˢ
0°	7° 0′ 0″	10°20′ 58″	12°38′ 44″	13°17′ 35″	12°16′ 21″	9° 58′ 29″
1	7 7 5	10 26 52	12 41 43	13 17 5	12 12 48	9 52 58
2	7 14 10	10 32 42	12 44 35	13 16 28	12 9 11	9 47 24
3	7 21 15	10 38 27	12 47 20	13 15 44	12 5 29	9 41 48
4	7 28 19	10 44 8	12 49 59	13 14 53	12 1 41	9 36 10
5	7 35 23	10 49 43	12 52 30	13 13 56	11 57 49	9 30 29
6	7 42 26	10 55 14	12 54 55	13 12 52	11 53 52	9 24 46
7	7 49 28	11 0 39	12 57 12	13 11 41	11 49 50	9 19 1
8	7 56 28	11 6 0	12 59 23	13 10 24	11 45 44	9 13 13
9	8 3 28	11 11 15	13 1 26	13 9 1	11 41 33	9 7 24
10	8 10 26	11 16 24	13 3 23	13 7 31	11 37 17	9 1 32
11	8 17 22	11 21 29	13 5 12	13 5 54	11 32 57	8 55 39
12	8 24 17	11 26 27	13 6 55	13 4 12	11 28 33	8 49 44
13	8 31 10	11 31 20	13 8 30	13 2 23	11 24 5	8 43 47
14	8 38 1	11 36 8	13 9 59	13 0 27	11 19 32	8 37 49
15	8 44 50	11 40 49	13 11 20	12 58 26	11 14 55	8 31 49
16	8 51 36	11 45 25	13 12 34	12 56 18	11 10 14	8 25 48
17	8 58 20	11 49 54	13 13 41	12 54 5	11 5 30	8 19 46
18	9 5 1	11 54 18	13 14 41	12 51 45	11 0 41	8 13 42
19	9 11 39	11 58 35	13 15 34	12 49 19	10 55 49	8 7 38
20	9 18 15	12 2 47	13 16 20	12 46 47	10 50 53	8 1 32
21	9 24 47	12 6 52	13 16 59	12 44 10	10 45 53	7 55 26
22	9 31 16	12 10 50	13 17 31	12 41 27	10 40 50	7 49 18
23	9 37 42	12 14 42	13 17 56	12 38 38	10 35 43	7 43 10
24	9 44 4	12 18 28	13 18 14	12 35 43	10 30 33	7 37 1
25	9 50 23	12 22 7	13 18 24	12 32 43	10 25 20	7 30 52
26	9 56 38	12 25 40	13 18 28	12 29 37	10 20 4	7 24 42
27	10 2 49	12 29 6	13 18 25	12 26 26	10 14 45	7 18 32
28	10 8 56	12 32 25	13 18 16	12 23 10	10 9 22	7 12 21
29	10 14 59	12 35 38	13 17 59	12 19 48	10 3 57	7 6 11
30	10 20 58	12 38 44	13 17 35	12 16 21	9 58 29	7 0 0

Equation of Moon's Centre.

Argument. Anomaly, corrected.

	VI^s	VII^s	VIII^s	IX^s	X^s	XI^s
0°	7° 0′ 0″	4° 1′ 31″	1°43′ 39″	0°42′ 25″	1°21′ 16″	3°39′ 2′
1	6 53 49	3 56 3	1 40 12	0 42 1	1 24 22	3 45 1
2	6 47 39	3 50 38	1 36 50	0 41 44	1 27 35	3 51 4
3	6 41 28	3 45 15	1 33 34	0 41 35	1 30 54	3 57 11
4	6 35 18	3 39 56	1 30 23	0 41 32	1 34 20	4 3 22
5	6 29 8	3 34 40	1 27 17	0 41 36	1 37 53	4 9 37
6	6 22 59	3 29 26	1 24 17	0 41 46	1 41 32	4 15 55
7	6 16 50	3 24 17	1 21 22	0 42 4	1 45 18	4 22 18
8	6 10 42	3 19 10	1 18 33	0 42 29	1 49 10	4 28 44
9	6 4 34	3 14 7	1 15 50	0 43 1	1 53 8	4 35 13
10	5 58 28	3 9 7	1 13 12	0 43 40	1 57 13	4 41 45
11	5 52 22	3 4 11	1 10 41	0 44 26	2 1 24	4 48 21
12	5 46 17	2 59 19	1 8 15	0 45 19	2 5 42	4 54 59
13	5 40 14	2 54 30	1 5 55	0 46 19	2 10 5	5 1 40
14	5 34 12	2 49 46	1 3 42	0 47 26	2 14 35	5 8 24
15	5 28 11	2 45 5	1 1 34	0 48 40	2 19 11	5 15 10
16	5 22 11	2 40 28	0 59 33	0 50 1	2 23 52	5 21 59
17	5 16 13	2 35 55	0 57 37	0 51 30	2 28 39	5 28 50
18	5 10 16	2 31 27	0 55 48	0 53 5	2 33 32	5 35 43
19	5 4 21	2 27 3	0 54 6	0 54 47	2 38 31	5 42 37
20	4 58 28	2 22 43	0 52 29	0 56 37	2 43 35	5 49 34
21	4 52 36	2 18 27	0 50 59	0 58 33	2 48 45	5 56 32
22	4 46 47	2 14 16	0 49 36	1 0 37	2 54 0	6 3 31
23	4 40 59	2 10 10	0 48 19	1 2 48	2 59 21	6 10 32
24	4 35 14	2 6 8	0 47 8	1 5 5	3 4 46	6 17 34
25	4 29 31	2 2 11	0 46 4	1 7 30	3 10 17	6 24 37
26	4 23 50	1 58 19	0 45 7	1 10 1	3 15 52	6 31 41
27	4 18 11	1 54 31	0 44 16	1 12 40	3 21 33	6 38 45
28	4 12 35	1 50 49	0 43 32	1 15 25	3 27 18	6 45 50
29	4 7 2	1 47 11	0 42 55	1 18 17	3 33 8	6 52 55
30	4 1 31	1 43 39	0 42 25	1 21 16	3 39 2	7 0 0

Variation.

ARGUMENT. Variation, corrected.

	0^s	I^s	II^s	III^s	IV^s	V^s
0°	0° 38′ 0″	1° 8′ 1″	1° 6′ 58″	0° 35′ 54″	0° 5′ 29″	0° 6′ 2″
1	0 39 13	1 8 35	1 6 18	0 34 40	0 4 54	0 6 42
2	0 40 26	1 9 7	1 5 36	0 33 27	0 4 21	0 7 24
3	0 41 39	1 9 36	1 4 52	0 32 13	0 3 51	0 8 8
4	0 42 52	1 10 3	1 4 5	0 31 0	0 3 22	0 8 55
5	0 44 4	1 10 28	1 3 17	0 29 47	0 2 56	0 9 44
6	0 45 16	1 10 50	1 2 27	0 28 34	0 2 33	0 10 34
7	0 46 28	1 11 9	1 1 35	0 27 22	0 2 12	0 11 27
8	0 47 38	1 11 26	1 0 42	0 26 11	0 1 54	0 12 22
9	0 48 48	1 11 41	0 59 46	0 25 1	0 1 38	0 13 19
10	0 49 57	1 11 53	0 58 49	0 23 51	0 1 24	0 14 17
11	0 51 6	1 12 2	0 57 50	0 22 42	0 1 14	0 15 17
12	0 52 13	1 12 9	0 56 50	0 21 34	0 1 5	0 16 19
13	0 53 19	1 12 13	0 55 48	0 20 28	0 1 0	0 17 22
14	0 54 24	1 12 15	0 54 45	0 19 22	0 0 57	0 18 27
15	0 55 27	1 12 14	0 53 41	0 18 18	0 0 57	0 19 33
16	0 56 30	1 12 10	0 52 35	0 17 15	0 0 59	0 20 41
17	0 57 31	1 12 4	0 51 28	0 16 13	0 1 4	0 21 50
18	0 58 30	1 11 55	0 50 21	0 15 13	0 1 11	0 23 0
19	0 59 28	1 11 44	0 49 12	0 14 15	0 1 22	0 24 11
20	1 0 24	1 11 30	0 48 2	0 13 17	0 1 34	0 25 23
21	1 1 19	1 11 14	0 46 52	0 12 22	0 1 50	0 26 36
22	1 2 11	1 10 55	0 45 40	0 11 28	0 2 8	0 27 50
23	1 3 2	1 10 34	0 44 29	0 10 37	0 2 28	0 29 4
24	1 3 51	1 10 10	0 43 16	0 9 47	0 2 51	0 30 20
25	1 4 38	1 9 44	0 42 3	0 8 59	0 3 17	0 31 36
26	1 5 23	1 9 15	0 40 50	0 8 13	0 3 45	0 32 52
27	1 6 6	1 8 44	0 39 36	0 7 29	0 4 16	0 34 9
28	1 6 47	1 8 11	0 38 22	0 6 47	0 4 48	0 35 26
29	1 7 25	1 7 36	0 37 8	0 6 7	0 5 24	0 36 43
30	1 8 1	1 6 58	0 35 54	0 5 29	0 6 2	0 38 0

Variation.

Argument. Variation, corrected.

	VIs	VIIs	VIIIs	IXs	Xs	XIs
0°	0°38′ 0″	1° 9′ 58″	1°10′ 30″	0°40′ 6″	0° 9′ 2″	0° 7′ 58″
1	0 39 17	1 10 36	1 9 53	0 38 52	0 8 24	0 8 35
2	0 40 34	1 11 11	1 9 13	0 37 38	0 7 49	0 9 13
3	0 41 51	1 11 44	1 8 31	0 36 24	0 7 15	0 9 54
4	0 43 8	1 12 15	1 7 47	0 35 10	0 6 45	0 10 37
5	0 44 24	1 12 43	1 7 1	0 33 57	0 6 16	0 11 22
6	0 45 40	1 13 9	1 6 13	0 32 44	0 5 50	0 12 9
7	0 46 55	1 13 32	1 5 23	0 31 31	0 5 26	0 12 58
8	0 48 10	1 13 52	1 4 31	0 30 19	0 5 5	0 13 49
9	0 49 24	1 14 10	1 3 38	0 29 8	0 4 46	0 14 41
10	0 50 37	1 14 26	1 2 42	0 27 58	0 4 29	0 15 36
11	0 51 49	1 14 38	1 1 45	0 26 48	0 4 16	0 16 32
12	0 53 0	1 14 48	1 0 47	0 25 39	0 4 4	0 17 30
13	0 54 10	1 14 56	0 59 47	0 24 31	0 3 56	0 18 29
14	0 55 19	1 15 1	0 58 45	0 23 25	0 3 50	0 19 30
15	0 56 27	1 15 3	0 57 42	0 22 19	0 3 46	0 20 32
16	0 57 33	1 15 3	0 56 38	0 21 15	0 3 45	0 21 36
17	0 58 38	1 15 0	0 55 32	0 20 12	0 3 47	0 22 41
18	0 59 41	1 14 54	0 54 25	0 19 10	0 3 51	0 23 47
19	1 0 43	1 14 46	0 53 18	0 18 10	0 3 58	0 24 54
20	1 1 43	1 14 35	0 52 9	0 17 11	0 4 7	0 26 3
21	1 2 41	1 14 22	0 50 59	0 16 14	0 4 19	0 27 12
22	1 3 38	1 14 6	0 49 49	0 15 18	0 4 34	0 28 22
23	1 4 33	1 13 48	0 48 38	0 14 25	0 4 51	0 29 32
24	1 5 25	1 13 27	0 47 26	0 13 33	0 5 10	0 30 44
25	1 6 16	1 13 3	0 46 13	0 12 43	0 5 32	0 31 55
26	1 7 5	1 12 38	0 45 0	0 11 54	0 5 57	0 33 8
27	1 7 52	1 12 9	0 43 47	0 11 8	0 6 23	0 34 20
28	1 8 36	1 11 39	0 42 33	0 10 24	0 6 53	0 35 33
29	1 9 18	1 11 6	0 41 20	0 9 42	0 7 24	0 36 47
30	1 9 58	1 10 30	0 40 6	0 9 2	0 7 58	0 38 0

Reduction.

ARGUMENT. Suppl. of Node + Moon's Orbit Longitude.

	0^s VI^s	I^s VII^s	II^s VIII^s	III^s IX^s	IV^s X^s	V^s XI^s
0°	7′ 0″	1′ 3″	1′ 3″	7′ 0″	12′ 57″	12′ 57″
1	6 46	0 56	1 10	7 14	13 4	12 50
2	6 31	0 49	1 18	7 29	13 10	12 42
3	6 17	0 43	1 26	7 43	13 17	12 33
4	6 3	0 38	1 35	7 57	13 22	12 25
5	5 48	0 33	1 44	8 12	13 27	12 16
6	5 34	0 28	1 54	8 26	13 32	12 6
7	5 20	0 24	2 3	8 40	13 36	11 56
8	5 6	0 20	2 14	8 54	13 40	11 46
9	4 53	0 17	2 24	9 7	13 43	11 36
10	4 39	0 14	2 35	9 21	13 46	11 25
11	4 26	0 12	2 46	9 34	13 48	11 14
12	4 12	0 10	2 58	9 48	13 50	11 2
13	3 59	0 9	3 9	10 1	13 51	10 50
14	3 46	0 8	3 22	10 13	13 52	10 38
15	3 34	0 8	3 34	10 26	13 52	10 26
16	3 22	0 8	3 46	10 38	13 52	10 13
17	3 9	0 9	3 59	10 50	13 51	10 1
18	2 58	0 10	4 12	11 2	13 50	9 48
19	2 46	0 12	4 26	11 14	13 48	9 34
20	2 35	0 14	4 39	11 25	13 46	9 21
21	2 24	0 17	4 53	11 36	13 43	9 7
22	2 14	0 20	5 6	11 46	13 40	8 54
23	2 3	0 24	5 20	11 56	13 36	8 40
24	1 54	0 28	5 34	12 6	13 32	8 26
25	1 44	0 33	5 48	12 16	13 27	8 12
26	1 35	0 38	6 3	12 25	13 22	7 57
27	1 26	0 43	6 17	12 33	13 17	7 43
28	1 18	0 49	6 31	12 42	13 10	7 29
29	1 10	0 56	6 46	12 50	13 4	7 14
30	1 3	1 3	7 0	12 57	12 57	7 0

Moon's distance from the North Pole of the Ecliptic.

ARGUMENT. Suppl. of Node + Moon's Orbit Longitude.

	III^s	IV^s	V^s	VI^s	VII^s	VIII^s	
0°	84° 39′ 16″	85° 20′ 43″	87° 13′ 47″	89° 48′ 0″	92° 22′ 13″	94° 15′ 17″	30°
1	84 39 19	85 23 27	87 18 28	89 53 23	92 26 52	94 17 57	29
2	84 39 27	85 26 16	87 23 12	89 58 46	92 31 27	94 20 31	28
3	84 39 41	85 29 10	87 27 58	90 4 8	92 36 0	94 23 1	27
4	84 40 1	85 32 9	87 32 48	90 9 31	92 40 30	94 25 25	26
5	84 40 27	85 35 12	87 37 39	90 14 52	92 44 56	94 27 45	25
6	84 40 58	85 38 20	87 42 33	90 20 14	92 49 19	94 29 59	24
7	84 41 34	85 41 33	87 47 30	90 25 35	92 53 39	94 32 8	23
8	84 42 17	85 44 50	87 52 28	90 30 55	92 57 56	94 34 12	22
9	84 43 5	85 48 11	87 57 29	90 36 14	93 2 9	94 36 11	21
10	84 43 58	85 51 37	88 2 31	90 41 33	93 6 18	94 38 4	20
11	84 44 57	85 55 7	88 7 36	90 46 50	93 10 24	94 39 52	19
12	84 46 2	85 58 42	88 12 42	90 52 7	93 14 27	94 41 35	18
13	84 47 12	86 2 20	88 17 50	90 57 22	93 18 25	94 43 13	17
14	84 48 27	86 6 3	88 23 0	91 2 36	93 22 20	94 44 45	16
15	84 49 49	86 9 50	88 28 11	91 7 49	93 26 10	94 46 11	15
16	84 51 15	86 13 40	88 33 24	91 13 0	93 29 57	94 47 32	14
17	84 52 47	86 17 35	88 38 38	91 18 10	93 33 40	94 48 48	13
18	84 54 25	86 21 33	88 43 53	91 23 18	93 37 18	94 49 58	12
19	84 56 7	86 25 36	88 49 10	91 28 24	93 40 53	94 51 3	11
20	84 57 56	86 29 42	88 54 27	91 33 29	93 44 23	94 52 2	10
21	84 59 49	86 33 51	88 59 46	91 38 31	93 47 49	94 52 55	9
22	85 1 48	86 38 4	89 5 5	91 43 32	93 51 10	94 53 43	8
23	85 3 52	86 42 21	89 10 25	91 48 30	93 54 27	94 54 26	7
24	85 6 1	86 46 41	89 15 46	91 53 27	93 57 40	94 55 2	6
25	85 8 15	86 51 4	89 21 7	91 58 21	94 0 48	94 55 33	5
26	85 10 35	86 55 30	89 26 29	92 3 12	94 3 51	94 55 59	4
27	85 12 59	87 0 0	89 31 52	92 8 1	94 6 50	94 56 18	3
28	85 15 29	87 4 32	89 37 14	92 12 48	94 9 44	94 56 33	2
29	85 18 3	87 9 8	89 42 37	92 17 32	94 12 33	94 56 41	1
30	85 20 43	87 13 47	89 48 0	92 22 13	94 15 17	94 56 44	0
	II^s	I^s	0^s	XI^s	X^s	IX^s	

TABLE XXXVI.

Equation II. *of the Moon's Polar Distance.*

Argument II, corrected.

	III^s	IV^s	V^s	VI^s	VII^s	VIII^s	
0°	0 14″	1′ 24″	4′ 37″	9′ 0″	13′ 23″	16′ 36″	30°
1	0 14	1 29	4 45	9 9	13 31	16 40	29
2	0 14	1 34	4 53	9 18	13 39	16 45	28
3	0 14	1 39	5 1	9 27	13 47	16 49	27
4	0 15	1 44	5 9	9 37	13 54	16 53	26
5	0 16	1 49	5 18	9 46	14 2	16 57	25
6	0 17	1 54	5 26	9 55	14 9	17 1	24
7	0 18	2 0	5 34	10 4	14 17	17 4	23
8	0 19	2 5	5 43	10 13	14 24	17 8	22
9	0 20	2 11	5 51	10 22	14 31	17 11	21
10	0 22	2 17	6 0	10 31	14 38	17 14	20
11	0 23	2 23	6 9	10 40	14 45	17 17	19
12	0 25	2 29	6 17	10 49	14 52	17 20	18
13	0 27	2 35	6 26	10 58	14 59	17 23	17
14	0 29	2 41	6 35	11 7	15 5	17 26	16
15	0 32	2 48	6 44	11 16	15 12	17 28	15
16	0 34	2 54	6 53	11 25	15 18	17 31	14
17	0 37	3 1	7 2	11 34	15 25	17 33	13
18	0 40	3 8	7 11	11 43	15 31	17 35	12
19	0 42	3 15	7 20	11 51	15 37	17 36	11
20	0 45	3 22	7 29	12 0	15 43	17 38	10
21	0 49	3 29	7 38	12 9	15 49	17 40	9
22	0 52	3 36	7 47	12 17	15 55	17 41	8
23	0 56	3 43	7 56	12 26	16 0	17 42	7
24	0 59	3 51	8 5	12 34	16 6	17 43	6
25	1 3	3 58	8 14	12 42	16 11	17 44	5
26	1 7	4 6	8 23	12 51	16 16	17 45	4
27	1 11	4 13	8 32	12 59	16 21	17 45	3
28	1 15	4 21	8 42	13 7	16 26	17 46	2
29	1 20	4 29	8 51	13 15	16 31	17 46	1
30	1 24	4 37	9 0	13 23	16 36	17 46	0
	II^s	I^s	0^s	XI^s	X^s	IX^s	

TABLE XXXVII.

Equation III. *of the Polar Distance.*

Argument. Moon's True Longitude.

	III^s	IV^s	V^s	VI^s	VII^s	VIII^s	
0°	16″	15″	12″	8″	4″	1″	30°
6	16	14	11	7	3	1	24
12	16	14	10	6	3	0	18
18	16	13	10	5	2	0	12
24	15	13	9	5	1	0	6
30	15	12	8	4	1	0	0
	II^s	I^s	0^s	XI^s	X^s	IX^s	

TABLE XXXVIII.

To convert Degrees and Minutes into Decimal Parts.

Degrees and Min.	Dec. parts.
1° 5′	003
1 26	4
1 48	5
2 10	6
2 31	7
2 53	8
3 14	9
3 36	10
3 58	11
4 19	12
4 41	13
5 2	14
5 24	15
5 46	16
6 7	17
6 29	18
6 50	19
7 12	20
7 34	21
7 55	22
8 17	23
8 38	24
9 0	25
9 22	26
9 43	27
10 5	28
10 26	29
10 48	30
11 10	31
11 31	32
11 53	33
12 14	34
12 36	35
12 58	36
13 19	37
13 41	38
14 2	39
14 24	40
14 46	41
15 7	42
15 29	43
15 50	44
16 12	45
16 34	46
16 55	47
17 17	48
17 38	49
18 0	50
18 22	51
18 43	52
19 5	53

TABLE XXXIX.

Equations of Polar Distance.

ARGUMENTS. 20 of Long.; V to IX, corrected; and X, not corrected.

Arg.	20	V.	VI.	VII.	VIII	IX.	X.	Arg.
250	0″	56″	6″	3″	25″	3″	11″	250
260	0	56	6	3	25	3	11	240
270	0	56	6	3	25	3	11	230
280	1	55	6	3	25	3	11	220
290	1	55	7	3	25	4	11	210
300	1	55	7	4	25	4	11	200
310	1	54	8	4	24	5	12	190
320	2	53	8	5	24	6	12	180
330	2	53	9	5	24	6	13	170
340	3	52	10	6	23	7	13	160
350	3	51	11	7	23	8	14	150
360	4	50	12	8	23	9	14	140
370	4	49	13	9	22	10	15	130
380	5	48	14	10	22	11	16	120
390	6	46	15	11	21	13	17	110
400	6	45	16	12	21	14	17	100
410	7	44	17	13	20	15	18	90
420	8	42	18	14	20	17	19	80
430	9	41	20	15	19	18	20	70
440	10	39	21	17	19	20	21	60
450	10	38	23	18	18	22	22	50
460	11	36	24	19	17	23	23	40
470	12	35	25	21	17	25	24	30
480	13	33	27	22	16	27	25	20
490	14	32	28	24	16	28	26	10
500	15	30	30	25	15	30	27	000
510	16	28	31	26	14	32	28	990
520	17	27	33	28	14	33	29	980
530	18	25	34	29	13	35	30	970
540	19	24	36	31	12	37	31	960
550	19	22	37	32	12	38	32	950
560	20	20	39	33	11	40	33	940
570	21	19	40	34	11	41	34	930
580	22	17	41	36	10	43	35	920
590	23	16	43	37	10	44	36	910
600	24	15	44	38	9	46	37	900
610	24	13	45	39	9	47	37	890
620	25	12	46	40	8	48	38	880
630	26	11	47	41	8	50	39	870
640	26	10	48	42	7	51	40	860
650	27	9	49	43	7	52	40	850
660	27	8	50	44	6	53	41	840
670	28	7	51	45	6	54	41	830
680	28	7	52	45	6	54	42	820
690	29	6	52	46	6	55	42	810
700	29	5	53	46	5	56	42	800
710	29	5	53	47	5	56	43	790
720	29	5	53	47	5	56	43	780
730	30	4	54	47	5	57	43	770
740	30	4	54	47	5	57	43	760
750	30	4	54	47	5	57	43	750

Moon's Equatorial Parallax.

Argument. Argument of the Evection.

	0^s	I^s	II^s	III^s	IV^s	V^s	
0°	1′ 28″	1′ 23″	1′ 9″	0′ 50″	0′ 32″	0′ 18″	30°
1	1 28	1 23	1 8	0 49	0 31	0 18	29
2	1 28	1 22	1 8	0 49	0 30	0 18	28
3	1 28	1 22	1 7	0 48	0 30	0 17	27
4	1 28	1 22	1 7	0 47	0 29	0 17	26
5	1 28	1 21	1 6	0 47	0 29	0 17	25
6	1 28	1 21	1 5	0 46	0 28	0 17	24
7	1 28	1 20	1 5	0 46	0 28	0 16	23
8	1 28	1 20	1 4	0 45	0 27	0 16	22
9	1 28	1 20	1 4	0 44	0 27	0 16	21
10	1 28	1 19	1 3	0 44	0 26	0 16	20
11	1 28	1 19	1 2	0 43	0 26	0 15	19
12	1 27	1 18	1 2	0 42	0 25	0 15	18
13	1 27	1 18	1 1	0 42	0 25	0 15	17
14	1 27	1 17	1 0	0 41	0 24	0 15	16
15	1 27	1 17	1 0	0 40	0 24	0 15	15
16	1 27	1 16	0 59	0 40	0 24	0 15	14
17	1 27	1 16	0 59	0 39	0 23	0 14	13
18	1 26	1 15	0 58	0 39	0 23	0 14	12
19	1 26	1 15	0 57	0 38	0 22	0 14	11
20	1 26	1 14	0 57	0 37	0 22	0 14	10
21	1 26	1 14	0 56	0 37	0 21	0 14	9
22	1 25	1 13	0 55	0 36	0 21	0 14	8
23	1 25	1 13	0 55	0 36	0 21	0 14	7
24	1 25	1 12	0 54	0 35	0 20	0 14	6
25	1 25	1 12	0 53	0 34	0 20	0 14	5
26	1 24	1 11	0 53	0 34	0 20	0 14	4
27	1 24	1 11	0 52	0 33	0 19	0 14	3
28	1 24	1 10	0 51	0 33	0 19	0 13	2
29	1 23	1 10	0 51	0 32	0 19	0 13	1
30	1 23	1 9	0 50	0 32	0 18	0 13	0
	XI^s	X^s	IX^s	$VIII^s$	VII^s	VI^s	

Moon's Equatorial Parallax.

ARGUMENT. Anomaly.

	0^s	I^s	II^s	III^s	IV^s	V^s	
0°	58′ 58″	58′ 27″	57′ 8″	55′ 30″	54′ 2″	53′ 3″	30°
1	58 58	58 25	57 5	55 27	53 59	53 2	29
2	58 58	58 23	57 2	55 23	53 57	53 0	28
3	58 57	58 21	56 58	55 20	53 54	52 59	27
4	58 57	58 19	56 55	55 17	53 52	52 58	26
5	58 57	58 16	56 52	55 14	53 50	52 57	25
6	58 56	58 14	56 49	55 11	53 47	52 56	24
7	58 56	58 12	56 45	55 7	53 45	52 55	23
8	58 55	58 10	56 42	55 4	53 43	52 54	22
9	58 55	58 7	56 39	55 1	53 41	52 53	21
10	58 54	58 5	56 36	54 58	53 38	52 52	20
11	58 53	58 2	56 32	54 55	53 36	52 51	19
12	58 53	58 0	56 29	54 52	53 34	52 50	18
13	58 52	57 57	56 26	54 49	53 32	52 49	17
14	58 51	57 55	56 22	54 46	53 30	52 49	16
15	58 50	57 52	56 19	54 43	53 28	52 48	15
16	58 49	57 49	56 16	54 40	53 26	52 47	14
17	58 48	57 46	56 13	54 37	53 24	52 47	13
18	58 46	57 44	56 9	54 34	53 22	52 46	12
19	58 45	57 41	56 6	54 31	53 21	52 45	11
20	58 44	57 38	56 3	54 29	53 19	52 45	10
21	58 42	57 35	55 59	54 26	53 17	52 45	9
22	58 41	57 32	55 56	54 23	53 15	52 44	8
23	58 39	57 29	55 53	54 20	53 14	52 44	7
24	58 38	57 26	55 49	54 18	53 12	52 43	6
25	58 36	57 23	55 46	54 15	53 10	52 43	5
26	58 34	57 20	55 43	54 12	53 9	52 43	4
27	58 33	57 17	55 40	54 10	53 7	52 43	3
28	58 31	57 14	55 36	54 7	53 6	52 43	2
29	58 29	57 11	55 33	54 4	53 4	52 43	1
30	58 27	57 8	55 30	54 2	53 3	52 43	0
	XI^s	X^s	IX^s	$VIII^s$	VII^s	VI^s	

TABLE XLI

Moon's Equatorial Parallax.

ARGUMENT. Argument of the Variation.

	0^s	I^s	II^s	III^s	IV^s	V^s	
0°	56″	42″	16″	4″	18″	44″	30°
1	56	41	15	4	18	45	29
2	55	41	14	4	19	46	28
3	55	40	14	4	20	46	27
4	55	39	13	4	21	47	26
5	55	38	12	4	22	48	25
6	55	37	12	4	23	48	24
7	55	36	11	5	24	49	23
8	55	35	10	5	24	50	22
9	54	35	10	5	25	50	21
10	54	34	9	6	26	51	20
11	54	33	9	6	27	51	19
12	53	32	8	6	28	52	18
13	53	31	8	7	29	53	17
14	52	30	7	7	30	53	16
15	52	29	7	8	31	53	15
16	51	28	6	8	32	54	14
17	51	27	6	9	33	54	13
18	50	26	6	9	34	55	12
19	50	25	5	10	35	55	11
20	49	24	5	10	35	55	10
21	49	24	5	11	36	56	9
22	48	23	4	12	37	56	8
23	47	22	4	12	38	56	7
24	47	21	4	13	39	56	6
25	46	20	4	14	40	57	5
26	45	19	4	14	41	57	4
27	45	18	4	15	42	57	3
28	44	18	4	16	42	57	2
29	43	17	4	17	43	57	1
30	42	16	4	18	44	57	0
	XI^s	X^s	IX^s	$VIII^s$	VII^s	VI^s	

TABLE XLIII.

Reduction of the Parallax, and also of the Latitude.

ARGUMENT. Latitude.

Lat.	Red. of Paral.	Red. of Lat.
0°	0″	0′ 0″
3	0	1 12
6	0	2 23
9	0	3 32
12	0	4 39
15	1	5 43
18	1	6 44
21	1	7 40
24	2	8 31
27	2	9 16
30	3	9 55
33	3	10 28
36	4	10 54
39	5	11 13
42	5	11 25
45	6	11 29
48	6	11 25
51	7	11 14
54	8	10 56
57	8	10 30
60	9	9 57
63	9	9 18
66	10	8 33
69	10	7 42
72	10	6 46
75	11	5 45
78	11	4 41
81	11	3 33
84	11	2 24
87	11	1 12
90	11	0 0

TABLE XLIV.

Moon's Semidiameter.

ARGUMENT. Equatorial Parallax.

Eq. Par.	Semidi.	Eq. Par.	Semidi.	Eq. Par.	Semidi.
53′ 0″	14′ 27″	56′ 0″	15′ 16″	59′ 0″	16′ 5″
53 10	14 29	56 10	15 18	59 10	16 7
53 20	14 32	56 20	15 21	59 20	16 10
53 30	14 35	56 30	15 24	59 30	16 13
53 40	14 37	56 40	15 26	59 40	16 16
53 50	14 40	56 50	15 29	59 50	16 18
54 0	14 43	57 0	15 32	60 0	16 21
54 10	14 46	57 10	15 35	60 10	16 24
54 20	14 48	57 20	15 37	60 20	16 26
54 30	14 51	57 30	15 40	60 30	16 29
54 40	14 54	57 40	15 43	60 40	16 32
54 50	14 57	57 50	15 46	60 50	16 35
55 0	14 59	58 0	15 48	61 0	16 37
55 10	15 2	58 10	15 51	61 10	16 40
55 20	15 5	58 20	15 54	61 20	16 43
55 30	15 7	58 30	15 56	61 30	16 46
55 40	15 10	58 40	15 59	61 40	16 48
55 50	15 13	58 50	16 2	61 50	16 51
56 0	15 16	59 0	16 5	62 0	16 54

TABLE XLV.

Augmentation of Moon's Semidiameter.

ARGUMENT. Appar. Alt.

Ap. Alt	Augm.
6°	2″
12	3
18	5
24	6
30	8
36	9
42	11
48	12
54	13
60	14
66	15
72	15
78	16
84	16
90	16

TABLE XLVI.

Moon's Horary Motion in Longitude.

ARGUMENTS. 2, 3, 4, and 5 of Long.

Arg.	2	3	4	5	Arg.
0	6″	1″	3″	3″	100
5	5	2	3	3	95
10	5	2	3	3	90
15	4	2	3	3	85
20	4	3	2	2	80
25	3	3	2	2	75
30	2	3	2	2	70
35	2	4	1	1	65
40	1	4	1	1	60
45	1	4	1	1	55
50	0	5	1	1	50

Moon's Horary Motion in Longitude.

ARGUMENT. Argument of the Evection.

	0^s	I^s	II^s	III^s	IV^s	V^s	
0°	1' 20"	1' 15"	1' 0"	0' 39"	0' 20"	0' 6"	30°
1	1 20	1 14	0 59	0 39	0 19	0 6	29
2	1 20	1 14	0 58	0 38	0 19	0 5	28
3	1 20	1 14	0 58	0 37	0 18	0 5	27
4	1 20	1 13	0 57	0 37	0 18	0 5	26
5	1 20	1 13	0 56	0 36	0 17	0 4	25
6	1 20	1 12	0 56	0 35	0 16	0 4	24
7	1 20	1 12	0 55	0 35	0 16	0 4	23
8	1 20	1 11	0 54	0 34	0 15	0 4	22
9	1 20	1 11	0 54	0 33	0 15	0 3	21
10	1 20	1 11	0 53	0 33	0 14	0 3	20
11	1 20	1 10	0 52	0 32	0 14	0 3	19
12	1 10	1 10	0 52	0 31	0 13	0 3	18
13	1 19	1 9	0 51	0 31	0 13	0 3	17
14	1 19	1 9	0 50	0 30	0 12	0 2	16
15	1 19	1 8	0 50	0 29	0 12	0 2	15
16	1 19	1 8	0 49	0 29	0 11	0 2	14
17	1 18	1 7	0 48	0 28	0 11	0 2	13
18	1 18	1 7	0 48	0 27	0 11	0 2	12
19	1 18	1 6	0 47	0 27	0 10	0 2	11
20	1 18	1 5	0 46	0 26	0 10	0 1	10
21	1 18	1 5	0 46	0 25	0 9	0 1	9
22	1 17	1 4	0 45	0 25	0 9	0 1	8
23	1 17	1 4	0 44	0 24	0 8	0 1	7
24	1 17	1 3	0 44	0 23	0 8	0 1	6
25	1 16	1 3	0 43	0 23	0 8	0 1	5
26	1 16	1 2	0 42	0 22	0 7	0 1	4
27	1 16	1 1	0 41	0 22	0 7	0 1	3
28	1 15	1 1	0 41	0 21	0 7	0 1	2
29	1 15	1 0	0 40	0 20	0 6	0 1	1
30	1 15	1 0	0 39	0 20	0 6	0 1	0
	XI^s	X^s	IX^s	VIII^s	VII^s	VI^s	

Moon's Horary Motion in Longitude.

Arguments. Sum of preceding equations, and Anomaly, corrected.

	0″	10″	20″	30″	40″	50″	60″	70″	80″	90″	100″	
0ˢ 0°	4″	5″	6″	8″	9″	10″	11″	12″	14″	15″	16″	XIIˢ 0°
5	4	5	6	8	9	10	11	12	14	15	16	25
10	4	5	7	8	9	10	11	12	13	15	16	20
15	4	5	7	8	9	10	11	12	13	15	16	15
20	5	6	7	8	9	10	11	12	13	14	15	10
25	5	6	7	8	9	10	11	12	13	14	15	5
I 0	5	6	7	8	9	10	11	12	13	14	15	XI 0
5	5	6	7	8	9	10	11	12	13	14	15	25
10	6	7	7	8	9	10	11	12	13	13	14	20
15	6	7	8	8	9	10	11	12	12	13	14	15
20	7	7	8	9	9	10	11	11	12	13	13	10
25	7	8	8	9	9	10	11	11	12	12	13	5
II 0	7	8	8	9	9	10	11	11	12	12	13	X 0
5	8	8	9	9	10	10	10	11	11	12	12	25
10	8	9	9	9	10	10	10	11	11	11	12	20
15	9	9	9	10	10	10	10	10	11	11	11	15
20	9	10	10	10	10	10	10	10	10	10	11	10
25	10	10	10	10	10	10	10	10	10	10	10	5
III 0	10	10	10	10	10	10	10	10	10	10	10	IX 0
5	11	11	11	10	10	10	10	10	9	9	9	25
10	11	11	11	11	10	10	10	9	9	9	9	20
15	12	11	11	11	10	10	10	9	9	9	8	15
20	12	12	11	11	10	10	10	9	9	8	8	10
25	13	12	12	11	11	10	9	9	8	8	7	5
IV 0	13	12	12	11	11	10	9	9	8	8	7	VIII 0
5	13	13	12	11	11	10	9	9	8	7	7	25
10	14	13	12	11	11	10	9	9	8	7	6	20
15	14	13	12	12	11	10	9	8	8	7	6	15
20	14	13	12	12	11	10	9	8	8	7	6	10
25	14	13	13	12	11	10	9	8	7	7	6	5
V 0	15	14	13	12	11	10	9	8	7	6	5	VII 0
5	15	14	13	12	11	10	9	8	7	6	5	25
10	15	14	13	12	11	10	9	8	7	6	5	20
15	15	14	13	12	11	10	9	8	7	6	5	15
20	15	14	13	12	11	10	9	8	7	6	5	10
25	15	14	13	12	11	10	9	8	7	6	5	5
VI 0	15	14	13	12	11	10	9	8	7	6	5	VI 0
	0″	10″	20″	30″	40″	50″	60″	70″	80″	90″	100″	

Moon's Horary Motion in Longitude.

ARGUMENT. Anomaly, corrected.

	0^s	I^s	II^s	III^s	IV^s	V^s	
0°	34′ 51″	34′ 14″	32′ 39″	30′ 45″	29′ 6″	28′ 1″	30°
1	34 51	34 12	32 36	30 42	29 3	27 59	29
2	34 51	34 9	32 32	30 38	29 0	27 58	28
3	34 51	34 7	32 28	30 34	28 58	27 56	27
4	34 51	34 4	32 24	30 31	28 55	27 55	26
5	34 50	34 1	32 21	30 27	28 52	27 54	25
6	34 50	33 59	32 17	30 23	28 50	27 53	24
7	34 49	33 56	32 13	30 20	28 47	27 51	23
8	34 49	33 53	32 9	30 16	28 45	27 50	22
9	34 48	33 50	32 5	30 13	28 42	27 49	21
10	34 47	33 47	32 2	30 9	28 40	27 48	20
11	34 46	33 44	31 58	30 6	28 37	27 47	19
12	34 45	33 41	31 54	30 2	28 35	27 46	18
13	34 44	33 38	31 50	29 59	28 33	27 45	17
14	34 43	33 35	31 46	29 56	28 30	27 45	16
15	34 42	33 32	31 42	29 52	28 28	27 44	15
16	34 41	33 28	31 38	29 49	28 26	27 43	14
17	34 39	33 25	31 35	29 46	28 24	27 42	13
18	34 38	33 22	31 31	29 42	28 22	27 42	12
19	34 36	33 18	31 27	29 39	28 20	27 41	11
20	34 34	33 15	31 23	29 36	28 18	27 41	10
21	34 33	33 12	31 19	29 33	28 16	27 40	9
22	34 31	33 8	31 15	29 30	28 14	27 40	8
23	34 29	33 5	31 12	29 26	28 12	27 39	7
24	34 27	33 1	31 8	29 23	28 10	27 39	6
25	34 25	32 58	31 4	29 20	28 9	27 39	5
26	34 23	32 54	31 0	29 17	28 7	27 39	4
27	34 21	32 50	30 57	29 14	28 5	27 38	3
28	34 19	32 47	30 53	29 12	28 4	27 38	2
29	34 16	32 43	30 49	29 9	28 2	27 38	1
30	34 14	32 39	30 45	29 6	28 1	27 38	0
	XI^s	X^s	IX^s	$VIII^s$	VII^s	VI^s	

Moon's Horary Motion in Longitude.

Arguments. Sum of preceding equations, and Arg. of Variation.

		27′	28′	29′	30′	31′	32′	33′	34′	35′	36′	37′		
0^s	0°	0″	1″	2″	4″	5″	6″	7″	8″	10″	11″	12″	XII^s	0°
	5	0	1	2	4	5	6	7	8	10	11	12		25
	10	0	1	3	4	5	6	7	8	9	11	12		20
	15	1	2	3	4	5	6	7	8	9	10	11		15
	20	1	2	3	4	5	6	7	8	9	10	11		10
	25	2	3	4	4	5	6	7	8	8	9	10		5
I	0	3	4	4	5	5	6	7	7	8	8	9	XI	0
	5	4	4	5	5	6	6	6	7	7	8	8		25
	10	5	5	5	6	6	6	6	6	7	7	7		20
	15	6	6	6	6	6	6	6	6	6	6	6		15
	20	7	7	7	7	6	6	6	5	5	5	5		10
	25	8	8	7	7	6	6	6	5	5	4	4		5
II	0	9	9	8	7	7	6	5	5	4	3	3	X	0
	5	10	9	8	8	7	6	5	4	4	3	2		25
	10	11	10	9	8	7	6	5	4	3	2	1		20
	15	11	10	9	8	7	6	5	4	3	2	1		15
	20	12	11	10	8	7	6	5	4	2	1	0		10
	25	12	11	10	8	7	6	5	4	2	1	0		5
III	0	12	11	10	8	7	6	5	4	2	1	0	IX	0
	5	12	11	10	8	7	6	5	4	2	1	0		25
	10	12	11	10	8	7	6	5	4	2	1	0		20
	15	11	10	9	8	7	6	5	4	3	2	1		15
	20	11	10	9	8	7	6	5	4	3	2	1		10
	25	10	9	8	8	7	6	5	4	4	3	2		5
IV	0	9	8	8	7	7	6	5	5	4	4	3	VIII	0
	5	8	8	7	7	6	6	6	5	5	4	4		25
	10	7	7	7	6	6	6	6	6	5	5	5		20
	15	6	6	6	6	6	6	6	6	6	6	6		15
	20	5	5	5	6	6	6	6	6	7	7	7		10
	25	4	4	5	5	6	6	6	7	7	8	8		5
V	0	3	3	4	5	5	6	7	7	8	9	9	VII	0
	5	2	3	3	4	5	6	7	8	9	9	10		25
	10	1	2	3	4	5	6	7	8	9	10	11		20
	15	0	2	3	4	5	6	7	8	9	10	12		15
	20	0	1	2	4	5	6	7	8	10	11	12		10
	25	0	1	2	3	5	6	7	9	10	11	12		5
VI	0	0	1	2	3	5	6	7	9	10	11	12	VI	0
		27′	28′	29′	30′	31′	32′	33′	34′	35′	36′	37′		

Moon's Horary Motion in Longitude.

ARGUMENT. Argument of the Variation.

	0s	Is	IIs	IIIs	IVs	Vs	
0°	1′ 17″	0′ 58″	0′ 20″	0′ 2″	0′ 22″	1′ 0″	30°
1	1 17	0 57	0 19	0 2	0 23	1 1	29
2	1 17	0 55	0 18	0 3	0 24	1 2	28
3	1 17	0 54	0 17	0 3	0 25	1 3	27
4	1 17	0 53	0 16	0 3	0 26	1 4	26
5	1 17	0 52	0 15	0 3	0 27	1 5	25
6	1 16	0 51	0 14	0 3	0 29	1 6	24
7	1 16	0 49	0 13	0 4	0 30	1 7	23
8	1 16	0 48	0 12	0 4	0 31	1 8	22
9	1 15	0 47	0 11	0 4	0 33	1 9	21
10	1 15	0 45	0 11	0 5	0 34	1 10	20
11	1 14	0 44	0 10	0 5	0 35	1 11	19
12	1 14	0 43	0 9	0 6	0 37	1 12	18
13	1 13	0 41	0 8	0 6	0 38	1 13	17
14	1 13	0 40	0 8	0 7	0 39	1 13	16
15	1 12	0 39	0 7	0 8	0 40	1 14	15
16	1 11	0 38	0 6	0 8	0 42	1 15	14
17	1 11	0 36	0 6	0 9	0 43	1 15	13
18	1 10	0 35	0 5	0 0	0 44	1 16	12
19	1 9	0 34	0 5	01 1	0 46	1 16	11
20	1 8	0 32	0 4	0 1	0 47	1 17	10
21	1 7	0 31	0 4	0 12	0 48	1 17	9
22	1 6	0 30	0 4	0 13	0 50	1 18	8
23	1 5	0 29	0 3	0 14	0 51	1 18	7
24	1 4	0 27	0 3	0 15	0 52	1 18	6
25	1 3	0 26	0 3	0 16	0 54	1 19	5
26	1 2	0 25	0 3	0 17	0 55	1 19	4
27	1 1	0 24	0 3	0 18	0 56	1 19	3
28	1 0	0 23	0 2	0 19	0 57	1 19	2
29	0 59	0 21	0 2	0 20	0 59	1 19	1
30	0 58	0 20	0 2	0 22	1 0	1 19	0
	XIs	Xs	IXs	VIIIs	VIIs	VIs	

Moon's Horary Motion in Longitude.

ARGUMENT. Argument of the Reduction.

	0^{s}	I^{s}	II^{s}	III^{s}	IV^{s}	V^{s}	
0°	2″	6″	14″	18″	14″	6″	30°
1	2	6	14	18	14	6	29
2	2	7	14	18	13	6	28
3	2	7	15	18	13	5	27
4	2	7	15	18	13	5	26
5	2	7	15	18	13	5	25
6	2	8	15	18	12	5	24
7	2	8	16	18	12	4	23
8	2	8	16	18	12	4	22
9	2	8	16	18	12	4	21
10	3	9	16	17	11	4	20
11	3	9	16	17	11	4	19
12	3	9	16	17	11	4	18
13	3	9	17	17	11	3	17
14	3	10	17	17	10	3	16
15	3	10	17	17	10	3	15
16	3	10	17	17	10	3	14
17	3	11	17	17	9	3	13
18	4	11	17	16	9	3	12
19	4	11	17	16	9	3	11
20	4	11	17	16	9	3	10
21	4	12	18	16	8	2	9
22	4	12	18	16	8	2	8
23	4	12	18	16	8	2	7
24	5	12	18	15	8	2	6
25	5	13	18	15	7	2	5
26	5	13	18	15	7	2	4
27	5	13	18	15	7	2	3
28	6	13	18	14	7	2	2
29	6	14	18	14	6	2	1
30	6	14	18	14	6	2	0
	XI^{s}	X^{s}	IX^{s}	$VIII^{s}$	VII^{s}	VI^{s}	

TABLE LIII.

Moon's Horary Motion in Latitude.

ARGUMENT. Arg. I, of Latitude.

	0^s+	I^s+	II^s+	III^s—	IV^s—	V^s—	
0°	2′ 58″	2′ 34″	1′ 29″	0′ 0″	1′ 29″	2′ 34″	30°
1	2 58	2 33	1 27	0 3	1 32	2 36	29
2	2 58	2 31	1 24	0 6	1 35	2 37	28
3	2 58	2 29	1 21	0 9	1 37	2 39	27
4	2 58	2 28	1 18	0 12	1 40	2 40	26
5	2 57	2 26	1 15	0 16	1 42	2 41	25
6	2 57	2 24	1 13	0 19	1 45	2 43	24
7	2 57	2 22	1 10	0 22	1 47	2 44	23
8	2 56	2 20	1 7	0 25	1 50	2 45	22
9	2 56	2 19	1 4	0 28	1 52	2 46	21
10	2 55	2 17	1 1	0 31	1 55	2 47	20
11	2 55	2 15	0 58	0 34	1 57	2 48	19
12	2 54	2 12	0 55	0 37	1 59	2 49	18
13	2 53	2 10	0 52	0 40	2 2	2 50	17
14	2 53	2 8	0 49	0 43	2 4	2 51	16
15	2 52	2 6	0 46	0 46	2 6	2 52	15
16	2 51	2 4	0 43	0 49	2 8	2 53	14
17	2 50	2 2	0 40	0 52	2 10	2 53	13
18	2 49	1 59	0 37	0 55	2 12	2 54	12
19	2 48	1 57	0 34	0 58	2 15	2 55	11
20	2 47	1 55	0 31	1 1	2 17	2 55	10
21	2 46	1 52	0 28	1 4	2 19	2 56	9
22	2 45	1 50	0 25	1 7	2 20	2 56	8
23	2 44	1 47	0 22	1 10	2 22	2 57	7
24	2 43	1 45	0 19	1 13	2 24	2 57	6
25	2 41	1 42	0 16	1 15	2 26	2 57	5
26	2 40	1 40	0 12	1 18	2 28	2 58	4
27	2 39	1 37	0 9	1 21	2 29	2 58	3
28	2 37	1 35	0 6	1 24	2 31	2 58	2
29	2 36	1 32	0 3	1 27	2 33	2 58	1
30	2 34	1 29	0 0	1 29	2 34	2 58	0
	XI^s+	X^s+	IX^s+	VIII^s—	VII^s—	VI^s—	

TABLE LIV.

Moon's Horary Motion in Latitude.

ARGUMENT. Arg. II, of Latitude.

	0^s+	I^s+	II^s+	III^s—	IV^s—	V^s—	
0°	4″	4″	2″	0″	2″	4″	30°
6	4	3	2	0	3	4	24
12	4	3	1	1	3	4	18
18	4	3	1	1	3	4	12
24	4	3	0	2	3	4	6
30	4	2	0	2	4	4	0
	XI^s+	X^s+	IX^s+	VIII^s—	VII^s—	VI^s—	

1. MAY, 1836.

AT APPARENT NOON.

Day of the Week.	Day of the Month.	THE SUN'S Right Ascension.	THE SUN'S Diff. for 1 hour.	THE SUN'S Declination.	THE SUN'S Diff. for 1 hour.	Sidereal Time of the Semidiam. passing the Meredian.*	Equation of Time, *to be subtracted from Apparent Time.*	Diff. for 1 hour.
	.	h. m. s.	s.	° ′ ″ .	″	m. s.	m. s.	s.
Sun.	1	2 34 39 ·57	9 ·551	N.15 10 19 ·0	44 ·85	1 6 ·00	3 6 ·12	0 ·305
Mon.	2	2 38 28 ·79	9 ·574	15 28 15 ·4	44 ·22	1 6 ·08	3 13 ·43	0 ·283
Tues.	3	2 42 18 ·56	9 ·598	15 45 56 ·6	43 ·57	1 6 ·16	3 20 ·21	0 ·259
Wed.	4	2 46 8 ·90	9 ·622	16 3 22 ·2	42 ·91	1 6 ·24	3 26 ·42	0 ·234
Thur.	5	2 49 59 ·82	9 ·645	16 20 32 ·1	42 ·24	1 6 ·32	3 32 ·04	0 ·211
Frid.	6	2 53 51 ·31	9 ·670	16 37 25 ·8	41 ·55	1 6 ·40	3 37 ·10	0 ·186
Sat.	7	2 57 43 ·38	9 ·695	16 54 3 ·1	40 ·86	1 6 ·48	3 41 ·57	0 ·161
Sun.	8	3 1 36 ·05	9 ·719	17 10 23 ·7	40 ·15	1 6 ·56	3 45 ·44	0 ·137
Mon.	9	3 5 29 ·31	9 ·744	17 26 27 ·2	39 ·43	1 6 ·64	3 48 ·73	0 ·112
Tues.	10	3 9 23 ·16	9 ·769	17 42 13 ·4	38 ·69	1 6 ·72	3 51 ·42	0 ·088
Wed.	11	3 13 17 ·61	9 ·793	17 57 41 ·9	37 ·94	1 6 ·81	3 53 ·53	0 ·063
Thur.	12	3 17 12 ·64	9 ·818	18 12 52 ·5	37 ·18	1 6 ·89	3 55 ·04	0 ·038
Frid.	13	3 21 8 ·27	9 ·842	18 27 44 ·9	36 ·40	1 6 ·97	3 55 ·96	0 ·014
Sat.	41	3 25 4 ·48	9 ·867	18 42 18 ·6	35 ·62	1 7 ·05	3 56 ·30	0 ·010
Sun.	15	3 29 1 ·28	9 ·890	18 56 33 ·6	34 ·82	1 7 ·13	3 56 ·05	0 ·034
Mon.	16	3 32 58 ·65	9 ·915	19 10 29 ·4	34 ·01	1 7 ·21	3 55 ·24	0 ·058
Tues.	17	3 36 56 ·61	9 ·937	19 24 5 ·6	33 ·20	1 7 ·29	3 53 ·86	0 ·080
Wed.	18	3 40 55 ·10	9 ·960	19 37 22 ·3	32 ·36	1 7 ·37	3 51 ·94	0 ·103
Thur.	19	3 44 54 ·14	9 ·983	19 50 18 ·9	31 ·51	1 7 ·45	3 49 ·46	0 ·126
Frid.	20	3 48 53 ·73	10 ·005	20 2 55 ·1	30 ·66	1 7 ·53	3 46 ·44	0 ·148
Sat.	21	3 52 53 ·85	10 ·026	20 15 10 ·9	29 ·79	1 7 ·60	3 42 ·89	0 ·169
Sun.	22	3 56 54 ·47	10 ·047	20 27 5 ·8	28 ·91	1 7 ·68	3 38 ·64	0 ·190
Mon.	23	4 0 55 ·60	10 ·068	20 38 39 ·7	28 ·02	1 7 ·75	3 34 ·28	0 ·211
Tues.	24	4 4 57 ·22	10 ·089	20 49 52 ·2	27 ·13	1 7 ·83	3 29 ·22	0 ·232
Wed.	25	4 8 59 ·35	10 ·108	21 0 43 ·3	26 ·22	1 7 ·90	3 23 ·66	0 ·252
Thur.	26	4 13 1 ·95	10 ·128	21 11 12 ·6	25 ·30	1 7 ·96	3 17 ·62	0 ·270
Frid.	27	4 17 5 ·02	10 ·148	21 21 19 ·9	24 ·38	1 8 ·03	3 11 ·13	0 ·290
Sat.	28	4 21 8 ·56	10 ·166	21 31 5 ·1	23 ·45	1 8 ·10	3 4 ·17	0 ·309
Sun.	29	4 25 12 ·55	10 ·185	21 40 28 ·0	22 ·52	1 8 ·16	2 56 ·76	0 ·328
Mon.	30	4 29 16 ·99	10 ·203	21 49 28 ·4	21 ·58	1 8 ·22	2 48 ·90	0 ·346
Tues.	31	4 33 21 ·86	10 ·221	21 58 6 ·2	20 ·62	1 8 ·28	2 40 ·60	0 ·363
Wed.	32	4 37 27 ·17		N.22 6 21 ·1		1 8 ·34	2 31 ·89	

* *Mean* Time of the Semidiameter passing may be found by subtracting 0s ·18 from the *Sidereal* Time.

MAY, 1836. II.

AT MEAN NOON.

Day of the Week.	Day of the Month.	THE SUN'S			Equation of Time, *to be added to Mean Time.*	Sidereal Time.
		Right Ascension.	Declination.	Semi-diam.*		
		h. m. s.	° ′ ″	′ ″	m. s.	h. m. s.
Sun.	1	2 34 40 ·06	N.15 10 21 ·3	15 52 ·9	3 6 ·14	2 37 46 ·20
Mon.	2	2 38 29 ·30	15 28 17 ·8	15 52 ·7	3 13 45	2 41 42 ·75
Tues.	3	2 42 19 ·09	15 45 59 ·0	15 52 ·5	3 20 ·22	2 45 39 ·31
Wed.	4	2 46 9 ·45	16 3 24 ·7	15 52 ·2	3 26 ·43	2 49 35 ·88
Thur.	5	2 50 0 ·39	16 20 34 ·6	15 52 ·0	3 32 ·05	2 53 32 ·44
Frid.	6	2 53 51 ·89	16 37 28 ·3	15 51 ·8	3 37 ·11	2 57 29 ·00
Sat.	7	2 57 43 ·98	16 54 5 ·6	15 51 ·5	3 41 ·58	3 1 25 ·56
Sun.	8	3 1 36 ·66	17 10 26 ·2	15 51 ·3	3 45 ·45	3 5 22 ·11
Mon.	9	3 5 29 ·93	17 26 29 ·7	15 51 ·1	3 48 ·74	3 9 18 ·67
Tues.	10	3 9 23 ·79	17 42 15 ·9	15 50 ·9	3 51 ·43	3 13 15 ·22
Wed.	11	3 13 18 ·24	17 57 44 ·4	15 50 ·7	3 53 ·53	3 17 11 ·77
Thur.	12	3 17 13 ·28	18 12 55 ·0	15 50 ·5	3 55 ·04	3 21 8 ·32
Frid.	13	3 21 8 ·91	18 27 47 ·3	15 50 ·3	3 55 ·96	3 25 4 ·87
Sat.	14	3 25 5 ·13	18 42 21 ·0	15 50 ·1	3 56 ·30	3 29 1 ·43
Sun.	15	3 29 1 ·93	18 56 35 ·9	15 49 ·9	3 56 ·05	3 32 57 ·98
Mon.	16	3 32 59 ·30	19 10 31 ·6	15 49 ·7	3 55 ·24	3 36 54 ·54
Tues.	17	3 36 57 ·25	19 24 7 ·8	15 49 ·5	3 53 ·86	3 40 51 ·11
Wed.	18	3 40 55 ·74	19 37 24 ·4	15 49 ·3	3 51 ·93	3 44 47 ·67
Thur.	19	3 44 54 ·78	19 50 20 ·9	15 49 ·1	3 49 ·45	3 48 44 ·23
Frid.	20	3 48 54 ·36	20 2 57 ·1	15 48 ·9	3 46 ·43	3 52 40 ·79
Sat.	21	3 52 54 ·47	20 15 12 ·8	15 48 ·8	3 42 ·88	3 56 37 ·35
Sun.	22	3 56 55 ·08	20 27 7 ·6	15 48 ·6	3 38 ·83	4 0 33 ·91
Mon.	23	4 0 56 ·20	20 38 41 ·4	15 48 ·5	3 34 ·27	4 4 30 ·47
Tues.	24	4 4 57 ·81	20 49 53 ·8	15 48 ·3	3 29 ·21	4 8 27 ·02
Wed.	25	4 8 59 ·92	21 0 44 ·8	15 48 ·1	3 23 ·65	4 12 23 ·57
Thur.	26	4 13 2 ·51	21 11 14 ·0	15 48 ·0	3 17 ·61	4 16 20 ·12
Frid.	27	4 17 5 ·56	21 21 21 ·2	15 47 ·8	3 11 ·12	4 20 16 ·68
Sat.	28	4 21 9 ·08	21 31 6 ·3	15 47 ·7	3 4 ·15	4 24 13 ·23
Sun.	29	4 25 13 ·05	21 40 29 ·1	15 47 ·5	2 56 ·74	4 28 9 ·79
Mon.	30	4 29 17 ·47	21 49 29 ·4	15 47 ·4	2 48 ·88	4 32 6 ·35
Tues.	31	4 33 22 ·32	21 58 7 ·1	15 47 ·3	2 40 ·59	4 36 2 ·91
Wed.	32	4 37 27 ·60	N.22 6 21 ·9	15 47 ·1	2 31 ·88	4 39 59 ·48

* The Semidiameter for *Apparent* Noon may be assumed the same as that for *Mean* Noon.

III. MAY, 1836.

MEAN TIME.

Day of the Month.	THE SUN'S		Logarithm of the Radius Vector of the Earth.	THE MOON'S			
	Longitude.	Latitude.		Semidiameter.		Horizontal Parallax.	
	Noon.	Noon.	Noon.	Noon.	Midnight.	Noon.	Midnight.
	° ′ ″	″		′ ″	′ ″	′ ″	′ ″
1	41 5 56 ·6	N.0 ·40	0 ·0036278	16 27 ·2	16 29 ·8	60 22 ·7	60 32 ·2
2	42 4 3 ·3	0 ·27	0 ·0037345	16 31 ·3	16 31 ·8	60 37 ·9	60 39 ·5
3	43 2 8 ·4	0 ·14	0 ·0038404	16 31 ·2	16 29 ·5	60 37 ·3	60 31 ·2
4	44 0 12 ·0	N.0 ·02	0 ·0039458	16 26 ·9	16 23 ·5	60 21 ·7	60 9 ·1
5	44 58 14 ·3	S.0 ·08	0 ·0040506	16 19 ·4	16 14 ·7	59 54 ·1	59 36 ·8
6	45 56 15 ·0	0 ·15	0 ·0041544	16 9 ·5	16 4 ·1	59 17 ·8	58 57 ·9
7	46 54 14 ·4	0 ·20	0 ·0042572	15 58 ·5	15 52 ·8	58 37 ·6	58 16 ·5
8	47 52 12 ·4	0 ·23	0 ·0043587	15 47 ·2	15 41 ·6	57 55 ·8	57 35 ·4
9	48 50 9 ·2	0 ·22	0 ·0044588	15 36 ·2	15 31 ·0	57 15 ·7	56 56 ·6
10	49 48 4 ·7	0 ·18	0 ·0045575	15 26 ·0	15 21 ·3	56 38 ·2	56 20 ·8
11	50 45 58 ·8	0 ·10	0 ·0046545	15 16 ·8	15 12 ·5	56 4 ·4	55 48 ·6
12	51 43 51 ·7	S.0 ·01	0 ·0047499	15 8 ·5	15 4 ·7	55 33 ·9	55 20 ·1
13	52 41 43 ·3	N.0 ·10	0 ·0048433	15 1 ·2	14 57 ·9	55 7 ·3	54 55 ·2
14	53 39 33 ·6	0 ·23	0 ·0049349	14 55 ·0	14 52 ·3	54 44 ·3	54 34 ·5
15	54 37 22 ·6	0 ·37	0 ·0050244	14 49 ·9	14 47 ·7	54 25 ·6	54 17 ·7
16	55 35 10 ·2	0 ·50	0 ·0051119	14 45 ·9	14 44 ·5	54 11 ·1	54 5 ·7
17	56 32 56 ·5	0 ·63	0 ·0051972	14 43 ·4	14 42 ·7	54 1 ·8	53 59 ·4
18	57 30 41 ·2	0 ·74	0 ·0052805	14 42 ·5	14 42 ·7	53 58 ·4	53 59 ·4
19	58 28 24 ·5	0 ·83	0 ·0053616	14 43 ·5	14 44 ·7	54 2 ·0	54 6 ·6
20	59 26 6 ·2	0 ·90	0 ·0054407	14 46 ·5	14 48 ·9	54 13 ·2	54 22 ·1
21	60 23 46 ·4	0 ·94	0 ·0055181	14 52 ·0	14 55 ·7	54 33 ·4	54 46 ·8
22	61 21 25 ·0	0 ·95	0 ·0055936	14 59 ·9	15 4 ·9	55 2 ·4	55 20 ·6
23	62 19 2 ·0	0 ·93	0 ·0056674	15 10 ·4	15 16 ·5	55 41 ·0	56 3 ·4
24	63 16 37 ·4	0 ·88	0 ·0057396	15 23 ·2	15 30 ·3	56 27 ·8	56 54 ·0
25	64 14 11 ·5	0 ·80	0 ·0058103	15 37 ·8	15 45 ·5	57 21 ·5	57 49 ·9
26	65 11 44 ·1	0 ·70	0 ·0058798	15 53 ·4	16 1 ·2	58 18 ·7	58 47 ·3
27	66 9 15 ·4	0 ·58	0 ·0059480	16 8 ·8	16 15 ·9	59 15 ·2	59 41 ·4
28	67 6 45 ·4	0 ·45	0 ·0060147	16 22 ·6	16 28 ·4	60 5 ·7	60 27 ·1
29	68 4 14 ·3	0 ·32	0 ·0060804	16 33 ·3	16 37 ·1	60 45 ·1	60 59 ·1
30	69 1 42 ·2	0 ·18	0 ·0061448	16 39 ·7	16 41 ·0	61 8 ·7	61 13 ·5
31	69 59 9 ·1	N.0 ·06	0 ·0062079	16 41 ·1	16 39 ·9	61 13 ·8	61 9 ·3
32	70 56 35 ·1	S.0 ·05	0 ·0062700	16 37 ·4	16 33 ·8	61 0 ·0	60 46 ·9

MAY, 1836. IV.

MEAN TIME.							
Day of the Week.	Day of the Month.	THE MOON'S					
		Longitude.		Latitude.		Age.	Meridian Passage.
		Noon.	Midnight.	Noon.	Midnight.	Noon.	
		° ′ ″	° ′ ″	° ′ ″	° ′ ″	d.	h. m.
Sun.	1	223 24 44 ·6	230 48 24 ·7	N.0 36 44 ·1	S.0 4 13 ·5	15 .5	12 35 ·0
Mon.	2	238 14 0 ·4	245 40 32 ·3	S.0 45 18 ·0	1 25 41 ·7	16 ·5	13 34 ·1
Tues.	3	253 7 3 ·7	260 32 39 ·3	2 4 37 ·8	2 41 22 ·2	17 ·5	14 37 ·1
Wed.	4	267 56 26 ·2	275 17 38 ·7	3 15 14 ·8	3 45 41 ·1	18 ·5	15 41 ·7
Thur.	5	282 35 36 ·1	289 49 43 ·9	4 12 12 ·5	4 34 27 ·4	19 ·5	16 45 ·1
Frid.	6	296 59 36 ·8	304 4 54 ·5	4 52 10 ·9	5 5 14 ·4	20 ·5	17 44 ·6
Sat.	7	311 5 24 ·5	318 1 1 ·4	5 13 34 ·8	5 17 14 ·5	21 ·5	18 39 ·5
Sun.	8	324 51 42 ·1	331 37 31 ·7	5 16 20 ·2	5 11 2 ·0	22 ·5	19 29 ·6
Mon.	9	338 18 35 ·0	344 55 4 ·2	5 1 33 ·1	4 48 10 ·1	23 ·5	20 15 ·8
Tues.	10	351 27 10 ·2	357 55 6 ·3	4 31 10 ·3	4 10 53 ·2	24 ·5	20 59 ·5
Wed.	11	4 19 6 ·3	10 39 26 ·4	3 47 39 ·3	3 21 50 0	25 ·5	21 41 ·8
Thur.	12	16 56 20 ·2	23 10 2 ·7	2 53 47 ·4	2 23 54 ·4	26 ·5	22 24 ·0
Frid.	13	29 20 48 ·0	35 28 50 ·3	1 52 33 ·8	1 20 8 ·3	27 ·5	23 7 ·0
Sat.	14	41 34 23 ·5	47 37 41 ·1	S.0 47 0 ·9	S.0 13 33 ·9	28 ·5	23 51 ·5
Sun.	15	53 38 56 ·6	59 38 24 ·2	N.0 19 50 ·8	N.0 52 51 ·9	29 ·5	☌
Mon.	16	65 36 18 ·6	71 32 55 ·4	1 25 8 ·9	1 56 23 ·0	0 ·9	0 38 ·0
Tues.	17	77 28 30 ·4	83 23 21 ·6	2 26 15 ·0	2 54 28 ·7	1 ·9	1 26 ·6
Wed.	18	89 17 47 ·4	95 12 9 ·2	3 20 47 ·4	3 44 56 ·7	2 ·9	2 16 ·7
Thur.	19	101 6 48 ·1	107 2 8 ·7	4 6 42 ·4	4 25 52 ·1	3 ·9	3 7 ·5
Frid.	20	112 58 35 ·7	118 56 36 ·5	4 42 13 ·3	4 55 36 ·2	4 ·9	3 58 ·0
Sat.	21	124 56 39 ·4	130 59 14 ·3	5 5 49 ·6	5 12 45 ·1	5 ·9	4 47 ·3
Sun.	22	137 4 51 ·3	143 14 2 ·9	5 16 13 ·2	5 16 7 ·2	6 ·9	5 35 ·0
Mon.	23	149 27 19 ·5	155 45 12 ·4	5 12 19 ·7	5 4 46 ·3	7 ·9	6 21 ·1
Tues.	24	162 8 10 ·8	168 36 42 ·7	4 53 22 ·1	4 38 5 ·5	8 ·9	7 6 ·2
Wed.	25	175 11 12 ·1	181 52 0 ·5	4 18 57 ·4	3 56 2 ·2	9 ·9	7 51 ·1
Thur.	26	188 39 21 ·7	195 33 24 ·6	3 29 26 ·4	2 59 23 ·7	10 ·9	8 37 ·2
Frid.	27	202 34 10 ·3	209 41 29 ·8	2 26 11 ·1	1 50 12 ·6	11 ·9	9 25 ·6
Sat.	28	216 55 6 ·2	224 14 29 ·8	N.1 11 57 ·4	N.0 32 2 ·0	12 ·9	10 17 ·8
Sun.	29	231 39 3 ·1	239 7 55 ·3	S.0 8 53 ·2	S.0 50 1 ·5	13 ·9	11 14 ·9
Mon.	30	246 40 8 ·5	254 14 37 ·0	1 30 34 ·1	2 9 41 ·7	14 ·9	12 17 ·2
Tues.	31	261 50 9 ·7	269 25 32 ·6	2 46 35 ·6	3 20 29 ·8	15 ·9	13 23 ·1
Wed.	32	276 59 33 ·8	284 31 2 ·3	S.3 50 45 ·6	S.4 16 49 ·7	16 ·9	14 29 ·7

VIII. MAY, 1836.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Declination.	Diff. Dec. for 10m.	Hour.	Right Ascension.	Declination.	Diff. Dec. for 10m.
	FRIDAY 13.				*SUNDAY* 15.		
	h. m. s.	° ′ ″	″		h. m. s.	° ′ ″	″
0	1 51 49 ·41	N. 9 29 48 ·4	132 ·95	0	3 24 41 ·69	N.19 1 24 ·7	101 ·38
1	1 53 42 ·44	9 43 6 ·1	132 ·47	1	3 26 41 ·88	19 11 33 ·0	100 ·52
2	1 55 35 ·55	9 56 20 ·9	132 ·02	2	3 28 42 ·26	19 21 36 ·1	99 ·65
3	1 57 28 ·75	10 9 33 ·0	131 ·53	3	3 30 42 ·84	19 31 34 ·0	98 ·77
4	1 59 22 ·05	10 22 42 ·2	131 ·05	4	3 32 43 ·62	19 41 26 ·6	97 ·88
5	2 1 15 ·44	10 35 48 ·5	130 ·55	5	3 34 44 ·60	19 51 13 ·9	97 ·00
6	2 3 8 ·93	10 48 51 ·8	130 ·05	6	3 36 45 ·77	20 0 55 ·9	96 ·10
7	2 5 2 ·53	11 1 52 ·1	129 ·53	7	3 38 47 ·14	20 10 32 ·5	95 ·18
8	2 6 56 ·23	11 14 49 ·3	129 ·02	8	3 40 48 ·72	20 20 3 ·6	94 ·27
9	2 8 50 ·04	11 27 43 ·4	128 ·48	9	3 42 50 ·49	20 29 29 ·2	93 ·35
10	2 10 43 ·96	11 40 34 ·3	127 ·95	10	3 44 52 ·46	20 38 49 ·3	92 ·42
11	2 12 37 ·99	11 53 22 ·0	127 ·42	11	3 46 54 ·64	20 48 3 ·8	91 ·48
12	2 14 32 ·14	12 6 6 ·5	126 ·85	12	3 48 57 ·01	20 57 12 ·7	90 ·53
13	2 16 26 ·42	12 18 47 ·6	126 ·28	13	3 50 59 ·59	21 6 15 ·9	89 ·58
14	2 18 20 ·81	12 31 25 ·3	125 ·73	14	3 53 2 ·37	21 15 13 ·4	88 ·62
15	2 20 15 .34	12 43 59 .7	125 ·13	15	3 55 5 ·34	21 24 5 ·1	87 ·63
16	2 22 9 ·99	12 56 30 ·5	124 ·55	16	3 57 8 ·52	21 32 50 ·9	86 ·68
17	2 24 4 ·78	13 8 57 ·8	123 ·97	17	3 59 11 ·91	21 41 31 ·0	85 ·68
18	2 25 59 ·70	13 21 21 ·6	123 ·35	18	4 1 15 ·49	21 50 5 ·1	84 ·70
19	2 27 54 ·76	13 33 41 ·7	122 ·75	19	4 3 19 ·27	21 58 33 ·3	83 ·68
20	2 29 49 ·97	13 45 58 ·2	122 ·12	20	4 5 23 ·25	22 6 55 ·4	82 ·70
21	2 31 45 ·31	13 58 10 ·9	121 ·48	21	4 7 27 ·44	22 15 11 ·6	81 ·67
22	2 33 40 ·80	14 10 19 ·8	120 ·87	22	4 9 31 ·82	22 23 21 ·6	80 ·67
23	2 35 36 ·44	N.14 22 25 ·0	120 ·20	23	4 11 36 ·40	N.22 31 25 ·6	79 ·63
	SATURDAY 14.				*MONDAY* 16.		
0	2 37 32 ·22	N.14 34 26 ·2	119 ·57	0	4 13 41 ·17	N.22 39 23 ·4	78 ·58
1	2 39 28 ·16	14 46 23 ·6	118 ·88	1	4 15 46 ·15	22 47 14 ·9	77 ·57
2	2 41 24 ·25	14 58 16 ·9	118 ·23	2	4 17 51 ·32	22 55 0 ·3	76 ·50
3	2 43 20 ·51	15 10 6 ·3	117 ·53	3	4 19 56 ·69	23 2 39 ·3	75 ·45
4	2 45 16 ·92	15 21 51 ·5	116 ·87	4	4 22 2 ·25	23 10 12 ·0	74 ·38
5	2 47 13 ·49	15 33 32 ·7	116 ·15	5	4 24 8 ·01	23 17 38 ·3	73 ·32
6	2 49 10 ·23	15 45 9 ·6	115 ·47	6	4 26 13 ·96	23 24 58 ·2	72 ·25
7	2 51 7 ·14	15 56 42 ·4	114 ·75	7	4 28 20 ·10	23 32 11 ·7	71 ·15
8	2 53 4 ·21	16 8 10 ·9	114 ·02	8	4 30 26 ·43	23 39 18 ·6	70 ·07
9	2 55 1 ·45	16 19 35 ·0	113 ·30	9	4 32 32 ·95	23 46 19 ·0	68 ·97
10	2 56 58 ·87	16 30 54 ·8	112 ·55	10	4 34 39 ·65	23 53 12 ·8	67 ·87
11	2 58 56 ·46	16 42 10 ·1	111 ·82	11	4 36 46 ·55	24 0 0 ·0	66 ·75
12	3 0 54 ·22	16 53 21 ·0	111 ·07	12	4 38 53 ·62	24 6 40 ·5	65 ·63
13	3 2 52 ·16	17 4 27 ·4	110 ·30	13	4 41 0 ·88	24 13 14 ·3	64 ·52
14	3 4 50 ·28	17 15 29 ·2	109 ·52	14	4 43 8 ·32	24 19 41 ·4	63 ·38
15	3 6 48 ·58	17 26 26 ·3	108 ·75	15	4 45 15 ·93	24 26 1 ·7	62 ·25
16	3 8 47 ·07	17 37 18 ·8	107 ·97	16	4 47 23 ·72	24 32 15 ·2	61 ·10
17	3 10 45 ·73	17 48 6 ·6	107 ·17	17	4 49 31 ·69	24 38 21 ·8	59 ·95
18	3 12 44 ·59	17 58 49 ·6	106 ·37	18	4 51 39 ·83	24 44 21 ·5	58 ·82
19	3 14 43 ·63	18 9 27 ·8	105 ·55	19	4 53 48 ·14	24 50 14 ·4	57 ·65
20	3 16 42 ·86	18 20 1 ·1	104 ·73	20	4 55 56 ·62	24 56 0 ·3	56 ·48
21	3 18 42 ·28	18 30 29 ·5	103 ·90	21	4 58 5 ·27	25 1 39 ·2	55 ·32
22	3 20 41 ·89	18 40 52 ·9	103 ·08	22	5 0 14 ·07	25 7 11 ·1	54 ·13
23	3 22 41 ·69	18 51 11 ·4	102 ·22	23	5 2 23 ·04	25 12 35 ·9	52 ·97
24	3 24 41 ·69	N.19 1 24 ·7		24	5 4 32 ·17	N.25 17 53 ·7	

Second Differences.

Hours & Min.		1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	11′
h. m.	h. m.	″	″	″	″	″	″	″	″	″	″	″
0 0	12 0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0 10	11 50	0.4	0.8	1.2	1.6	2.0	2.4	2.9	3.3	3.7	4.1	4.5
0 20	11 40	0.8	1.6	2.4	3.2	4.1	4.9	5.7	6.5	7.3	8.1	8.9
0 30	11 30	1.2	2.4	3.6	4.8	6.0	7.2	8.4	9.6	10.8	12.0	13.2
0 40	11 20	1.6	3.1	4.7	6.3	7.9	9.4	11.0	12.6	14.2	15.7	17.3
0 50	11 10	1.9	3.9	5.8	7.8	9.7	11.6	13.6	15.5	17.4	19.4	21.4
1 0	11 0	2.3	4.6	6.9	9.2	11.5	13.8	16.0	18.3	20.6	22.9	25.2
1 10	10 50	2.6	5.3	7.9	10.5	13.2	15.8	18.4	21.1	23.7	26.3	29.0
1 2 0	10 40	3.0	5.9	8.9	11.9	14.8	17.8	20.7	23.7	26.7	29.6	32.6
1 30	10 30	3.3	6.6	9.8	13.1	16.4	19.7	23.0	26.3	29.5	32.8	36.1
1 40	10 20	3.6	7.2	10.8	14.4	17.9	21.5	25.1	28.7	32.3	35.9	39.5
1 50	10 10	3.9	7.8	11.6	15.5	19.4	23.3	27.2	31.0	34.9	38.8	42.7
2 0	10 0	4.2	8.3	12.5	16.7	20.8	25.0	29.2	33.3	37.5	41.7	45.8
2 10	9 50	4.4	8.9	13.3	17.8	22.2	26.6	31.1	35.5	40.0	44.4	48.8
2 20	9 40	4.7	9.4	14.1	18.8	23.5	28.2	32.9	37.6	42.3	47.0	51.7
2 30	9 30	4.9	9.9	14.8	19.8	24.7	29.7	34.6	39.6	44.5	49.5	54.4
2 40	9 20	5.2	10.4	15.6	20.7	25.9	31.1	36.3	41.5	46.7	51.9	57.0
2 50	9 10	5.4	10.8	16.2	21.6	27.1	32.5	37.9	43.3	48.7	54.1	59.5
3 0	9 0	5.6	11.3	16.9	22.5	28.1	33.8	39.4	45.0	50.6	56.3	61.9
3 10	8 50	5.8	11.7	17.5	23.3	29.1	35.0	40.8	46.6	52.4	58.3	64.1
3 20	8 40	6.0	12.0	18.1	24.1	30.1	36.1	42.1	48.1	54.2	60.2	66.2
3 30	8 30	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8	62.0	68.2
3 40	8 20	6.4	12.7	19.1	25.5	31.8	38.2	44.6	50.9	57.3	63.7	70.0
3 50	8 10	6.5	13.0	19.6	26.1	32.6	39.1	45.7	52.2	58.7	65.2	71.7
4 0	8 0	6.7	13.3	20.0	26.7	33.3	40.0	46·7	53.3	60.0	66.7	73.3
4 10	7 50	6.8	13.6	20.4	27.2	34.0	40.8	47·6	54.4	61.2	68.0	74.8
4 20	7 40	6.9	13.8	20.8	27.7	34.6	41.5	48·4	55.4	62.3	69.2	76.1
4 30	7 30	7.0	14.1	21.1	28.1	35.2	42.2	49.2	56.2	63.3	70.3	77.3
4 40	7 20	7.1	14.3	21.4	28.5	35.6	42.8	49.9	57.0	64.2	71.3	78.4
4 50	7 10	7.2	14.4	21.6	28.9	36.1	43.3	50.5	57.7	64.9	72.2	79.4
5 0	7 0	7.3	14.6	21.9	29.2	36.5	43.8	51·0	58.3	65.6	72.9	80.2
5 10	6 50	7.4	14.7	22.1	29.4	36.8	44.1	51·5	58.8	66.2	73.6	80.9
5 20	6 40	7.4	14.8	22.2	29·6	37.0	44.4	51·9	59.3	66.7	74.1	81.5
5 30	6 30	7.4	14.9	22.3	29.8	37.2	44.7	52.1	59.6	67.0	74.5	81.9
5 40	6 20	7.5	15.0	22.4	29.9	37.4	44.9	52.3	59.8	67.3	74.8	82.2
5 50	6 10	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.4	74.9	82.4
6 0	6 0	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	75.0	82.5

Second Differences.

Hours & Min.		10″	20″	30″	40″	50″	1″	2″	3″	4″	5″	6″	7″	8″	9″
h. m.	h. m.	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0 0	12 0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0 10	11 50	0.1	0.1	0.2	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1
0 20	11 40	0.1	0.3	0.4	0.5	0.7	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1
0 30	11 30	0.2	0.4	0.6	0.8	1.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.2	0.2
0 40	11 20	0.3	0.5	0.8	1.0	1.3	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2
0 50	11 10	0.3	0.6	1.0	1.3	1.6	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3
1 0	11 0	0.4	0·8	1.1	1.5	1.9	0.0	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.3
1 10	10 50	0.4	0.9	1.3	1.8	2.2	0.0	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4
1 20	10 40	0.5	1.0	1.5	2.0	2.5	0.0	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4
1 30	10 30	0.5	1.1	1.6	2.2	2.7	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	0.5
1 40	10 20	0.6	1.2	1.8	2.4	3.0	0.1	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.5
1 50	10 10	0.6	1.3	1.9	2.6	3.2	0.1	0.1	0.2	0.3	0.3	0.4	0.5	0.5	0.6
2 0	10 0	0.7	1.4	2.1	2.8	3.5	0.1	0.1	0.2	0.3	0.3	0.4	0.5	0.6	0.6
2 10	9 50	0.7	1.5	2.2	3.0	3.7	0.1	0.1	0.2	0.3	0.4	0.4	0.5	0.6	0.7
2 20	9 40	0.8	1.6	2.3	3.1	3.9	0.1	0.2	0.2	0.3	0.4	0.5	0.5	0.6	0.7
2 30	9 30	0.8	1.6	2.5	3.3	4.1	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.7
2 40	9 20	0.9	1.7	2.6	3.5	4.3	0.1	0.2	0.3	0.3	0.4	0.5	0.6	0.7	0.8
2 50	9 10	0.9	1.8	2.7	3.6	4.5	0.1	0.2	0.3	0.4	0.5	0.5	0.6	0.7	0.8
3 0	9 0	0.9	1.9	2.8	3.8	4.7	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8
3 10	8 50	1.0	1.9	2.9	3.9	4.9	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
3 20	8 40	1.0	2.0	3.0	4.0	5.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
3 30	8 30	1.0	2.1	3.1	4.1	5.2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
3 40	8 20	1.1	2.1	3.2	4.2	5.3	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	1.0
3 50	8 10	1.1	2.2	3.3	4.3	5.4	0.1	0.2	0.3	0.4	0.5	0.7	0.8	0.9	1.0
4 0	8 0	1.1	2.2	3.3	4.4	5.6	0.1	0.2	0.3	0.4	0.6	0.7	0.8	0.9	1.0
4 10	7 50	1.1	2.3	3.4	4.5	5.7	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.9	1.0
4 20	7 40	1.2	2.3	3.5	4.6	5.8	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.9	1.0
4 30	7 30	1.2	2.3	3.5	4.7	5.9	0.1	0.2	0.4	0.5	0.6	0.7	0.8	0.9	1.1
4 40	7 20	1.2	2.4	3.6	4.8	5.9	0.1	0.2	0.4	0.5	0.6	0.7	0.8	1.0	1.1
4 50	7 10	1.2	2.4	3.6	4.8	6.0	0.1	0.2	0.4	0.5	0.6	0.7	0.8	1.0	1.1
5 0	7 0	1.2	2.4	3.6	4.9	6.1	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1
5 10	6 50	1.2	2.5	3.7	4.9	6.1	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1
5 20	6 40	1.2	2.5	3.7	4.9	6.1	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1
5 30	6 30	1.2	2.5	3.7	5.0	6.2	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1
5 40	6 20	1.2	2.5	3.7	5.0	6.2	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1
5 50	6 10	1.2	2.5	3.7	5.0	6.2	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1
6 0	6 0	1.3	2.6	3.8	5.0	6.3	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1

TABLE LVII.

Third Differences.

Time after noon or midnight.	10″	20″	30″	40″	50″	1′	2′	3′	4′	5′	Time after noon or midnight.
+	″	″	″	″	″	″	″	″	″	″	—
0h. 0m.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12h. 0m.
0 30	0.0	0.1	0.1	0.1	0.2	0.2	0.4	0.5	0.7	0.9	11 30
1 0	0.1	0.1	0.2	0.2	0.3	0.3	0.6	1.0	1.3	1.5	11 0
1 30	0.1	0.1	0.2	0.3	0.3	0.4	0.8	1.2	1.6	2.1	10 30
2 0	0.1	0.2	0.2	0.3	0.4	0.5	0.9	1.4	1.9	2.3	10 0
2 30	0.1	0.2	0.2	0.3	0.4	0.5	1.0	1.4	1.9	2.4	9 30
3 0	0.1	0.2	0.2	0.3	0.4	0.5	0.9	1.4	1.9	2.3	9 0
3 30	0.1	0.1	0.2	0.3	0.4	0.4	0.9	1.3	1.7	2.2	8 30
4 0	0.1	0.1	0.2	0.2	0.3	0.4	0.7	1.1	1.5	1.9	8 0
4 30	0.0	0.1	0.1	0.2	0.2	0.3	0.6	0.9	1.2	1.5	7 30
5 0	0.0	0.1	0.1	0.1	0.2	0.2	0.4	0.6	0.8	1.0	7 0
5 30	0.0	0.0	0.1	0.1	0.1	0.1	0.2	0.3	0.4	0.5	6 30
6 0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	6 0
+											—

TABLE LVIII.

Fourth Differences.

Time after noon or midnight.	10″	20″	30″	40″	50″	1′	2′	3′	Time after noon or midnight.
h. m.	″	″	″	″	″	″	″	″	h. m.
0 0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12 0
0 30	0.0	0.1	0.1	0.1	0.2	0.2	0.4	0.6	11 30
1 0	0.1	0.1	0.2	0.3	0.3	0.4	0.8	1.2	11 0
1 30	0.1	0.2	0.3	0.4	0.5	0.6	1.2	1.7	10 30
2 0	0.1	0.2	0.4	0.5	0.6	0.7	1.5	2.2	10 0
2 30	0.1	0.3	0.4	0.6	0.7	0.9	1.8	2.7	9 30
3 0	0.2	0.3	0.5	0.7	0.9	1.0	2.1	3.1	9 0
3 30	0.2	0.4	0.6	0.8	0.9	1.1	2.3	3.4	8 30
4 0	0.2	0.4	0.6	0.8	1.0	1.2	2.5	3.7	8 0
4 30	0.2	0.4	0.7	0.9	1.1	1.3	2.6	3.9	7 30
5 0	0.2	0.5	0.7	0.9	1.1	1.4	2.7	4.1	7 0
5 30	0.2	0.5	0.7	0.9	1.2	1.4	2.8	4.2	6 30
6 0	0.2	0.5	0.7	0.9	1.2	1.4	2.8	4.2	6 0

Mercury's Epochs.

	M. Long.				Aphelion.				Node.				II.	III.
	s	°	′	″	s	°	′	″	s	°	′	″		
1835	7	11	3	34	8	14	52	47	1	16	21	44	11	1559
1836 B.	9	8	52	11	8	14	53	43	1	16	22	26	377	1925
1837	11	2	35	14	8	14	54	39	1	16	23	8	336	228
1838	0	26	18	18	8	14	55	35	1	16	23	50	296	592
1839	2	20	1	21	8	14	56	31	1	16	24	32	255	955
1840 B.	4	17	49	58	8	14	57	27	1	16	25	15	217	1320
1841	6	11	33	1	8	14	58	23	1	16	25	57	176	1682
1842	8	5	16	5	8	14	59	19	1	16	26	39	135	2047
1843	9	28	59	8	8	15	0	15	1	16	27	21	95	351
1844 B.	11	26	47	45	8	15	1	11	1	16	28	3	56	717
1845	1	10	30	48	8	15	2	7	1	16	28	45	15	1083
1846	3	14	13	52	8	15	3	3	1	16	29	27	380	1450
1847	5	7	56	55	8	15	3	59	1	16	30	10	340	1816
1848 B.	7	5	45	32	8	15	4	55	1	16	30	52	300	113
1849	8	29	28	35	8	15	5	51	1	16	31	34	260	478
1850	10	23	11	39	8	15	6	47	1	16	32	16	220	844
1851	0	16	54	42	8	15	7	43	1	16	32	58	180	1209
1852 B.	2	14	43	19	8	15	8	39	1	16	33	40	140	1575
1853	4	8	26	22	8	15	9	35	1	16	34	23	100	1940
1854	6	2	9	26	8	15	10	31	1	16	35	5	59	235
1855	7	25	52	29	8	15	11	27	1	16	35	47	19	600
1856 B.	9	23	41	6	8	15	12	23	1	16	36	29	385	966
1857	11	17	24	9	8	15	13	19	1	16	37	11	345	1331
1858	1	11	7	13	8	15	14	15	1	16	37	53	304	1696
1859	3	4	50	16	8	15	15	11	1	16	38	35	264	2061
1860 B.	5	2	38	53	8	15	16	7	1	16	39	18	224	357
1861	6	26	21	56	8	15	17	4	1	16	40	0	184	722
1862	8	20	5	0	8	15	18	0	1	16	40	42	144	1087
1863	10	13	48	3	8	15	18	56	1	16	41	24	103	1452
1864 B.	0	11	36	40	8	15	19	52	1	16	42	6	64	1818
1865	2	5	19	43	8	15	20	48	1	16	42	48	24	113
1866	3	29	2	47	8	15	21	44	1	16	43	30	389	478
1867	5	22	45	50	8	15	22	40	1	16	44	13	348	843
1868 B.	7	20	34	27	8	15	23	36	1	16	44	55	309	1209
1869	9	14	17	30	8	15	24	32	1	16	45	37	268	1574
1870	11	8	0	34	8	15	25	28	1	16	46	19	228	1939
1871	1	1	43	37	8	15	26	24	1	16	47	1	188	234
1872 B.	2	29	32	14	8	15	27	20	1	16	47	43	148	599
1873	4	23	15	17	8	15	28	16	1	16	48	26	108	965
1874	6	16	58	21	8	15	29	12	1	16	49	8	67	1330
1875	8	10	41	24	8	15	30	8	1	16	49	50	27	1695
1876 B.	10	8	30	1	8	15	31	4	1	16	50	32	393	2061
1877	0	2	13	4	8	15	32	0	1	16	51	14	353	356
1878	1	25	56	8	8	15	32	56	1	16	51	56	312	720
1879	3	19	39	11	8	15	33	52	1	16	52	38	272	1086
1880 B.	5	17	27	48	8	15	34	48	1	16	53	21	232	1452
1881	7	11	10	51	8	15	35	44	1	16	54	3	192	1817
1882	9	4	53	55	8	15	36	40	1	16	54	45	152	112
1883	10	28	36	58	8	15	37	36	1	16	55	27	111	477
1884 B.	0	26	25	35	8	15	38	32	1	16	56	9	72	843
1865	2	20	8	38	8	15	39	28	1	16	56	51	32	1208

TABLE LX.

Mercury's Motions for Months.

Months.		D.	Longitude.				Aph.	Nod.
			s	°	′	″	″	″
Jan.	Bis.	0	0	0	0	0	0	0
	Com.	1	0	4	5	33	0	0
Feb.	Bis.	31	4	6	51	49	5	4
	Com.	32	4	10	57	22	5	4
March,		60	8	5	32	33	9	7
April,		91	0	12	24	23	14	11
May,		121	4	15	10	39	19	14
June,		152	8	22	2	29	23	18
July,		182	0	24	48	46	28	21
August,		213	5	1	40	35	33	25
September,		244	9	8	32	24	38	28
October.		274	1	11	18	41	42	32
November,		305	5	18	10	30	47	35
December,		335	9	20	56	47	52	39

TABLE LXI.

Motions for Days and Hours.

Days.	Longitude.				Ap.	No.	Hrs.	Long.		
	s	°	′	″	″	″		°	′	″
1	0	0	0	0	0	0	1	0	10	14
2	0	4	5	33	0	0	2	0	20	28
3	0	8	11	5	0	0	3	0	30	42
4	0	12	16	38	1	0	4	0	40	55
5	0	16	22	10	1	0	5	0	51	9
6	0	20	27	43	1	1	6	1	1	23
7	0	24	33	15	1	1	7	1	11	37
8	0	28	38	48	1	1	8	1	21	51
9	1	2	44	20	1	1	9	1	32	5
10	1	6	49	53	2	1	10	1	42	19
11	1	10	55	26	2	1	11	1	52	32
12	1	15	0	58	2	1	12	2	2	46
13	1	19	6	31	2	1	13	2	13	0
14	1	23	12	3	2	2	14	2	23	14
15	1	27	17	36	2	2	15	2	33	28
16	2	1	23	8	2	2	16	2	43	42
17	2	5	28	41	3	2	17	2	53	56
18	2	9	34	13	3	2	18	3	4	9
19	2	13	39	46	3	2	19	3	14	23
20	2	17	45	19	3	2	20	3	24	37
21	2	21	50	51	3	2	21	3	34	51
22	2	25	56	24	3	2	22	3	45	5
23	3	0	1	56	4	3	23	3	55	19
24	3	4	7	29	4	3	24	4	5	33
25	3	8	13	1	4	3				
26	3	12	18	34	4	3				
27	3	16	24	6	4	3				
28	3	20	29	39	4	3				
29	3	24	35	12	4	3				
30	3	28	40	44	4	3				
31	4	2	46	17	5	3				

TABLE LXII.

Mots. Min. & Sec.

Min.	Long.		Sec.	Lon.
	′	″		″
1	0	10	1	0
2	0	20	2	0
3	0	31	3	1
4	0	41	4	1
5	0	51	5	1
6	1	1	6	1
7	1	12	7	1
8	1	22	8	1
9	1	32	9	2
10	1	42	10	2
11	1	53	11	2
12	2	3	12	2
13	2	13	13	2
14	2	23	14	2
15	2	33	15	3
16	2	44	16	3
17	2	54	17	3
18	3	4	18	3
19	3	14	19	3
20	3	25	20	3
21	3	35	21	4
22	3	45	22	4
23	3	55	23	4
24	4	6	24	4
25	4	16	25	4
26	4	26	26	4
27	4	36	27	5
28	4	46	28	5
29	4	57	29	5
30	5	7	30	5
31	5	17	31	5
32	5	27	32	5
33	5	38	33	6
34	5	48	34	6
35	5	58	35	6
36	6	8	36	6
37	6	18	37	6
38	6	29	38	6
39	6	39	39	7
40	6	49	40	7
41	6	59	41	7
42	7	10	42	7
43	7	20	43	7
44	7	30	44	8
45	7	40	45	8
46	7	51	46	8
47	8	1	47	8
48	8	11	48	8
49	8	21	49	8
50	8	31	50	9
51	8	42	51	9
52	8	52	52	9
53	9	2	53	9
54	9	12	54	9
55	9	23	55	9
56	9	33	56	10
57	9	43	57	10
58	9	53	58	10
59	10	4	59	10
60	10	14	60	10

Equation of Mercury's Centre.

ARGUMENT. Mean Anomaly.

	0^s—			I^s—			II^s—			III^s—			IV^s—			V^s—			
°	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°
0	0	0	0	9	34	57	17	47	21	22	56	6	22	46	14	14	55	46	30
1	0	19	36	9	53	11	18	1	15	23	1	46	22	38	36	14	31	30	29
2	0	39	12	10	11	19	18	14	57	23	7	3	22	30	26	14	6	42	28
3	0	58	48	10	29	22	18	28	26	23	11	59	22	21	44	13	41	25	27
4	1	18	23	10	47	18	18	41	42	23	16	33	22	12	29	13	15	39	26
5	1	37	57	11	5	9	18	54	45	23	20	45	22	2	42	12	49	23	25
6	1	57	31	11	22	53	19	7	35	23	24	33	21	52	23	12	22	39	24
7	2	17	3	11	40	30	19	20	11	23	27	59	21	41	30	11	55	27	23
8	2	36	35	11	58	1	19	32	33	23	31	1	21	30	4	11	27	48	22
9	2	56	5	12	15	24	19	44	41	23	33	39	21	18	5	10	59	44	21
10	3	15	33	12	32	41	19	56	34	23	35	52	21	5	31	10	31	14	20
11	3	34	59	12	49	50	20	8	13	23	37	41	20	52	25	10	2	20	19
12	3	54	24	13	6	51	20	19	37	23	39	5	20	38	44	9	33	2	18
13	4	13	47	13	23	45	20	30	45	23	40	4	20	24	29	9	3	22	17
14	4	33	7	13	40	30	20	41	38	23	40	37	20	9	40	8	33	20	16
15	4	52	25	13	57	8	20	52	15	23	40	44	19	54	17	8	2	58	15
16	5	11	40	14	13	37	21	2	36	23	40	24	19	38	20	7	32	16	14
17	5	30	53	14	29	57	21	12	40	23	39	38	19	21	48	7	1	17	13
18	5	50	2	14	46	8	21	22	27	23	38	24	19	4	43	6	30	0	12
19	6	9	8	15	2	10	21	31	57	23	36	43	18	47	3	5	58	27	11
20	6	28	11	15	18	3	21	41	10	23	34	34	18	28	49	5	26	40	10
21	6	47	11	15	33	46	21	50	5	23	31	57	18	10	0	4	54	39	9
22	7	6	7	15	49	19	21	58	42	23	28	51	17	50	38	4	22	26	8
23	7	24	58	16	4	42	22	7	0	23	25	16	17	30	42	3	50	1	7
24	7	43	46	16	19	55	22	15	0	23	21	12	17	10	13	3	17	28	6
25	8	2	30	16	34	57	22	22	41	23	16	38	16	49	10	2	44	46	5
26	8	21	9	16	49	48	22	30	2	23	11	34	16	27	34	2	11	57	4
27	8	39	43	17	4	29	22	37	3	23	6	0	16	5	25	1	39	3	3
28	8	58	13	17	18	58	22	43	45	22	59	56	15	42	44	1	6	4	2
29	9	16	37	17	33	16	22	50	6	22	53	21	15	19	31	0	33	3	1
30	9	34	57	17	47	21	22	56	6	22	46	14	14	55	46	0	0	0	0
	XI^s+			X^s+			IX^s+			VIII^s+			VII^s+			VI^s+			

TABLE LXIV.

Equations 2 *and* 3. *Always Affirmative.*

Arg.	2	Arg.	2	Arg.	2	Arg.	3	Arg.	3	Arg.	3	Arg.	3
0	9″	260	1″	520	5″	0	5″	650	5″	1300	19″	1950	8″
20	9	280	2	540	4	50	4	700	6	1350	19	2000	7
40	9	300	4	560	3	100	3	750	7	1400	19	2050	6
60	8	320	5	580	2	150	2	800	8	1450	19	2100	5
80	7	340	6	600	1	200	2	850	10	1500	19	2150	4
120	6	360	7	620	1	250	1	900	11	1550	18	2200	3
140	5	380	8	640	1	300	1	950	12	1600	17	2250	2
160	4	400	9	660	1	350	1	1000	14	1650	16	2300	1
180	2	420	9	680	2	400	1	1050	15	1700	15	2350	1
200	1	440	9	700	3	450	1	1100	16	1750	14	2400	1
220	1	460	9	720	5	500	2	1150	17	1800	13	2450	1
240	1	480	8	740	6	550	3	1200	18	1850	11		
260	1	500	7	760	7	600	4	1250	18	1900	10		
280	1	520	5	780	8	650	5	1300	19	1950	8		

Mercury's Radius Vector.

ARGUMENT. Mean Anomaly.

	0^s	I^s	II^s	III^s	IV^s	V^s	
0°	0.46669	0.45923	0.43734	0.40303	0.36138	0.32362	30°
1	0.46668	0.45872	0.43638	0.40171	0.35997	0.32263	29
2	0.46666	0.45820	0.43540	0.40039	0.35857	0.32167	28
3	0.46662	0.45767	0.43441	0.39907	0.35717	0.32073	27
4	0.46656	0.45712	0.43341	0.39773	0.35578	0.31982	26
5	0.46649	0.45655	0.43240	0.39639	0.35440	0.31893	25
6	0.46639	0.45597	0.43137	0.39504	0.35302	0.31807	24
7	0.46628	0.45537	0.43032	0.39368	0.35164	0.31725	23
8	0.46616	0.45475	0.42927	0.39231	0.35028	0.31645	22
9	0.46602	0.45412	0.42820	0.39094	0.34892	0.31568	21
10	0.46586	0.45347	0.42712	0.38956	0.34757	0.31495	20
11	0.46569	0.45281	0.42602	0.38818	0.34624	0.31424	19
12	0.46549	0.45213	0.42492	0.38679	0.34491	0.31357	18
13	0.46529	0.45144	0.42380	0.38540	0.34359	0.31293	17
14	0.46506	0.45073	0.42266	0.38400	0.34229	0.31232	16
15	0.46482	0.45001	0.42152	0.38260	0.34099	0.31175	15
16	0.46456	0.44927	0.42036	0.38120	0.33972	0.31121	14
17	0.46429	0.44851	0.41919	0.37979	0.33845	0.31071	13
18	0.46400	0.44774	0.41802	0.37838	0.33720	0.31024	12
19	0.46369	0.44696	0.41682	0.37696	0.33597	0.30981	11
20	0.46337	0.44616	0.41562	0.37555	0.33475	0.30941	10
21	0.46302	0.44534	0.41441	0.37413	0.33355	0.30905	9
22	0.46267	0.44451	0.41318	0.37271	0.33236	0.30873	8
23	0.46229	0.44366	0.41195	0.37129	0.33120	0.30844	7
24	0.46190	0.44280	0.41070	0.36987	0.33005	0.30819	6
25	0.46150	0.44193	0.40945	0.36845	0.32893	0.30798	5
26	0.46108	0.44104	0.40818	0.36704	0.32782	0.30781	4
27	0.46064	0.44014	0.40691	0.36562	0.32674	0.30768	3
28	0.46018	0.43922	0.40562	0.36420	0.32567	0.30758	2
29	0.45971	0.43829	0.40433	0.36279	0.32464	0.30752	1
30	0.45923	0.43734	0.40303	0.36138	0.32362	0.30750	0
	XI^s	X^s	IX^s	$VIII^s$	VII^s	VI^s	

Reduction to the Ecliptic.

ARGUMENT. Orbit Long. of Mercury—Long. of Node.

	0^s , VI^s	I^s , VII^s	II^s , VIII^s	III^s , IX^s	IV^s , X^s	V^s , XI^s
0°	— 0′ 15″	— 11′ 22″	— 11′ 25″	— 0′ 15″	+ 10′ 55″	+ 10′ 52″
1	0 42	11 35	11 11	+ 0 12	11 8	10 38
2	1 9	11 48	10 56	0 39	11 19	10 24
3	1 35	11 59	10 41	1 6	11 31	10 8
4	2 2	12 10	10 25	1 33	11 42	9 52
5	2 29	12 20	10 8	1 59	11 51	9 35
6	— 2 55	— 12 28	— 9 50	+ 2 26	+ 12 0	+ 9 17
7	3 21	12 36	9 32	2 52	12 8	8 59
8	3 47	12 43	9 13	3 18	12 15	8 40
9	4 13	12 49	8 53	3 44	12 21	8 20
10	4 38	12 55	8 33	4 10	12 26	8 0
11	— 5 3	— 12 59	— 8 12	+ 4 35	+ 12 30	+ 7 39
12	5 28	13 2	7 50	5 0	12 33	7 17
13	5 52	13 5	7 28	5 24	12 35	6 55
14	6 16	13 6	7 5	5 49	12 37	6 33
15	6 40	13 7	6 42	6 12	12 37	6 10
16	— 7 3	— 13 7	— 6 19	+ 6 35	+ 12 36	+ 5 46
17	7 25	13 5	5 54	6 58	12 35	5 22
18	7 47	13 3	5 30	7 20	12 32	4 58
19	8 9	13 0	5 5	7 42	12 29	4 33
20	8 30	12 56	4 40	8 3	12 25	4 8
21	— 8 50	— 12 51	— 4 14	+ 8 23	+ 12 19	+ 3 43
22	9 10	12 45	3 48	8 43	12 13	3 17
23	9 29	12 38	3 22	9 2	12 6	2 51
24	9 47	12 30	2 56	9 20	11 58	2 25
25	10 5	12 21	2 29	9 38	11 49	1 59
26	— 10 22	— 12 12	— 2 3	+ 9 55	+ 11 40	+ 1 32
27	10 38	12 1	1 36	10 11	11 29	0 55
28	10 54	11 49	1 9	10 26	11 18	0 38
29	11 8	11 38	0 42	10 41	11 5	+ 0 12
30	— 11 22	— 11 25	— 0 15	+ 10 55	+ 10 52	— 0 15

TABLE LXVII.

Heliocentric Longitudes, &c. of the Planet Venus, at the times of the next two Transits over the Sun's Disc.

Times.	Hel. Long. from true Equinox.	Hel. Lat.	Rad. Vector.
1874, Dec. 8th, at 12h.	76° 41′ 36.6″	4′ 6.3″N.	0.7203632
16h.	76 57 44.1	5 3.5	0.7203449
20h.	77 13 51.5	6 1.0	0.7203268
1882, Dec. 6th, at noon.	74 12 55.7	4 58.1S.	0.7205502
4h.	74 29 2.5	4 0.8	0.7205315
8h.	74 45 9.7	3 3.5	0.7205127

NOTE.—The Aberration in Longitude at the time of each Transit, is 3.4″; to be *added* to the true geocentric longitude, to obtain the apparent longitude.

Mercury's Heliocentric Latitude for the year 1800, with the Secular Variation.

Argument. Orbit Longitude of Mercury—Long. of Node.

	Latitude. 0^s N. VI^s S.	Sec. Var. +	Log. Cos. Lat.	Latitude. I^s N. VII^s S.	Sec. Var. +	Log. Cos. Lat.	Latitude. II^s N. VIII^s S.	Sec. Var. +	Log. Cos. Lat.	
°	° ′ ″	″		° ′ ″	″		° ′ ″	″		°
0	0 0 0	0	10.00000	3 29 39	9	9.99919	6 3 35	16	9.99757	30
1	0 7 19	0	10.00000	3 35 58	9	9.99914	6 7 13	16	9.99752	29
2	0 14 37	1	10.00000	3 42 13	10	9.99909	6 10 43	16	9.99747	28
3	0 21 56	1	9.99999	3 48 24	10	9.99904	6 14 7	16	9.99742	27
4	0 29 14	1	9.99998	3 54 31	10	9.99899	6 17 24	16	9.99738	26
5	0 36 31	2	9.99998	4 0 33	10	9.99894	6 20 34	17	9.99733	25
6	0 43 48	2	9.99996	4 6 31	11	9.99888	6 23 37	17	9.99729	24
7	0 51 4	2	9.99995	4 12 25	11	9.99883	6 26 33	17	9.99725	23
8	0 58 19	2	9.99994	4 18 14	11	9.99877	6 29 22	17	9.99721	22
9	1 5 33	3	9.99992	4 23 59	11	9.99872	6 32 4	17	9.99717	21
10	1 12 46	3	9.99990	4 29 38	12	9.99866	6 34 39	17	9.99713	20
11	1 19 58	3	9.99988	4 35 13	12	9.99861	6 37 6	17	9.99710	19
12	1 27 8	4	9.99986	4 40 43	12	9.99855	6 39 27	17	9.99706	18
13	1 34 17	4	9.99984	4 46 8	12	9.99849	6 41 39	17	9.99703	17
14	1 41 24	4	9.99981	4 51 27	13	9.99844	6 43 45	18	9.99700	16
15	1 48 29	5	9.99978	4 56 41	13	9.99838	6 45 43	18	9.99697	15
16	1 55 32	5	9.99975	5 1 50	13	9.99832	6 47 34	18	9.99694	14
17	2 2 33	5	9.99972	5 6 53	13	9.99827	6 49 17	18	9.99691	13
18	2 9 31	6	9.99969	5 11 51	14	9.99821	6 50 52	18	9.99689	12
19	2 16 28	6	9.99966	5 16 43	14	9.99815	6 52 21	18	9.99687	11
20	2 23 22	6	9.99962	5 21 29	14	9.99810	6 53 41	18	9.99685	10
21	2 30 13	6	9.99959	5 26 9	14	9.99804	6 54 54	18	9.99683	9
22	2 37 2	7	9.99955	5 30 44	14	9.99799	6 55 59	18	9.99681	8
23	2 43 48	7	9.99951	5 35 12	15	9.99793	6 56 57	18	9.99680	7
24	2 50 31	7	9.99947	5 39 34	15	9.99788	6 57 47	18	9.99678	6
25	2 57 11	8	9.99942	5 43 51	15	9.99782	6 58 29	18	9.99677	5
26	3 3 47	8	9.99938	5 48 0	15	9.99777	6 59 4	18	9.99676	4
27	3 10 21	8	9.99933	5 52 4	15	9.99772	6 59 31	18	9.99676	3
28	3 16 50	9	9.99929	5 56 1	15	9.99767	6 59 51	18	9.99675	2
29	3 23 17	9	9.99924	5 59 51	16	9.99762	7 0 2	18	9.99675	1
30	3 29 39	9	9.99919	6 3 35	16	9.99757	7 0 6	18	9.99675	0
	XI^s S. V^s N.	+		X^s S. IV^s N.	+		IX^s S. III^s N.	+		

Multiplier for Aber. in Lat. Arg. Geoc. Lat.

Arg.	Mult.
0°	0.00
1	0.02
2	0.03
3	0.05
4	0.07
5	0.09
6	0.10
7	0.12

Aberration in Latitude. Part IV.

Argument. Arg. of Latitude.

Arg.	Aber.	Arg.	Arg.	Aber.	Arg.
0°	— 5″	360°	90°	— 1″	270°
10	5	350	100	0	260
20	5	340	110	+ 1	250
30	4	330	120	1	240
40	4	320	130	2	230
50	3	310	140	2	220
60	3	300	150	3	210
70	2	290	160	3	200
80	1	280	170	3	190
90	— 1	270	180	+ 3	180

Aberration of Mercury in Longitude.

Part. I. Arg. Elong.			Part II. Arg. Ann. Par.			Part. III. Arg. Geoc. Long.		
Elon.	Aberrat.	Elon.	Ann. Par.	Aberrat.	Ann. Par.	Geoc. Long.	Aberrat.	Geoc. Long.
°	″	°	°	″	°	°	″	°
0	—20—	360	0	— 33 —	360	0	—2+	180
4	20	356	4	33	356	4	2	184
8	20	352	8	32	352	8	3	188
12	20	348	12	32	348	12	3	192
16	19	344	16	31	344	16	4	196
20	19	340	20	31	340	20	4	200
24	18	336	24	30	336	24	4	204
28	18	332	28	29	332	28	5	208
32	17	328	32	28	328	32	5	212
36	16	324	36	26	324	36	5	216
40	15	320	40	25	320	40	6	220
44	14	316	44	24	316	44	6	224
48	13	312	48	22	312	48	6	228
52	12	308	52	20	308	52	6	232
56	11	304	56	18	304	56	7	236
60	10	300	60	16	300	60	7	240
64	9	296	64	14	296	64	7	244
68	8	292	68	12	292	68	7	248
72	6	288	72	10	288	72	7	252
76	5	284	76	8	284	76	7	256
80	3	280	80	6	280	80	7	260
84	2	276	84	3	276	84	7	264
88	— 1 —	272	88	— 1 —	272	88	7	268
92	+ 1 +	268	92	+ 1 +	268	92	7	272
96	2	264	96	3	264	96	7	276
100	3	260	100	6	260	100	6	280
104	5	256	104	8	256	104	6	284
108	6	252	108	10	252	108	6	288
112	8	248	112	12	248	112	5	292
116	9	244	116	14	244	116	5	296
120	10	240	120	16	240	120	5	300
124	11	236	124	18	236	124	4	304
128	12	232	128	20	232	128	4	308
132	13	228	132	22	228	132	4	312
136	14	224	136	24	224	136	3	316
140	15	220	140	25	220	140	3	320
144	16	216	144	26	216	144	2	324
148	17	212	148	28	212	148	2	328
152	18	208	152	29	208	152	1	332
156	18	204	156	30	204	156	1	336
160	19	200	160	31	200	160	—0+	340
164	19	196	164	31	196	164	+0—	344
168	20	192	168	32	192	168	0	348
172	20	188	172	32	188	172	1	352
176	20	184	176	33	184	176	1	356
180	+20+	180	180	+ 33 +	180	180	+2—	360

Logistical Logarithms.

	0′	1′	2′	3′	4′	5′	6′	7′
0″	00000	17782	14771	13010	11761	10792	10000	9331
1	35563	17710	14735	12986	11743	10777	9988	9320
2	32553	17639	14699	12962	11725	10763	9976	9310
3	30792	17570	14664	12939	11707	10749	9964	9300
4	29542	17501	14629	12915	11689	10734	9952	9289
5	23573	17434	14594	12891	11671	10720	9940	9279
6	27782	17368	14559	12868	11654	10706	9928	9269
7	27112	17302	14525	12845	11636	10692	9916	9259
8	26532	17238	14491	12821	11619	10678	9905	9249
9	26021	17175	14457	12798	11601	10663	9893	9238
10	25563	17112	14424	12775	11584	10649	9881	9228
11	25149	17050	14390	12753	11566	10635	9869	9218
12	24771	16990	14357	12730	11549	10621	9858	9208
13	24424	16930	14325	12707	11532	10608	9846	9198
14	24102	16871	14292	12685	11515	10594	9834	9188
15	23802	16812	14260	12663	11498	10580	9823	9178
16	23522	16755	14228	12640	11481	10566	9811	9168
17	23259	16698	14196	12618	11464	10552	9800	9158
18	23010	16642	14165	12596	11447	10539	9788	9148
19	22775	16587	14133	12574	11430	10525	9777	9138
20	22553	16532	14102	12553	11413	10512	9765	9128
21	22341	16478	14071	12531	11397	10498	9754	9119
22	22139	16425	14040	12510	11380	10484	9742	9109
23	21946	16372	14010	12488	11363	10471	9731	9099
24	21761	16320	13979	12467	11347	10458	9720	9089
25	21584	16269	13949	12445	11331	10444	9708	9079
26	21413	16218	13919	12424	11314	10431	9697	9070
27	21249	16168	13890	12403	11298	10418	9686	9060
28	21091	16118	13860	12382	11282	10404	9675	9050
29	20939	16069	13831	12362	11266	10391	9664	9041
30	20792	16021	13802	12341	11249	10378	9652	9031
31	20649	15973	13773	12320	11233	10365	9641	9021
32	20512	15925	13745	12300	11217	10352	9630	9012
33	20378	15878	13716	12279	11201	10339	9619	9002
34	20248	15832	13688	12259	11186	10326	9608	8992
35	20122	15786	13660	12239	11170	10313	9597	8983
36	20000	15740	13632	12218	11154	10300	9586	8973
37	19881	15695	13604	12198	11138	10287	9575	8964
38	19765	15651	13576	12178	11123	10274	9564	8954
39	19652	15607	13549	12159	11107	10261	9553	8945
40	19542	15563	13522	12139	11091	10248	9542	8935
41	19435	15520	13495	12119	11076	10235	9532	8926
42	19331	15477	13468	12099	11061	10223	9521	8917
43	19228	15435	13441	12080	11045	10210	9510	8907
44	19128	15393	13415	12061	11030	10197	9499	8898
45	19031	15351	13388	12041	11015	10185	9488	8888
46	18935	15310	13362	12022	10999	10172	9478	8879
47	18842	15269	13336	12003	10984	10160	9467	8870
48	18751	15229	13310	11984	10969	10147	9456	8861
49	18661	15189	13284	11965	10954	10135	9446	8851
50	18573	15149	13259	11946	10939	10122	9436	8842
51	18487	15110	13233	11927	10924	10110	9425	8833
52	18403	15071	13208	11908	10909	10098	9414	8824
53	18320	15032	13183	11889	10894	10085	9404	8814
54	18239	14994	13158	11871	10880	10073	9393	8805
55	18159	14956	13133	11852	10865	10061	9383	8796
56	18081	14918	13108	11834	10850	10049	9372	8787
57	18004	14881	13083	11816	10835	10036	9362	8778
58	17929	14844	13059	11797	10821	10024	9351	8769
59	17855	14808	13034	11779	10806	10012	9341	8760
60	17782	14771	13010	11761	10792	10000	9331	8751

Logistical Logarithms.

	8′	9′	10′	11′	12′	13′	14′	15′	16′
0″	8751	8239	7782	7368	6990	6642	6320	6021	5740
1	8742	8231	7774	7361	6984	6637	6315	6016	5736
2	8733	8223	7767	7354	6978	6631	6310	6011	5731
3	8724	8215	7760	7348	6972	6625	6305	6006	5727
4	8715	8207	7753	7341	6966	6620	6300	6001	5722
5	8706	8199	7745	7335	6960	6614	6294	5997	5718
6	8697	8191	7738	7328	6954	6609	6289	5992	5713
7	8688	8183	7731	7322	6948	6603	6284	5987	5709
8	8679	8175	7724	7315	6942	6598	6279	5982	5704
9	8670	8167	7717	7309	6936	6592	6274	5977	5700
10	8661	8159	7710	7302	6930	6587	6269	5973	5695
11	8652	8152	7703	7296	6924	6581	6264	5968	5691
12	8643	8144	7696	7289	6918	6576	6259	5963	5686
13	8635	8136	7688	7283	6912	6570	6254	5958	5682
14	8626	8128	7681	7276	6906	6565	6248	5954	5677
15	8617	8120	7674	7270	6900	6559	6243	5949	5673
16	8608	8112	7667	7264	6894	6554	6238	5944	5669
17	8599	8104	7660	7257	6888	6548	6233	5939	5664
18	8591	8097	7653	7251	6882	6543	6228	5935	5660
19	8582	8089	7646	7244	6877	6538	6223	5930	5655
20	8573	8081	7639	7238	6871	6532	6218	5925	5651
21	8565	8073	7632	7232	6865	6527	6213	5920	5646
22	8556	8066	7625	7225	6859	6521	6208	5916	5642
23	8547	8058	7618	7219	6853	6516	6203	5911	5637
24	8539	8050	7611	7212	6847	6510	6198	5906	5633
25	8530	8043	7604	7206	6841	6505	6193	5902	5629
26	8522	8035	7597	7200	6836	6500	6188	5897	5624
27	8513	8027	7590	7193	6830	6494	6183	5892	5620
28	8504	8020	7583	7187	6824	6489	6178	5888	5615
29	8496	8012	7577	7181	6818	6484	6173	5883	5611
30	8487	8004	7570	7175	6812	6478	6168	5878	5607
31	8479	7997	7563	7168	6807	6473	6163	5874	5602
32	8470	7989	7556	7162	6801	6467	6158	5869	5598
33	8462	7981	7549	7156	6795	6462	6153	5864	5594
34	8453	7974	7542	7149	6789	6457	6148	5860	5589
35	8445	7966	7535	7143	6784	6451	6143	5855	5585
36	8437	7959	7528	7137	6778	6446	6138	5850	5580
37	8428	7951	7522	7131	6772	6441	6133	5846	5576
38	8420	7944	7515	7124	6766	6435	6128	5841	5572
39	8411	7936	7508	7118	6761	6430	6123	5836	5567
40	8403	7929	7501	7112	6755	6425	6118	5832	5563
41	8395	7921	7494	7106	6749	6420	6113	5827	5559
42	8386	7914	7488	7100	6743	6414	6108	5823	5554
43	8378	7906	7481	7093	6738	6409	6103	5818	5550
44	8370	7899	7474	7087	6732	6404	6099	5813	5546
45	8361	7891	7467	7081	6726	6398	6094	5809	5541
46	8353	7884	7461	7075	6721	6393	6089	5804	5537
47	8345	7877	7454	7069	6715	6388	6084	5800	5533
48	8337	7869	7447	7063	6709	6383	6079	5795	5528
49	8328	7862	7441	7057	6704	6377	6074	5790	5524
50	8320	7855	7434	7050	6698	6372	6069	5786	5520
51	8312	7847	7427	7044	6692	6367	6064	5781	5516
52	8304	7840	7421	7038	6687	6362	6059	5777	5511
53	8296	7832	7414	7032	6681	6357	6055	5772	5507
54	8288	7825	7407	7026	6676	6351	6050	5768	5503
55	8279	7818	7401	7020	6670	6346	6045	5763	5498
56	8271	7811	7394	7014	6664	6341	6040	5758	5494
57	8263	7803	7387	7008	6659	6336	6035	5754	5490
58	8255	7796	7381	7002	6653	6331	6030	5749	5486
59	8247	7789	7374	6996	6648	6325	6025	5745	5481
60	8239	7782	7368	6990	6642	6320	6021	5740	5477

Logistical Logarithms.

	17′	18′	19′	20′	21′	22′	23′	24′	25′
0″	5477	5229	4994	4771	4559	4357	4164	3979	3802
1	5473	5225	4990	4768	4556	4354	4161	3976	3799
2	5469	5221	4986	4764	4552	4351	4158	3973	3796
3	5464	5217	4983	4760	4549	4347	4155	3970	3793
4	5460	5213	4979	4757	4546	4344	4152	3967	3791
5	5456	5209	4975	4753	4542	4341	4149	3964	3788
6	5452	5205	4971	4750	4539	4338	4145	3961	3785
7	5447	5201	4967	4746	4535	4334	4142	3958	3782
8	5443	5197	4964	4742	4532	4331	4139	3955	3779
9	5439	5193	4960	4739	4528	4328	4136	3952	3776
10	5435	5189	4956	4735	4525	4325	4133	3949	3773
11	5430	5185	4952	4732	4522	4321	4130	3946	3770
12	5426	5181	4949	4728	4518	4318	4127	3943	3768
13	5422	5177	4945	4724	4515	4315	4124	3940	3765
14	5418	5173	4941	4721	4511	4311	4120	3937	3762
15	5414	5169	4937	4717	4508	4308	4117	3934	3759
16	5409	5165	4933	4714	4505	4305	4114	3931	3756
17	5405	5161	4930	4710	4501	4302	4111	3928	3753
18	5401	5157	4926	4707	4498	4298	4108	3925	3750
19	5397	5153	4922	4703	4494	4295	4105	3922	3747
20	5393	5149	4918	4699	4491	4292	4102	3919	3745
21	5389	5145	4915	4696	4488	4289	4039	3917	3742
22	5384	5141	4911	4692	4484	4285	4096	3914	3739
23	5380	5137	4907	4689	4481	4282	4092	3911	3736
24	5376	5133	4903	4685	4477	4279	4089	3908	3733
25	5372	5129	4900	4682	4474	4276	4086	3905	3730
26	5368	5125	4896	4678	4471	4273	4083	3902	3727
27	5364	5122	4892	4675	4467	4269	4080	3899	3725
28	5359	5118	4889	4671	4464	4266	4077	3896	3722
29	5355	5114	4885	4668	4460	4263	4074	3893	3719
30	5351	5110	4881	4664	4457	4260	4071	3890	3716
31	5347	5106	4877	4660	4454	4256	4068	3887	3713
32	5343	5102	4874	4657	4450	4253	4065	3884	3710
33	5339	5098	4870	4653	4447	4250	4062	3881	3708
34	5335	5094	4866	4650	4444	4247	4059	3878	3705
35	5331	5090	4863	4646	4440	4244	4055	3875	3702
36	5326	5086	4859	4643	4437	4240	4052	3872	3699
37	5322	5082	4855	4639	4434	4237	4049	3869	3696
38	5318	5079	4852	4636	4430	4234	4046	3866	3693
39	5314	5075	4848	4632	4427	4231	4043	3863	3691
40	5310	5071	4844	4629	4424	4228	4040	3860	3688
41	5306	5067	4841	4625	4420	4224	4037	3857	3685
42	5302	5063	4837	4622	4417	4221	4034	3855	3682
43	5298	5059	4833	4618	4414	4218	4031	3852	3679
44	5294	5055	4830	4615	4410	4215	4028	3849	3677
45	5290	5051	4826	4611	4407	4212	4025	3846	3674
46	5285	5048	4822	4608	4404	4209	4022	3843	3671
47	5281	5044	4819	4604	4400	4205	4019	3840	3668
48	5277	5040	4815	4601	4397	4202	4016	3837	3665
49	5273	5036	4811	4597	4394	4199	4013	3834	3663
50	5269	5032	4808	4594	4390	4196	4010	3831	3660
51	5265	5028	4804	4590	4387	4193	4007	3828	3657
52	5261	5025	4800	4587	4384	4189	4004	3825	3654
53	5257	5021	4797	4584	4380	4186	4001	3822	3651
54	5253	5017	4793	4580	4377	4183	3998	3820	3649
55	5249	5013	4789	4577	4374	4180	3995	3817	3646
56	5245	5009	4786	4573	4370	4177	3991	3814	3643
57	5241	5005	4782	4570	4367	4174	3988	3811	3640
58	5237	5002	4778	4566	4364	4171	3985	3808	3637
59	5233	4998	4775	4563	4361	4167	3982	3805	3635
60	5229	4994	4771	4559	4357	4164	3979	3802	3632

Logistical Logarithms.

	26′	27′	28′	29′	30′	31′	32′	33′	34′
0″	3632	3468	3310	3158	3010	2868	2730	2596	2467
1	3629	3465	3307	3155	3008	2866	2728	2594	2465
2	3626	3463	3305	3153	3005	2863	2725	2592	2462
3	3623	3460	3302	3150	3003	2861	2723	2590	2460
4	3621	3457	3300	3148	3001	2859	2721	2588	2458
5	3618	3454	3297	3145	2998	2856	2719	2585	2456
6	3615	3452	3294	3143	2996	2854	2716	2583	2454
7	3612	3449	3292	3140	2993	2852	2714	2581	2452
8	3610	3446	3289	3138	2991	2849	2712	2579	2450
9	3607	3444	3287	3135	2989	2847	2710	2577	2448
10	3604	3441	3284	3133	2986	2845	2707	2574	2445
11	3601	3438	3282	3130	2984	2842	2705	2572	2443
12	3598	3436	3279	3128	2981	2840	2703	2570	2441
13	3596	3433	3276	3125	2979	2838	2701	2568	2439
14	3593	3431	3274	3123	2977	2835	2698	2566	2437
15	3590	3428	3271	3120	2974	2833	2696	2564	2435
16	3587	3425	3269	3118	2972	2831	2694	2561	2433
17	3585	3423	3266	3115	2969	2828	2692	2559	2431
18	3582	3420	3264	3113	2967	2826	2689	2557	2429
19	3579	3417	3261	3110	2965	2824	2687	2555	2426
20	3576	3415	3259	3108	2962	2821	2685	2553	2424
21	3574	3412	3256	3105	2960	2819	2683	2551	2422
22	3571	3409	3253	3103	2958	2817	2681	2548	2420
23	3568	3407	3251	3101	2955	2815	2678	2546	2418
24	3565	3404	3248	3098	2953	2812	2676	2544	2416
25	3563	3401	3246	3096	2950	2810	2674	2542	2414
26	3560	3399	3243	3093	2948	2808	2672	2540	2412
27	3557	3396	3241	3091	2946	2805	2669	2538	2410
28	3555	3393	3238	3088	2943	2803	2667	2535	2408
29	3552	3391	3236	3086	2941	2801	2665	2533	2405
30	3549	3388	3233	3083	2939	2798	2663	2531	2403
31	3546	3386	3231	3081	2936	2796	2660	2529	2401
32	3544	3383	3228	3078	2934	2794	2658	2527	2399
33	3541	3380	3225	3076	2931	2792	2656	2525	2397
34	3538	3378	3223	3073	2929	2789	2654	2522	2395
35	3535	3375	3220	3071	2927	2787	2652	2520	2393
36	3533	3372	3218	3069	2924	2785	2649	2518	2391
37	3530	3370	3215	3066	2922	2782	2647	2516	2389
38	3527	3367	3213	3064	2920	2780	2645	2514	2387
39	3525	3365	3210	3061	2917	2778	2643	2512	2384
40	3522	3362	3208	3059	2915	2775	2640	2510	2382
41	3519	3359	3205	3056	2912	2773	2638	2507	2380
42	3516	3357	3203	3054	2910	2771	2636	2505	2378
43	3514	3354	3200	3052	2908	2769	2634	2503	2376
44	3511	3351	3198	3049	2905	2766	2632	2501	2374
45	3508	3349	3195	3047	2903	2764	2629	2499	2372
46	3506	3346	3193	3044	2901	2762	2627	2497	2370
47	3503	3344	3190	3042	2898	2760	2625	2494	2368
48	3500	3341	3188	3039	2896	2757	2623	2492	2366
49	3497	3338	3185	3037	2894	2755	2621	2490	2364
50	3495	3336	3183	3034	2891	2753	2618	2488	2362
51	3492	3333	3180	3032	2889	2750	2616	2486	2359
52	3489	3331	3178	3030	2887	2748	2614	2484	2357
53	3487	3328	3175	3027	2884	2746	2612	2482	2355
54	3484	3325	3173	3025	2882	2744	2610	2480	2353
55	3481	3323	3170	3022	2880	2741	2607	2477	2351
56	3479	3320	3168	3020	2877	2739	2605	2475	2349
57	3476	3318	3165	3018	2875	2737	2603	2473	2347
58	3473	3315	3163	3015	2873	2735	2601	2471	2345
59	3471	3313	3160	3013	2870	2732	2599	2469	2343
60	3468	3310	3158	3010	2868	2730	2596	2467	2341

Logistical Logarithms.

	35′	36′	37′	38′	39′	40′	41′	42′	43′
0″	2341	2218	2099	1984	1871	1761	1654	1549	1447
1	2339	2216	2098	1982	1869	1759	1652	1547	1445
2	2337	2214	2096	1980	1867	1757	1650	1546	1443
3	2335	2212	2094	1978	1865	1755	1648	1544	1442
4	2333	2210	2092	1976	1863	1754	1647	1542	1440
5	2331	2208	2090	1974	1862	1752	1645	1540	1438
6	2328	2206	2088	1972	1860	1750	1643	1539	1437
7	2326	2204	2086	1970	1858	1748	1641	1537	1435
8	2324	2202	2084	1968	1856	1746	1640	1535	1433
9	2322	2200	2082	1967	1854	1745	1638	1534	1432
10	2320	2198	2080	1965	1852	1743	1636	1532	1430
11	2318	2196	2078	1963	1850	1741	1634	1530	1428
12	2316	2194	2076	1961	1849	1739	1633	1528	1427
13	2314	2192	2074	1959	1847	1737	1631	1527	1425
14	2312	2190	2072	1957	1845	1736	1629	1525	1423
15	2310	2188	2070	1955	1843	1734	1627	1523	1422
16	2308	2186	2068	1953	1841	1732	1626	1522	1420
17	2306	2184	2066	1951	1839	1730	1624	1520	1418
18	2304	2182	2064	1950	1838	1728	1622	1518	1417
19	2302	2180	2062	1948	1836	1727	1620	1516	1415
20	2300	2178	2061	1946	1834	1725	1619	1515	1413
21	2298	2176	2059	1944	1832	1723	1617	1513	1412
22	2296	2174	2057	1942	1830	1721	1615	1511	1410
23	2294	2172	2055	1940	1828	1719	1613	1510	1408
24	2291	2170	2053	1938	1827	1718	1612	1508	1407
25	2289	2169	2051	1936	1825	1716	1610	1506	1405
26	2287	2167	2049	1934	1823	1714	1608	1504	1403
27	2285	2165	2047	1933	1821	1712	1606	1503	1402
28	2283	2163	2045	1931	1819	1711	1605	1501	1400
29	2281	2161	2043	1929	1817	1709	1603	1499	1398
30	2279	2159	2041	1927	1816	1707	1601	1498	1397
31	2277	2157	2039	1925	1814	1705	1599	1496	1395
32	2275	2155	2037	1923	1812	1703	1598	1494	1393
33	2273	2153	2035	1921	1810	1702	1596	1493	1392
34	2271	2151	2033	1919	1808	1700	1594	1491	1390
35	2269	2149	2032	1918	1806	1698	1592	1489	1388
36	2267	2147	2030	1916	1805	1696	1591	1487	1387
37	2265	2145	2028	1914	1803	1694	1589	1486	1385
38	2263	2143	2026	1912	1801	1693	1587	1484	1383
39	2261	2141	2024	1910	1799	1691	1585	1482	1382
40	2259	2139	2022	1908	1797	1689	1584	1481	1380
41	2257	2137	2020	1906	1795	1687	1582	1479	1378
42	2255	2135	2018	1904	1794	1686	1580	1477	1377
43	2253	2133	2016	1903	1792	1684	1578	1476	1375
44	2251	2131	2014	1901	1790	1682	1577	1474	1373
45	2249	2129	2012	1899	1788	1680	1575	1472	1372
46	2247	2127	2010	1897	1786	1678	1573	1470	1370
47	2245	2125	2009	1895	1785	1677	1571	1469	1368
48	2243	2123	2007	1893	1783	1675	1570	1467	1367
49	2241	2121	2005	1891	1781	1673	1568	1465	1365
50	2239	2119	2003	1889	1779	1671	1566	1464	1363
51	2237	2117	2001	1888	1777	1670	1565	1462	1362
52	2235	2115	1999	1886	1775	1668	1563	1460	1360
53	2233	2113	1997	1884	1774	1666	1561	1459	1359
54	2231	2111	1995	1882	1772	1664	1559	1457	1357
55	2229	2109	1993	1880	1770	1663	1558	1455	1355
56	2227	2107	1991	1878	1768	1661	1556	1454	1354
57	2225	2105	1989	1876	1766	1659	1554	1452	1352
58	2223	2103	1987	1875	1765	1657	1552	1450	1350
59	2220	2101	1986	1873	1763	1655	1551	1449	1349
60	2218	2099	1984	1871	1761	1654	1549	1447	1347

Logistical Logarithms.

	44′	45′	46′	47′	48′	49′	50′	51′	52′
0″	1347	1249	1154	1061	969	880	792	706	621
1	1345	1248	1152	1059	968	878	790	704	620
2	1344	1246	1151	1057	966	877	789	703	619
3	1342	1245	1149	1056	965	875	787	702	617
4	1340	1243	1148	1054	963	874	786	700	616
5	1339	1241	1146	1053	962	872	785	699	615
6	1337	1240	1145	1051	960	871	783	697	613
7	1335	1238	1143	1050	959	869	782	696	612
8	1334	1237	1141	1048	957	868	780	694	610
9	1332	1235	1140	1047	956	866	779	693	609
10	1331	1233	1138	1045	954	865	777	692	608
11	1329	1232	1137	1044	953	863	776	690	606
12	1327	1230	1135	1042	951	862	774	689	605
13	1326	1229	1134	1041	950	860	773	687	603
14	1324	1227	1132	1039	948	859	772	686	602
15	1322	1225	1130	1037	947	857	770	685	601
16	1321	1224	1129	1036	945	856	769	683	599
17	1319	1222	1127	1034	944	855	767	682	598
18	1317	1221	1126	1033	942	853	766	680	596
19	1316	1219	1124	1031	941	852	764	679	595
20	1314	1217	1123	1030	939	850	763	678	594
21	1313	1216	1121	1028	938	849	762	676	592
22	1311	1214	1119	1027	936	847	760	675	591
23	1309	1213	1118	1025	935	846	759	673	590
24	1308	1211	1116	1024	933	844	757	672	588
25	1306	1209	1115	1022	932	843	756	670	587
26	1304	1208	1113	1021	930	841	754	669	585
27	1303	1206	1112	1019	929	840	753	668	584
28	1301	1205	1110	1018	927	838	751	666	583
29	1300	1203	1109	1016	926	837	750	665	581
30	1298	1201	1107	1015	924	835	749	663	580
31	1296	1200	1105	1013	923	834	747	662	579
32	1295	1198	1104	1012	921	833	746	661	577
33	1293	1197	1102	1010	920	831	744	659	576
34	1291	1195	1101	1008	918	830	743	658	574
35	1290	1193	1099	1007	917	828	741	656	573
36	1288	1192	1098	1005	915	827	740	655	572
37	1287	1190	1096	1004	914	825	739	654	570
38	1285	1189	1095	1002	912	824	737	652	569
39	1283	1187	1093	1001	911	822	736	651	568
40	1282	1186	1091	999	909	821	734	649	566
41	1280	1184	1090	998	908	819	733	648	565
42	1278	1182	1088	996	906	818	731	647	563
43	1277	1181	1087	995	905	816	730	645	562
44	1275	1179	1085	993	903	815	729	644	561
45	1274	1178	1084	992	902	814	727	642	559
46	1272	1176	1082	990	900	812	726	641	558
47	1270	1174	1081	989	899	811	724	640	557
48	1269	1173	1079	987	897	809	723	638	555
49	1267	1171	1078	986	896	808	721	637	554
50	1266	1170	1076	984	894	806	720	635	552
51	1264	1168	1074	983	893	805	719	634	551
52	1262	1167	1073	981	891	803	717	633	550
53	1261	1165	1071	980	890	802	716	631	548
54	1259	1163	1070	978	888	801	714	630	547
55	1257	1162	1068	977	887	799	713	628	546
56	1256	1160	1067	975	885	798	711	627	544
57	1254	1159	1065	974	884	796	710	626	543
58	1253	1157	1064	972	883	795	709	624	541
59	1251	1156	1062	971	881	793	707	623	540
60	1249	1154	1061	969	880	792	706	621	539

Logistical Logarithms.

	53′	54′	55′	56′	57′	58′	59′
0″	539	458	378	300	223	147	73
1	537	456	377	298	221	146	72
2	536	455	375	297	220	145	71
3	535	454	374	296	219	143	69
4	533	452	373	294	218	142	68
5	532	451	371	293	216	141	67
6	531	450	370	292	215	140	66
7	529	448	369	291	214	139	64
8	528	447	367	289	213	137	63
9	526	446	366	288	211	136	62
10	525	444	365	287	210	135	61
11	524	443	363	285	209	134	60
12	522	442	362	284	208	132	58
13	521	440	361	283	206	131	57
14	520	439	359	282	205	130	56
15	518	438	358	280	204	129	55
16	517	436	357	279	202	127	53
17	516	435	356	278	201	126	52
18	514	434	354	276	200	125	51
19	513	432	353	275	199	124	50
20	512	431	352	274	197	122	49
21	510	430	350	273	196	121	47
22	509	428	349	271	195	120	46
23	507	427	348	270	194	119	45
24	506	426	346	269	192	117	44
25	505	424	345	267	191	116	42
26	503	423	344	266	190	115	41
27	502	422	342	265	189	114	40
28	501	420	341	264	187	112	39
29	499	19	340	262	186	111	38
30	498	416	339	261	185	110	36
31	497	416	337	260	184	109	35
32	495	415	336	258	182	107	34
33	494	414	335	257	181	106	33
34	493	412	333	256	180	105	31
35	491	411	332	255	179	104	30
36	490	410	331	253	177	103	29
37	489	408	329	252	176	101	28
38	487	407	328	251	175	100	27
39	486	406	327	250	174	99	25
40	484	404	326	248	172	98	24
41	483	403	324	247	171	96	23
42	482	402	323	246	170	95	22
43	480	400	322	244	169	94	21
44	479	399	320	243	167	93	19
45	478	398	319	242	166	91	18
46	476	396	318	241	165	90	17
47	475	395	316	239	163	89	16
48	474	394	315	238	162	88	15
49	472	392	314	237	161	87	13
50	471	391	313	235	160	85	12
51	470	390	31	234	158	84	11
52	468	388	31	233	157	83	10
53	467	387	309	232	156	82	8
54	466	386	307	230	155	80	7
55	464	384	306	229	153	79	6
56	463	383	305	228	152	78	5
57	462	382	304	227	151	77	4
58	460	381	302	225	150	75	2
59	459	379	301	224	148	74	1
60	458	378	300	223	147	73	0

TABLE LXXI.

Reduction of Lat. and Moon's Hor. Par., and the logarithm of ϱ, for compression $\frac{1}{300}$.

ARGUMENT. Geographic Latitude.

Arg.	Reduction of Lat.		Reduc. of Hor. Par.			Log. ϱ
			53′	57′	61′	
°	″	″	″	″	″	
0	0	0.0	0.0	0.0	0.0	0.00000
2	0	47.9	0.0	0.0	0.0	0.00000
4	1	35.5	0.1	0.1	0.1	9.99999
6	2	22.7	0.1	0.1	0.1	9.99998
8	3	9.2	0.2	0.2	0.2	9.99997
10	3	54.8	0.3	0.3	0.4	9.99996
12	4	39.3	0.5	0.5	0.5	9.99994
14	5	22.4	0.6	0.7	0.7	9.99992
16	6	3.9	0.8	0.9	0.9	9.99989
18	6	43.7	1.0	1.1	1.2	9.99986
20	7	21.6	1.2	1.3	1.4	9.99983
22	7	57.3	1.5	1.6	1.7	9.99980
24	8	30.7	1.8	1.9	2.0	9.99976
26	9	1.6	2.0	2.2	2.3	9.99972
28	9	29.9	2.3	2.5	2.7	9.99968
30	9	55.4	2.7	2.9	3.1	9.99964
32	10	18.1	3.0	3.2	3.4	9.99960
34	10	37.8	3.3	3.6	3.8	9.99955
36	10	54.3	3.7	3.9	4.2	9.99950
38	11	7.7	4.0	4.3	4.6	9.99945
40	11	17.9	4.4	4.7	5.0	9.99940
42	11	24.7	4.7	5.1	5.5	9.99935
44	11	28.2	5.1	5.5	5.9	9.99930
46	11	28.4	5.5	5.9	6.3	9.99925
48	11	25.2	5.9	6.3	6.7	9.99920
50	11	18.6	6.2	6.7	7.2	9.99915
52	11	8.8	6.6	7.1	7.6	9.99910
54	10	55.7	6.9	7.5	8.0	9.99905
56	10	39.4	7.3	7.8	8.4	9.99901
58	10	19.9	7.6	8.2	8.8	9.99896
60	9	57.4	7.9	8.5	9.1	9.99892
62	9	32.0	8.3	8.9	9.5	9.99887
64	9	3.8	8.6	9.2	9.9	9.99883
66	8	32.9	8.8	9.5	10.2	9.99879
68	7	59.6	9.1	9.8	10.5	9.99876
70	7	23.8	9.4	10.1	10.8	9.99872
72	6	45.9	9.6	10.3	11.0	9.99869
74	6	6.0	9.8	10.5	11.3	9.99866
76	5	24.3	10.0	10.7	11.5	9.99864
78	4	41.0	10.1	10.9	11.7	9.99861
80	3	56.3	10.3	11.1	11.8	9.99859
			10.4	11.2	12.0	
82	3	10.4	10.5	11.3	12.1	9.99858
84	2	23.7	10.5	11.3	12.1	9.99857
86	1	36.2	10.5	11.3	12.1	9.99856
88	0	48.2	10.6	11 4	12.2	9.99855
90	0	0.0	10.6	11.4	12.2	9.99855

TABLE LXXI.*

Change of values of Z, the right ascen. of zenith, for hours, minutes, and tenths of a minute.

	Change of Z.		
H.	°	′	″
1	15	2	27.8
2	30	4	55.7
3	45	7	23.5
4	60	9	51.4
5	75	12	19.2
6	90	14	47.1
M	°	′	″
1	0	15	2.5
2	0	30	4.9
3	0	45	7.4
4	1	0	9.9
5	1	15	12.3
6	1	30	14.8
7	1	45	17.2
8	2	0	19.7
9	2	15	22.2
10	2	30	24.6
11	2	45	27.1
12	3	0	29.6
13	3	15	32.0
14	3	30	34.5
15	3	45	37.0
16	4	0	39.4
17	4	15	41.9
18	4	30	44.4
19	4	45	46.8
20	5	0	49.3
21	5	15	51.7
22	5	30	54.2
23	5	45	56.7
24	6	0	59.1
25	6	16	1.6
26	6	31	4.1
27	6	46	6.5
28	7	1	9.0
29	7	16	11.5
30	7	31	13.9
31	7	46	16.4
32	8	1	18.9
33	8	16	21.3
34	8	31	23.8
35	8	46	26.2
	°	′	″
35	8	46	26.2
36	9	1	28.7
37	9	16	31.2
38	9	31	33.6
39	9	46	36.1
40	10	1	38.6
41	10	16	41.0
42	10	31	43.5
43	10	46	46.0
44	11	1	48.4
45	11	16	50.9
46	11	31	53.3
47	11	46	55.8
48	12	1	58.3
49	12	17	0.7
50	12	32	3.2
51	12	47	5.7
52	13	2	8.1
53	13	17	10.6
54	13	32	13.1
55	13	47	15.5
56	14	2	18.0
57	14	17	20.5
58	14	32	22.9
59	14	47	25.4
60	15	2	27.8
M.	°	′	″
0.1	0	1	30.2
0.2	0	3	0.5
0.3	0	4	30.7
0.4	0	6	1.0
0.5	0	7	31.2
0.6	0	9	1.5
0.7	0	10	31.7
0.8	0	12	2.0
0.9	0	13	32.2
1.0	0	15	2.5

TABLE LXXII.

Logarithms x and y.

ARG. Geog. Latitude.

Arg.	Log. x	Log. y
0°	1.00000	0.99710
2	1.00000	0.99710
4	1.00001	0.99711
6	1.00002	0.99712
8	1.00003	0.99713
10	1.00004	0.99714
12	1.00006	0.99716
14	1.00008	0.99718
16	1.00011	0.99721
18	1.00014	0.99724
20	1.00017	0.99727
22	1.00020	0·99730
24	1.00024	0.99734
26	1.00028	0.99738
28	1.00032	0.99742
30	1.00036	0.99746
32	1.00041	0.99751
34	1.00045	0.99755
36	1.00050	0.99760
38	1.00055	0.99765
40	1.00060	0.99770
42	1.00065	0.99775
44	1.00070	0.99780
46	1.00075	0.99785
48	1.00080	0.99790
50	1.00085	0.99795
52	1.00090	0.99800
54	1.00095	0.99805
56	1.00100	0.99810
58	1.00105	0.99815
60	1.00109	0.99819
62	1.00113	0.99823
64	1.00117	0.99827
66	1.00121	0.99831
68	1.00125	0.99835
70	1.00128	0.99838
72	1.00131	0.99841
74	1.00134	0.99844
76	1.00137	0.99847
78	1.00139	0.99849
80	1.00141	0.99851
82	1.00143	0.99853
84	1.00143	0.99853
86	1.00144	0.99854
88	1.00145	0.99855
90	1.00145	0.99855

TABLE LXXIII.

Values of u' and v'.

ARG. b'' for u'. ARG. g' for v'.

Arg.	u'	v'	Arg.	u'	v'
0.0	0.000	0.000	5.0	1.309	1.312
0.1	0.026	0.026	5.1	1.335	1.339
0.2	0.052	0.052	5.2	1.361	1.365
0.3	0.079	0.079	5.3	1.388	1.391
0.4	0.105	0.105	5.4	1.414	1.417
0.5	0.131	0.131	5.5	1.440	1.444
0.6	0.157	0.157	5.6	1.466	1.470
0.7	0.183	0.184	5.7	1.492	1.496
0.8	0.209	0.210	5.8	1.518	1.522
0.9	0.236	0.236	5.9	1.545	1.549
1.0	0.262	0.262	6.0	1.571	1.575
1.1	0.288	0.289	6.1	1.597	1.601
1.2	0.314	0.315	6.2	1.623	1.627
1.3	0.340	0.341	6.3	1.649	1.654
1.4	0.367	0.367	6.4	1.676	1.680
1.5	0.393	0.394	6.5	1.702	1.706
1.6	0.419	0.420	6.6	1.728	1.732
1.7	0.445	0.446	6.7	1.754	1.759
1.8	0.471	0.472	6.8	1.780	1.785
1.9	0.497	0.499	6.9	1.806	1.811
2.0	0.524	0.525	7.0	1.833	1.837
2.1	0.550	0.551	7.1	1.859	1.864
2.2	0.576	0.577	7.2	1.885	1.890
2.3	0.602	0.604	7.3	1.911	1.916
2.4	0.628	0.630	7.4	1.937	1.942
2.5	0.654	0.656	7.5	1.963	1.969
2.6	0.681	0.682	7.6	1.990	1.995
2.7	0.707	0.709	7.7	2.016	2.021
2.8	0.733	0.735	7.8	2.042	2.047
2.9	0.759	0.761	7.9	2.068	2.074
3.0	0.785	0.787	8.0	2.094	2.100
3.1	0.812	0.814	8.1	2.121	2.126
3.2	0.838	0.840	8.2	2.147	2.152
3.3	0.864	0.866	8.3	2.173	2.179
3.4	0.890	0.892	8.4	2.199	2.205
3.5	0.916	0.919	8.5	2.225	2.231
3.6	0.942	0.945	8.6	2.251	2.257
3.7	0.969	0.971	8.7	2.278	2.284
3.8	0.995	0.997	8.8	2.304	2.310
3.9	1.021	1.024	8.9	2.330	2.336
4.0	1.047	1.050	9.0	2.356	2.362
4.1	1.073	1.076	9.1	2.382	2.389
4.2	1.100	1.102	9.2	2.409	2.415
4.3	1.126	1.129	9.3	2.435	2.441
4.4	1.152	1.155	9.4	2.461	2.467
4.5	1.178	1.181	9.5	2.487	2.494
4.6	1.204	1.207	9.6	2.513	2.520
4.7	1.230	1.234	9.7	2.539	2.546
4.8	1.257	1.260	9.8	2.566	2.572
4.9	1.283	1.286	9.9	2.592	2.599
5.0	1.309	1.312	10.0	2.618	2.625

TABLE LXXIV.

Log. of ϱ *sin.* φ' *sin.* ι, *or Log.* A.; *Log. of* ϱ *cos.* φ', *or Log.* B.; *Log. of* ϱ *sin.* φ', *or Log.* F.; *Log. of* ϱ *cos.* φ' *sin.* ι, *taken negative, or Log.* G.; *and value of* f $=$ ϱ *sin.* φ' *cos.* ι. *For the places in the table.*

Places.	Log. A	Log. B	Log. F	Log. G	*f*
Albany,	0.429	0.867	0.829	0.467n	+ 6.18
Baltimore,	0.399	0.889	0.799	0.489n	5.78
Boston,	0.426	0.869	0.826	0.469n	6.15
Charleston,	0.331	0.925	0.731	0.525n	4.94
Charlottesville,	0.387	0.897	0.787	0.497n	5.62
Cincinnati,	0.397	0.890	0.797	0.490n	+ 5.75
Lancaster,	0.406	0.886	0.806	0.486n	5.87
New Haven,	0.417	0.876	0.817	0.476n	6.02
New Orleans,	0.296	0.938	0.696	0.538n	4.55
New York,	0.412	0.880	0.812	0.480n	5.95
Philadelphia,	0.405	0.885	0.805	0.485n	+ 5.86
Pittsburg,	0.410	0.882	0.810	0.482n	5.92
Princeton,	0.409	0.883	0.809	0.482n	5.91
Providence,	0.422	0.873	0.822	0.473n	6.09
Richmond,	0.382	0.900	0.782	0.500n	5.56
Savannah,	0.323	0.928	0.723	0.528n	+ 4.84
Washington,	0.395	0.892	0.795	0.492n	5.73

TABLE LXXV.

Value of r = *moon's reduced semidiameter.*

Arg. Moon's Hor. Par.

Arg.	r
53	2.7324
54	2.7323
55	2.7321
56	2.7320
57	2.7319
58	2.7318
59	2.7316
60	2.7315
61	2.7314
62	2.7313

TABLE LXXVI.

Correction of R, *the Sun's reduced semidiameter.*

Arguments. Arg. E at top, and sun's semidiameter at the side.

		1	2	3	4	5	6	7	8	9	10
15′	46″	.0046	.0092	.0138	.0184	.0230	.0276	.0322	.0368	.0414	.0460
	48	46	92	138	184	230	276	323	369	415	461
15	50	.0046	.0092	.0139	.0185	.0231	.0277	.0323	.0369	.0416	.0462
	52	46	93	139	185	231	278	324	370	416	463
	54	46	93	139	185	232	278	325	371	417	464
	56	46	93	139	186	232	279	325	372	418	465
	58	47	93	140	186	233	279	326	372	419	466
16	0	.0047	.0093	.0140	.0187	.0233	.0280	.0327	.0373	.0420	.0467
	2	47	94	140	187	234	281	327	374	421	468
	4	47	94	141	187	234	281	328	375	422	468
	6	47	94	141	188	235	282	329	376	423	469
	8	47	94	141	188	235	282	329	376	423	470
16	10	.0047	.0094	.0141	.0189	.0236	.0283	.0330	.0377	.0424	.0471
	12	47	94	142	189	236	283	331	378	425	472
	14	47	95	142	189	237	284	331	379	426	473
	16	47	95	142	190	237	285	332	380	427	474
	18	48	95	143	190	238	285	833	380	428	475

Logarithmic Sines, Cosines, and Tangents, to three decimal figures.

Arc	Sine	Cos.	Tan.	Arc	Arc	Sine	Cos.	Tan.	Arc
0° 5′	7.163	10.000	7.163	179° 55′	9° 0′	9.194	9.995	9.200	171° 0′
10	7.464	10.000	7.464	50	10	9.202	9.994	9.208	50
20	7.765	10.000	7.765	40	20	9.210	9.994	9.216	40
30	7.941	10.000	7.941	30	30	9.218	9.994	9.224	30
40	8.066	10.000	8.066	20	40	9.225	9.994	9.231	20
50	8.163	10.000	8.163	10	50	9.232	9.994	9.239	10
1 0	8.242	10.000	8.242	179 0	10 0	9.240	9.993	9.246	170 0
10	8.309	10.000	8.309	50	10	9.247	9.993	9.254	50
20	8.367	10.000	8.367	40	20	9.254	9.993	9.261	40
30	8.418	10.000	8.418	30	30	9.261	9.993	9.268	30
40	8.464	10.000	8.464	20	40	9.267	9.992	9.275	20
50	8.505	10.000	8.505	10	50	9.274	9.992	9.282	10
2 0	8.543	10.000	8.543	178 0	11 0	9.281	9.992	9.289	169 0
10	8.578	10.000	8.578	50	10	9.287	9.992	9.295	50
20	8.610	10.000	8.610	40	20	9.293	9.991	9.302	40
30	8.640	10.000	8.640	30	30	9.300	9.991	9.308	30
40	8.668	10.000	8.668	20	40	9.306	9.991	9.315	20
50	8.694	9.999	8.695	10	50	9.312	9.991	9.321	10
3 0	8.719	9.999	8.719	177 0	12 0	9.318	9.990	9.327	168 0
10	8.742	9.999	8.743	50	10	9.324	9.990	9.334	50
20	8.765	9.999	8.765	40	20	9.330	9.990	9.340	40
30	8.786	9.999	8.786	30	30	9.335	9.990	9.346	30
40	8.806	9.999	8.807	20	40	9.341	9.989	9.352	20
50	8.825	9.999	8.826	10	50	9.347	9.969	9.358	10
4 0	8.844	9.999	8.845	176 0	13 0	9.352	9.989	9.363	167 0
10	8.861	9.999	8.862	50	10	9.358	9.988	9.369	50
20	8.878	9.999	8.880	40	20	9.363	9.988	9.375	40
30	8.895	9.999	8.896	30	30	9.368	9.988	9.380	30
40	8.910	9.999	8.912	20	40	9.373	9.988	9.386	20
50	8.926	9.998	8.927	10	50	9.379	9.987	9.391	10
5 0	8.940	9.998	8.942	175 0	14 0	9.384	9.987	9.397	166 0
10	8.954	9.998	8.956	50	10	9.389	9.987	9.402	50
20	8.968	9.998	8.970	40	20	9.394	9.986	9.407	40
30	8.982	9.998	8.984	30	30	9.399	9.986	9.413	30
40	8.994	9.998	8.997	20	40	9.403	9.986	9.418	20
50	9.007	9.998	9.009	10	50	9.408	9.985	9.423	10
6 0	9.019	9.998	9.022	174 0	15 0	9.413	9.985	9.428	165 0
10	9.031	9.997	9.034	50	10	9.418	9.985	9.433	50
20	9.043	9.997	9.045	40	20	9.422	9.984	9.438	40
30	9.054	9.997	9.057	30	30	9.427	9.984	9.443	30
40	9.065	9.997	9.068	20	40	9.431	9.984	9.448	20
50	9.075	9.997	9.079	10	50	9.436	9.983	9.453	10
7 0	9.086	9.997	9.089	173 0	16 0	9.440	9.983	9.457	164 0
10	9.096	9.997	9.099	50	10	9.445	9.982	9.462	50
20	9.106	9.996	9.110	40	20	9.449	9.982	9.467	40
30	9.116	9.996	9.119	30	30	9.453	9.982	9.472	30
40	9.125	9.996	9.129	20	40	9.458	9.981	9.476	20
50	9.134	9.996	9.139	10	50	9.462	9.981	9.481	10
8 0	9.144	9.996	9.148	172 0	17 0	9.466	9.981	9.485	163 0
10	9.152	9.996	9.157	50	10	9.470	9.980	9.490	50
20	9.161	9.995	9.166	40	20	9.474	9.980	9.494	40
30	9.170	9.995	9.174	30	30	9.478	9.979	9.499	30
40	9.178	9.995	9.183	20	40	9.482	9.979	9.503	20
50	9.186	9.995	9.191	10	50	9.486	9.979	9.507	10
9 0	9.134	9.995	9.200	171 0	18 0	9.490	9.978	9.512	162 0

Logarithmic Sines, Cosines, and Tangents, to three decimal figures.

Arc	Sin.	Cos.	Tan.	Arc.	Arc	Sin.	Cos.	Tan.	Arc
18° 0′	9.490	9.978	9.512	162° 0′	27° 0′	9.657	9.950	9.707	153° 0′
10	9.494	9.978	9.516	50	10	9.660	9.949	9.710	50
20	9.498	9.977	9.520	40	20	9.662	9.949	9.713	40
30	9.501	9.977	9.525	30	30	9.664	9.948	9.716	30
40	9.505	9.977	9.529	20	40	9.667	9.947	9.720	20
50	9.509	9.976	9.533	10	50	9.669	9.947	9.723	10
19 0	9.513	9.976	9.537	161 0	28 0	9.672	9.946	9.726	152 0
10	9.516	9.975	9.541	50	10	9.674	9.945	9.729	50
20	9.520	9.975	9.545	40	20	9.676	9.945	9.732	40
30	9.523	9.974	9.549	30	30	9.679	9.944	9.735	30
40	9.527	9.974	9.553	20	40	9.681	9.943	9.738	20
50	9.531	9.973	9.557	10	50	9.683	9.943	9.741	10
20 0	9.534	9.973	9.561	160 0	29 0	9.686	9.942	9.744	151 0
10	9.538	9.973	9.565	50	10	9.688	9.941	9.747	50
20	9.541	9.972	9.569	40	20	9.690	9.940	9.750	40
30	9.544	9.972	9.573	30	30	9.692	9.940	9.753	30
40	9.548	9.971	9.577	20	40	9.695	9.939	9.756	20
50	9.551	9.971	9.580	10	50	9.697	9.938	9.759	10
21 0	9.554	9.970	9.584	159 0	30 0	9.699	9.938	9.761	150 0
10	9.558	9.970	9.588	50	10	9.701	9.937	9.764	50
20	9.561	9.970	9.592	40	20	9.703	9.936	9.767	40
30	9.564	9.969	9.595	30	30	9.705	9.935	9.770	30
40	9.567	9.968	9.599	20	40	9.708	9.935	9.773	20
50	9.570	9.968	9.603	10	50	9.710	9.934	9.776	10
22 0	9.574	9.967	9.606	158 0	31 0	9.712	9.933	9.779	149 0
10	9.577	9.967	9.610	50	10	9.714	9.932	9.782	50
20	9.580	9.966	9.614	40	20	9.716	9.932	9.784	40
30	9.583	9.966	9.617	30	30	9.718	9.931	9.787	30
40	9.586	9.965	9.621	20	40	9.720	9.930	9.790	20
50	9.589	9.965	9.624	10	50	9.722	9.929	9.793	10
23 0	9.592	9.964	9.628	157 0	32 0	9.724	9.928	9.796	148 0
10	9.595	9.963	9.631	50	10	9.726	9.928	9.799	50
20	9.598	9.963	9.635	40	20	9.728	9.927	9.801	40
30	9.601	9.962	9.638	30	30	9.730	9.926	9.804	30
40	9.604	9.962	9.642	20	40	9.732	9.925	9.807	20
50	9.606	9.961	9.645	10	50	9.734	9.924	9.810	10
24 0	9.609	9.961	9.649	156 0	33 0	9.736	9.924	9.813	147 0
10	9.612	9.960	9.652	50	10	9.738	9.923	9.815	50
20	9.615	9.960	9.655	40	20	9.740	9.922	9.818	40
30	9.618	9.959	9.659	30	30	9.742	9.921	9.821	30
40	9.620	9.958	9.662	20	40	9.744	9.920	9.824	20
50	9.623	9.958	9.665	10	50	9.746	9.919	9.826	10
25 0	9.626	9.957	9.669	155 0	34 0	9.748	9.919	9.829	146 0
10	9.629	9.957	9.672	50	10	9.749	9.918	9.832	50
20	9.631	9.956	9.675	40	20	9.751	9.917	9.834	40
30	9.634	9.955	9.678	30	30	9.753	9.916	9.837	30
40	9.637	9.955	9.682	20	40	9.755	9.915	9.840	20
50	9.639	9.954	9.685	10	50	9.757	9.914	9.843	10
26 0	9.642	9.954	9.688	154 0	35 0	9.759	9.913	9.845	145 0
10	9.644	9.953	9.691	50	10	9.760	9.912	9.848	50
20	9.647	9.952	9.695	40	20	9.762	9.912	9.851	40
30	9.650	9.952	9.698	30	30	9.764	9.911	9.853	30
40	9.652	9.951	9.701	20	40	9.766	9.910	9.856	20
50	9.655	9.951	9.704	10	50	9.767	9.909	9.859	10
27 0	9.657	9.950	9.707	153 0	36 0	9.769	9.908	9.861	144 0

Logarithmic Sines, Cosines, and Tangents, to three decimal figures.

Arc	Sin.	Cos.	Tan.	Arc	Arc	Sin.	Cos.	Tan.	Arc
36° 0′	9.769	9.908	9.861	144° 0′	45° 0′	9.849	9.849	10.000	135° 0′
10	9.771	9.907	9.864	50	10	9.851	9.848	10.003	50
20	9.773	9.906	9.867	40	20	9.852	9.847	10.005	40
30	9.774	9.905	9.869	30	30	9.853	9.846	10.008	30
40	9.776	9.904	9.872	20	40	9.854	9.844	10.010	20
50	9.778	9.903	9.874	10	50	9.856	9.843	10.013	10
37 0	9.779	9.902	9.877	143 0	46 0	9.857	9.842	10.015	134 0
10	9.781	9.901	9.880	50	10	9.858	9.840	10.018	50
20	9.783	9.900	9.882	40	20	9.859	9.839	10.020	40
30	9.784	9.899	9.885	30	30	9.861	9.838	10.023	30
40	9.786	9.898	9.888	20	40	9.862	9.836	10.025	20
50	9.788	9.898	9.890	10	50	9.863	9.835	10.028	10
38 0	9.789	9.897	9.893	142 0	47 0	9.864	9.834	10.030	133 0
10	9.791	9.896	9.895	50	10	9.865	9.832	10.033	50
20	9.793	9.895	9.898	40	20	9.866	9.831	10.035	40
30	9.794	9.894	9.901	30	30	9.868	9.830	10.038	30
40	9.796	9.893	9.903	20	40	9.869	9.828	10.040	20
50	9.797	9.892	9.906	10	50	9.870	9.827	10.043	10
39 0	9.799	9.890	9.908	141 0	48 0	9.871	9.826	10.046	132 0
10	9.800	9.889	9.911	50	10	9.872	9.824	10.048	50
20	9.802	9.888	9.914	40	20	9.873	9.823	10.051	40
30	9.804	9.887	9.916	30	30	9.874	9.821	10.053	30
40	9.805	9.886	9.919	20	40	9.876	9.820	10.056	20
50	9.807	9.885	9.921	10	50	9.877	9.818	10.058	10
40 0	9.808	9.884	9.924	140 0	49 0	9.878	9.817	10.061	131 0
10	9.810	9.883	9.926	50	10	9.879	9.815	10.063	50
20	9.811	9.882	9.929	40	20	9.880	9.814	10.066	40
30	9.813	9.881	9.931	30	30	9.881	9.813	10.068	30
40	9.814	9.880	9.934	20	40	9.882	9.811	10.071	20
50	9.815	9.879	9.937	10	50	9.883	9.810	10.074	10
41 0	9.817	9.878	9.939	139 0	50 0	9.884	9.808	10.076	130 0
10	9.818	9.877	9.942	50	10	9.885	9.807	10.079	50
20	9.820	9.876	9.944	40	20	9.886	9.805	10.081	40
30	9.821	9.874	9.947	30	30	9.887	9.804	10.084	30
40	9.823	9.873	9.949	20	40	9.888	9.802	10.086	20
50	9.824	9.872	9.952	10	50	9.889	9.800	10.089	10
42 0	9.826	9.871	9.954	138 0	51 0	9.890	9.799	10.092	129 0
10	9.827	9.870	9.957	50	10	9.892	9.797	10.094	50
20	9.828	9.869	9.960	40	20	9.893	9.796	10.097	40
30	9.830	9.868	9.962	30	30	9.894	9.794	10.099	30
40	9.831	9.866	9.965	20	40	9.895	9.793	10.102	20
50	9.832	9.865	9.967	10	50	9.896	9.791	10.105	10
43 0	9.834	9.864	9.970	137 0	52 0	9.897	9.789	10.107	128 0
10	9.835	9.863	9.972	50	10	9.898	9.788	10.110	50
20	9.836	9.862	9.975	40	20	9.898	9.786	10.112	40
30	9.838	9.861	9.977	30	30	9.899	9.784	10.115	30
40	9.839	9.859	9.980	20	40	9.900	9.783	10.118	20
50	9.840	9.858	9.982	10	50	9.901	9.781	10.120	10
44 0	9.842	9.857	9.985	136 0	53 0	9.902	9.779	10.123	127 0
10	9.843	9.856	9.987	50	10	9.903	9.778	10.126	50
20	9.844	9.854	9.990	40	20	9.904	9.776	10.128	40
30	9.846	9.853	9.992	30	30	9.905	9.774	10.131	30
40	9.847	9.852	9.995	20	40	9.906	9.773	10.133	20
50	9.848	9.851	9.997	10	50	9.907	9.771	10.136	10
45 0	9.849	9.849	10.000	135 0	54 0	9.908	9.769	10.139	126 0

TABLE LXXVII.

Logarithmic Sines, Cosines, and Tangents, to three decimal figures.

Arc	Sin.	Cos.	Tan.	Arc	Arc	Sin.	Cos.	Tan.	Arc
54° 0′	9.908	9.769	10.139	126° 0′	63° 0′	9.950	9.657	10.293	117° 0′
10	9.909	9.767	10.141	50	10	9.951	9.655	10.296	50
20	9.910	9.766	10.144	40	20	9.951	9.652	10.299	40
30	9.911	9.764	10.147	30	30	9.952	9.650	10.302	30
40	9.912	9.762	10.149	20	40	9.952	9.647	10.305	20
50	9.912	9.760	10.152	10	50	9.953	9.644	10.309	10
55 0	9.913	9.759	10.155	125 0	64 0	9.954	9.642	10.312	116 0
10	9.914	9.757	10.157	50	10	9.954	9.639	10.315	50
20	9.915	9.755	10.160	40	20	9.955	9.637	10.318	40
30	9.916	9.753	10.163	30	30	9.955	9.634	10.321	30
40	9.917	9.751	10.166	20	40	9.956	9.631	10.325	20
50	9.918	9.749	10.168	10	50	9.957	9.629	10.328	10
56 0	9.919	9.748	10.171	124 0	65 0	9.957	9.626	10.331	115 0
10	9.919	9.746	10.174	50	10	9.958	9.623	10.335	50
20	9.920	9.744	10.176	40	20	9.958	9.621	10.338	40
30	9.921	9.742	10.179	30	30	9.959	9.618	10.341	30
40	9.922	9.740	10.182	20	40	9.960	9.615	10.345	20
50	9.923	9.738	10.185	10	50	9.960	9.612	10.348	10
57 0	9.924	9.736	10.187	123 0	66 0	9.961	9.609	10.351	114 0
10	9.924	9.734	10.190	50	10	9.961	9.606	10.355	50
20	9.925	9.732	10.193	40	20	9.962	9.604	10.358	40
30	9.926	9.730	10.196	30	30	9.962	9.601	10.362	30
40	9.927	9.728	10.199	20	40	9.963	9.598	10.365	20
50	9.928	9.726	10.201	10	50	9.963	9.595	10.369	10
58 0	9.928	9.724	10.204	122 0	67 0	9.964	9.592	10.372	113 0
10	9.929	9.722	10.207	50	10	9.965	9.589	10.376	50
20	9.930	9.720	10.210	40	20	9.965	9.586	10.379	40
30	9.931	9.718	10.213	30	30	9.966	9.583	10.383	30
40	9.932	9.716	10.216	20	40	9.966	9.580	10.386	20
50	9.932	9.714	10.218	10	50	9.967	9.577	10.390	10
59 0	9.933	9.712	10.221	121 0	68 0	9.967	9.574	10.394	112 0
10	9.934	9.710	10.224	50	10	9.968	9.570	10.397	50
20	9.935	9.708	10.227	40	20	9.968	9.567	10.401	40
30	9.935	9.705	10.230	30	30	9.969	9.564	10.405	30
40	9.936	9.703	10.233	20	40	9.969	9.561	10.408	20
50	9.937	9.701	10.236	10	50	9.970	9.558	10.412	10
60 0	9.938	9.699	10.239	120 0	69 0	9.970	9.554	10.416	111 0
10	9.938	9.697	10.241	50	10	9.971	9.551	10.420	50
20	9.939	9.695	10.244	40	20	9.971	9.548	10.423	40
30	9.940	9.692	10.247	30	30	9.972	9.544	10.427	30
40	9.940	9.690	10.250	20	40	9.972	9.541	10.431	20
50	9.941	9.688	10.253	10	50	9.973	9.538	10.435	10
61 0	9.942	9.686	10.256	119 0	70 0	9.973	9.534	10.439	110 0
10	9.943	9.683	10.259	50	10	9.973	9.531	10.443	50
20	9.943	9.681	10.262	40	20	9.974	9.527	10.447	40
30	9.944	9.679	10.265	30	30	9.974	9.523	10.451	30
40	9.945	9.676	10.268	20	40	9.975	9.520	10.455	20
50	9.945	9.674	10.271	10	50	9.975	9.516	10.459	10
62 0	9.946	9.672	10.274	118 0	71 0	9.976	9.513	10.463	109 0
10	9.947	9.669	10.277	50	10	9.976	9.509	10.467	50
20	9.947	9.667	10.280	40	20	9.977	9.505	10.471	40
30	9.948	9.664	10.284	30	30	9.977	9.501	10.475	30
40	9.949	9.662	10.287	20	40	9.977	9.498	10.480	20
50	9.949	9.660	10.290	10	50	9.978	9.494	10.484	10
63 0	9.950	9.657	10.293	117 0	72 0	9.978	9.490	10.488	108 0

Logarithmic Sines, Cosines, and Tangents, to three decimal figures.

Arc	Sin.	Cos.	Tan.	Arc	Arc	Sin.	Cos.	Tan.	Arc
72° 0′	9.978	9.490	10.488	108° 0′	81° 0′	9.995	9.194	10.800	99° 0′
10	9.979	9.486	10.493	50	10	9.995	9.186	10.809	50
20	9.979	9.482	10.497	40	20	9.995	9.178	10.817	40
30	9.979	9.478	10.501	30	30	9.995	9.170	10.825	30
40	9.980	9.474	10.506	20	40	9.995	9.161	10.834	20
50	9.980	9.470	10.510	10	50	9.996	9.152	10.843	10
73 0	9.981	9.466	10.515	107 0	82 0	9.996	9.144	10.852	98 0
10	9.981	9.462	10.519	50	10	9.996	9.134	10.861	50
20	9.981	9.458	10.524	40	20	9.996	9.125	10.871	40
30	9.982	9.453	10.528	30	30	9.996	9.116	10.881	30
40	9.982	9.449	10.533	20	40	9.996	9.106	10.890	20
50	9.982	9.445	10.538	10	50	9.997	9.096	10.901	10
74 0	9.983	9.440	10.542	106 0	83 0	9.997	9.086	10.911	97 0
10	9.983	9.436	10.547	50	10	9.997	9.075	10.921	50
20	9.984	9.431	10.552	40	20	9.997	9.064	10.932	40
30	9.984	9.427	10.557	30	30	9.997	9.054	10.943	30
40	9.984	9.422	10.562	20	40	9.997	9.043	10.955	20
50	9.985	9.418	10.567	10	50	9.997	9.031	10.966	10
75 0	9.985	9.413	10.572	105 0	84 0	9.998	9.019	10.978	96 0
10	9.985	9.408	10.577	50	10	9.998	9.007	10.991	50
20	9.986	9.403	10.582	40	20	9.998	8.994	11.003	40
30	9.986	9.399	10.587	30	30	9.998	8.982	11.016	30
40	9.986	9.394	10.593	20	40	9.998	8.968	11.030	20
50	9.987	9.389	10.598	10	50	9.998	8.954	11.044	10
76 0	9.987	9.384	10.603	104 0	85 0	9.998	8.940	11.058	95 0
10	9.987	9.379	10.609	50	10	9.998	8.926	11.073	50
20	9.988	9.373	10.614	40	20	9.999	8.910	11.088	40
30	9.988	9.368	10.620	30	30	9.999	8.895	11.104	30
40	9.988	9.363	10.625	20	40	9.999	8.878	11.120	20
50	9.988	9.358	10.631	10	50	9.999	8.861	11.138	10
77 0	9.989	9.352	10.637	103 0	86 0	9.999	8.844	11.155	94 0
10	9.989	9.347	10.642	50	10	9.999	8.825	11.174	50
20	9.989	9.341	10.648	40	20	9.999	8.806	11.193	40
30	9.990	9.335	10.654	30	30	9.999	8.786	11.214	30
40	9.990	9.330	10.660	20	40	9.999	8.765	11.235	20
50	9.990	9.324	10.666	10	50	9.999	8.742	11.257	10
78 0	9.990	9.318	10.673	102 0	87 0	9.999	8.719	11.281	93 0
10	9.991	9.312	10.679	50	10	9.999	8.694	11.305	50
20	9.991	9.306	10.685	40	20	10.000	8.668	11.332	40
30	9.991	9.300	10.692	30	30	10.000	8.640	11.360	30
40	9.991	9.293	10.698	20	40	10.000	8.610	11.390	20
50	9.992	9.287	10.705	10	50	10.000	8.578	11.422	10
79 0	9.992	9.281	10.711	101 0	88 0	10.000	8.543	11.457	92 0
10	9.992	9.274	10.718	50	10	10.000	8.505	11.495	50
20	9.992	9.267	10.725	40	20	10.000	8.464	11.536	40
30	9.993	9.261	10.732	30	30	10.000	8.418	11.582	30
40	9.993	9.254	10.739	20	40	10.000	8.367	11.633	20
50	9.993	9.247	10.746	10	50	10.000	8.309	11.691	10
80 0	9.993	9.240	10.754	100 0	89 0	10.000	8.242	11.758	91 0
10	9.994	9.232	10.761	50	10	10.000	8.171	11.829	50
20	9.994	9.225	10.769	40	20	10.000	8.076	11.923	40
30	9.994	9.218	10.776	30	30	10.000	7.955	12.045	30
40	9.994	9.210	10.784	20	40	10.000	7.786	12.214	20
50	9.994	9.202	10.792	10	50	10.000	7.505	12.495	10
81 0	9.995	9.194	10.800	99 0	89 55	10.000	7.163	12.837	90 5

Logarithms to three decimal figures, with the corresponding Natural Numbers.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
100	.000	112	.050	126	.100	141	.150	158	.200
100	.001	112	.051	126	.101	142	.151	159	.201
100	.002	113	.052	126	.102	142	.152	159	.202
101	.003	113	.053	127	.103	142	.153	160	.203
101	.004	113	.054	127	.104	143	.154	160	.204
101	.005	114	.055	127	.105	143	.155	160	.205
101	.006	114	.056	128	.106	143	.156	161	.206
102	.007	114	.057	128	.107	144	.157	161	.207
102	.008	114	.058	128	.108	144	.158	161	.208
102	.009	115	.059	129	.109	144	.159	162	.209
102	.010	115	.060	129	.110	145	.160	162	.210
103	.011	115	.061	129	.111	145	.161	163	.211
103	.012	115	.062	129	.112	145	.162	163	.212
103	.013	116	.063	130	.113	146	.163	163	.213
103	.014	116	.064	130	.114	146	.164	164	.214
104	.015	116	.065	130	.115	146	.165	164	.215
104	.016	116	.066	131	.116	147	.166	164	.216
104	.017	117	.067	131	.117	147	.167	165	.217
104	.018	117	.068	131	.118	147	.168	165	.218
104	.019	117	.069	132	.119	148	.169	166	.219
105	.020	117	.070	132	.120	148	.170	166	.220
105	.021	118	.071	132	.121	148	.171	166	.221
105	.022	118	.072	132	.122	149	.172	167	.222
105	.023	118	.073	133	.123	149	.173	167	.223
106	.024	119	.074	133	.124	149	.174	167	.224
106	.025	119	.075	133	.125	150	.175	168	.225
106	.026	119	.076	134	.126	150	.176	168	.226
106	.027	119	.077	134	.127	150	.177	169	.227
107	.028	120	.078	134	.128	151	.178	169	.228
107	.029	120	.079	135	.129	151	.179	169	.229
107	.030	120	.080	135	.130	151	.180	170	.230
107	.031	121	.081	135	.131	152	.181	170	.231
108	.032	121	.082	136	.132	152	.182	171	.232
108	.033	121	.083	136	.133	152	.183	171	.233
108	.034	121	.084	136	.134	153	.184	171	.234
108	.035	122	.085	136	.135	153	.185	172	.235
109	.036	122	.086	137	.136	153	.186	172	.236
109	.037	122	.087	137	.137	154	.187	173	.237
109	.038	122	.088	137	.138	154	.188	173	.238
109	.039	123	.089	138	.139	155	.189	173	.239
110	.040	123	.090	138	.140	155	.190	174	.240
110	.041	123	.091	138	.141	155	.191	174	.241
110	.042	124	.992	139	.142	156	.192	175	.242
110	.043	124	.093	139	.143	156	.193	175	.243
111	.044	124	.094	139	.144	156	.194	175	.244
111	.045	124	.095	140	.145	157	.195	176	.245
111	.046	125	.096	140	.146	157	.196	176	.246
111	.047	125	.097	140	.147	157	.197	177	.247
112	.048	125	.098	141	.148	158	.198	177	.248
112	.049	126	.099	141	.149	158	.199	178	.249
112	.050	126	.100	141	.150	158	.200	178	.250

Logarithms to three decimal figures, with the corresponding Natural Numbers.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
178	.250	200	.300	224	.350	251	.400	282	.450
178	.251	200	.301	224	.351	252	.401	282	.451
179	.252	200	.302	225	.352	252	.402	283	.452
179	.253	201	.303	225	.353	253	.403	284	.453
179	.254	201	.304	226	.354	254	.404	284	.454
180	.255	202	.305	226	.355	254	.405	285	.455
180	.256	202	.306	227	.356	255	.406	286	.456
181	.257	203	.307	228	.357	255	.407	286	.457
181	.258	203	.308	228	.358	256	.408	287	.458
182	.259	204	.309	229	.359	256	.409	288	.459
182	.260	204	.310	229	.360	257	.410	288	.460
182	.261	205	.311	230	.361	258	.411	289	.461
183	.262	205	.312	230	.362	258	.412	290	.462
183	.263	206	.313	231	.363	259	.413	290	.463
184	.264	206	.314	231	.364	259	.414	291	.464
184	.265	207	.315	232	.365	260	.415	292	.465
185	.266	207	.316	232	.366	261	.416	292	.466
185	.267	207	.317	233	.367	261	.417	293	.467
185	.268	208	.318	233	.368	262	.418	294	.468
186	.269	208	.319	234	.369	262	.419	294	.469
186	.270	209	.320	234	.370	263	.420	295	.470
187	.271	209	.321	235	.371	264	.421	296	.471
187	.272	210	.322	236	.372	264	.422	296	.472
187	.273	210	.323	236	.373	265	.423	297	.473
188	.274	211	.324	237	.374	265	.424	298	.474
188	.275	211	.325	237	.375	266	.425	299	.475
189	.276	212	.326	238	.376	267	.426	299	.476
189	.277	212	.327	238	.377	267	.427	300	.477
190	.278	213	.328	239	.378	268	.428	301	.478
190	.279	213	.329	239	.379	269	.429	301	.479
191	.280	214	.330	240	.380	269	.430	302	.480
191	.281	214	.331	240	.381	270	.431	303	.481
191	.282	215	.332	241	.382	270	.432	303	.482
192	.283	215	.333	242	.383	271	.433	304	.483
192	.284	216	.334	242	.384	272	.434	305	.484
193	.285	216	.335	243	.385	272	.435	305	.485
193	.286	217	.336	243	.386	273	.436	306	.486
194	.287	217	.337	244	.387	274	.437	307	.487
194	.288	218	.338	244	.388	274	.438	308	.488
195	.289	218	.339	245	.389	275	.439	308	.489
195	.290	219	.340	245	.390	275	.440	309	.490
195	.291	219	.341	246	.391	276	.441	310	.491
196	.292	220	.342	247	.392	277	.442	310	.492
196	.293	220	.343	247	.393	277	.443	311	.493
197	.294	221	.344	248	.394	278	.444	312	.494
197	.295	221	.345	248	.395	279	.445	313	.495
198	.296	222	.346	249	.396	279	.446	313	.496
198	.297	222	.347	249	.397	280	.447	314	.497
199	.298	223	.348	250	.398	281	.448	315	.498
199	.299	223	.349	251	.399	281	.449	315	.499
200	.300	224	.350	251	.400	282	.450	316	.500

TABLE LXXVIII.

Logarithms to three decimal figures, with the corresponding Natural Numbers.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
316	.500	355	.550	398	.600	447	.650	501	.700
317	.501	356	.551	399	.601	448	.651	502	.701
318	.502	356	.552	400	.602	449	.652	504	.702
318	.503	357	.553	401	.603	450	.653	505	.703
319	.504	358	.554	402	.604	451	.654	506	.704
320	.505	359	.555	403	·605	452	.655	507	.705
321	.506	360	.556	404	·606	453	.656	508	.706
321	.507	361	.557	405	·607	454	.657	509	.707
322	.508	361	.558	406	·608	455	.658	511	.708
323	.509	362	.559	406	·609	456	.659	512	.709
324	.510	363	.560	407	.610	457	.660	513	.710
324	.511	364	.561	408	.611	458	.661	514	.711
325	.512	365	.562	409	.612	459	.662	515	.712
326	.513	366	.563	410	.613	460	.663	516	.713
327	.514	366	.564	411	.614	461	.664	518	.714
327	.515	367	.565	412	.615	462	.665	519	.715
328	.516	368	.566	413	.616	463	.666	520	.716
329	.517	369	.567	414	.617	465	.667	521	.717
330	.518	370	.568	415	.618	466	.668	522	.718
330	.519	371	.569	416	.619	467	.669	524	.719
331	.520	372	.570	417	.620	468	.670	525	.720
332	.521	372	.571	418	.621	469	.671	526	.721
333	.522	373	.572	419	.622	470	.672	527	.722
333	.523	374	.573	420	.623	471	.673	528	.723
334	.524	375	.574	421	.624	472	.674	530	.724
335	.525	376	.575	422	.625	473	.675	531	.725
336	.526	377	.576	423	.626	474	.676	532	.726
337	.527	378	.577	424	.627	475	.677	533	.727
337	.528	378	.578	425	.628	476	.678	535	.728
338	.529	379	.579	426	.629	478	.679	536	.729
339	.530	380	.580	427	.630	479	.680	537	.730
340	.531	381	.581	428	.631	480	.681	538	.731
340	.532	382	.582	429	.632	481	.682	540	.732
341	.533	383	.583	430	.633	482	.683	541	.733
342	.534	384	.584	431	.634	483	.684	542	.734
343	.535	385	.585	432	.635	484	.685	543	.735
344	.536	385	.586	433	.636	485	.686	544	.736
344	.537	386	.587	434	.637	486	.687	546	.737
345	.538	387	.588	435	.638	488	.688	547	.738
346	.539	388	.589	436	.639	489	.689	548	.739
347	.540	389	.590	437	.640	490	.690	550	.740
348	.541	390	.591	438	.641	491	.691	551	.741
348	.542	391	.592	439	.642	492	.692	552	.742
349	.543	392	.593	440	.643	493	.693	553	.743
350	.544	393	.594	441	.644	494	.694	555	.744
351	.545	394	.595	442	.645	495	.695	556	.745
352	.546	394	.596	443	.646	497	.696	557	.746
352	.547	395	.597	444	.647	498	.697	558	.747
353	.548	396	.598	445	.648	499	.698	560	.748
354	.549	397	.599	446	.649	500	.699	561	.749
355	.550	398	.600	447	.650	501	.700	562	.750

Logarithms to three decimal figures, with the corresponding Natural Numbers.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
562	.750	631	.800	708	.850	794	.900	891	.950
564	.751	632	.801	710	.851	796	.901	893	.951
565	.752	634	.802	711	.852	798	.902	895	.952
566	.753	635	.803	713	.853	800	.903	897	.953
568	.754	637	.804	714	.854	802	.904	899	.954
569	.755	638	.805	716	.855	804	.905	902	.955
570	.756	640	.806	718	.856	805	.906	904	.956
571	.757	641	.807	719	.857	807	.907	906	.957
573	.758	643	.808	721	.858	809	.908	908	.958
574	.759	644	.809	723	.859	811	.909	910	.959
575	.760	646	.810	724	.860	813	.910	912	.960
577	.761	647	.811	726	.861	815	.911	914	.961
578	.762	649	.812	728	.862	817	.912	916	.962
579	.763	650	.813	729	.863	818	.913	918	.963
581	.764	652	.814	731	.864	820	.914	920	.964
582	.765	653	.815	733	.865	822	.915	923	.965
583	.766	655	.816	735	.866	824	.916	925	.966
585	.767	656	.817	736	.867	826	.917	927	.967
586	.768	658	.818	738	.868	828	.918	929	.968
587	.769	659	.819	740	.869	830	.919	931	.969
589	.770	661	.820	741	.870	832	.920	933	.970
590	.771	662	.821	743	.871	834	.921	935	.971
592	.772	664	.822	745	.872	836	.922	938	.972
593	.773	665	.823	746	.873	838	.923	940	.973
594	.774	667	.824	748	.874	839	.924	942	.974
596	.775	668	.825	750	.875	841	.925	944	.975
597	.776	670	.826	752	.876	843	.926	946	.976
598	.777	671	.827	753	.877	845	.927	948	.977
600	.778	673	.828	755	.878	847	.928	951	.978
601	.779	675	.829	757	.879	849	.929	953	.979
603	.780	676	.830	759	.880	851	.930	955	.980
604	.781	678	.831	760	.881	853	.931	957	.981
605	.782	679	.832	762	.882	855	.932	959	.982
607	.783	681	.833	764	.883	857	.933	962	.983
608	.784	682	.834	766	.884	859	.934	964	.984
610	.785	684	.835	767	.885	861	.935	966	.985
611	.786	685	.836	769	.886	863	.936	968	.986
612	.787	687	.837	771	.887	865	.937	971	.987
614	.788	689	.838	773	.888	867	.938	973	.988
615	.789	690	.839	774	.889	869	.939	975	.989
617	.790	692	.840	776	.890	871	.940	977	.990
618	.791	693	.841	778	.891	873	.941	980	.991
619	.792	695	.842	780	.892	875	.942	982	.992
621	.793	697	.843	782	.893	877	.943	984	.993
622	.794	698	.844	783	.894	879	.944	986	.994
624	.795	700	.845	785	.895	881	.945	989	.995
625	.796	701	.846	787	.896	883	.946	991	.996
627	.797	703	.847	789	.897	885	.947	993	.997
628	.798	705	.848	791	.898	887	.948	995	.998
630	.799	706	.849	793	.899	889	.949	998	.999
631	.800	708	.850	794	.900	891	.950	1000	.1000

TABLE LXXIX.

Squares of Numbers to two decimal places.

No.	Sq.	No.	Sq.	No.	Sq.	No.	Sq.	No.	Sq.	No.	Sq.
0.00	0.00	0.60	0.36	1.20	1.44	1.80	3.24	2.40	5.76	3.00	9.00
0.01	0.00	0.61	0.37	1.21	1.46	1.81	3.28	2.41	5.81	3.01	9.06
0.02	0.00	0.62	0.38	1.22	1.49	1.82	3.31	2.42	5.86	3.02	9.12
0.03	0.00	0.63	0.40	1.23	1.51	1.83	3.35	2.43	5.90	3.03	9.18
0.04	0.00	0.64	0.41	1.24	1.54	1.84	3.39	2.44	5.95	3.04	9.24
0.05	0.00	0.65	0.42	1.25	1.56	1.85	3.42	2.45	6.00	3.05	9.30
0.06	0.00	0.66	0.44	1.26	1.59	1.86	3.46	2.46	6.05	3.06	9.36
0.07	0.00	0.67	0.45	1.27	1.61	1.87	3.50	2.47	6.10	3.07	9.42
0.08	0.01	0.68	0.46	1.28	1.64	1.88	3.53	2.48	6.15	3.08	9.49
0.09	0.01	0.69	0.48	1.29	1.66	1.89	3.57	2.49	6.20	3.09	9.55
0.10	0.01	0.70	0.49	1.30	1.69	1.90	3.61	2.50	6.25	3.10	9.61
0.11	0.01	0.71	0.50	1.31	1.72	1.91	3.65	2.51	6.30	3.11	9.67
0.12	0.01	0.72	0.52	1.32	1.74	1.92	3.69	2.52	6.35	3.12	9.73
0.13	0.02	0.73	0.53	1.33	1.77	1.93	3.72	2.53	6.40	3.13	9.80
0.14	0.02	0.74	0.55	1.34	1.80	1.94	3.76	2.54	6.45	3.14	9.86
0.15	0.02	0.75	0.56	1.35	1.82	1.95	3.80	2.55	6.50	3.15	9.92
0.16	0.03	0.76	0.58	1.36	1.85	1.96	3.84	2.56	6.55	3.16	9.99
0.17	0.03	0.77	0.59	1.37	1.88	1.97	3.88	2.57	6.60	3.17	10.05
0.18	0.03	0.78	0.61	1.38	1.90	1.98	3.92	2.58	6.66	3.18	10.11
0.19	0.04	0.79	0.62	1.39	1.93	1.99	3.96	2.59	6.71	3.19	10.18
0.20	0.04	0.80	0.64	1.40	1.96	2.00	4.00	2.60	6.76	3.20	10.24
0.21	0.04	0.81	0.66	1.41	1.99	2.01	4.04	2.61	6.81	3.21	10.30
0.22	0.05	0.82	0.67	1.42	2.02	2.02	4.08	2.62	6.86	3.22	10.37
0.23	0.05	0.83	0.69	1.43	2.04	2.03	4.12	2.63	6.92	3.23	10.43
0.24	0.06	0.84	0.71	1.44	2.07	2.04	4.16	2.64	6.97	3.24	10.50
0.25	0.06	0.85	0.72	1.45	2.10	3.05	4.20	2.65	7.02	3.25	10.56
0.26	0.07	0.86	0.74	1.46	2.13	2.06	4.24	2.66	7.08	3.26	10.63
0.27	0.07	0.87	0.76	1.47	2.16	2.07	4.28	2.67	7.13	3.27	10.69
0.28	0.08	0.88	0.77	1.48	2.19	2.08	4.33	2.68	7.18	3.28	10.76
0.29	0.08	0.89	0.79	1.49	2.22	2.09	4.37	2.69	7.24	3.29	10.82
0.30	0.09	0.90	0.81	1.50	2.25	2.10	4.41	2.70	7.29	3.30	10.89
0.31	0.10	0.91	0.83	1.51	2.28	2.11	4.45	2.71	7.34	3.31	10.96
0.32	0.10	0.92	0.85	1.52	2.31	2.12	4.49	2.72	7.40	3.32	11.02
0.33	0.11	0.93	0.86	1.53	2.34	2.13	4.54	2.73	7.45	3.33	11.09
0.34	0.12	0.94	0.88	1.54	2.37	2.14	4.58	2.74	7.51	3.34	11.16
0.35	0.12	0.95	0.90	1.55	2.40	2.15	4.62	2.75	7.56	3.35	11.22
0.36	0.13	0.96	0.92	1.56	2.43	2.16	4.67	2.76	7.62	3.36	11.29
0.37	0.14	0.97	0.94	1.57	2.46	2.17	4.71	2.77	7.67	3.37	11.36
0.38	0.14	0.98	0.96	1.58	2.50	2.18	4.75	2.78	7.73	3.38	11.42
0.39	0.15	0.99	0.98	1.59	2.53	2.19	4.80	2.79	7.78	3.39	11.49
0.40	0.16	1.00	1.00	1.60	2.56	2.20	4.84	2.80	7.84	3.40	11.56
0.41	0.17	1.01	1.02	1.61	2.59	2.21	4.88	2.81	7.90	3.41	11.63
0.42	0.18	1.02	1.04	1.62	2.62	2.22	4.93	2.82	7.95	3.42	11.70
0.43	0.18	1.03	1.06	1.63	2.66	2.23	4.97	2.83	8.01	3.43	11.76
0.44	0.19	1.04	1.08	1.64	2.69	2.24	5.02	2.84	8.07	3.44	11.83
0.45	0.20	1.05	1.10	1.65	2.72	2.25	5.06	2.85	8.12	3.45	11.90
0.46	0.21	1.06	1.12	1.66	2.76	2.26	5.11	2.86	8.18	3.46	11.97
0.47	0.22	1.07	1.14	1.67	2.79	2.27	5.15	2.87	8.24	3.47	12.04
0.48	0.23	1.08	1.17	1.68	2.82	2.28	5.20	2.88	8.29	3.48	12.11
0.49	0.24	1.09	1.19	1.69	2.86	2.29	5.24	2.89	8.35	3.49	12.18
0.50	0.25	1.10	1.21	1.70	2.89	2.30	5.29	2.90	8.41	3.50	12.25
0.51	0.26	1.11	1.23	1.71	2.92	2.31	5.34	2.91	8.47	3.51	12.32
0.52	0.27	1.12	1.25	1.72	2.96	2.32	5.38	2.92	8.53	3.52	12.39
0.53	0.28	1.13	1.28	1.73	2.99	2.33	5.43	2.93	8.58	3.53	12.46
0.54	0.29	1.14	1.30	1.74	3.03	2.34	5.48	2.94	8.64	3.54	12.53
0.55	0.30	1.15	1.32	1.75	3.06	2.35	5.52	2.95	8.70	3.55	12.60
0.56	0.31	1.16	1.35	1.76	3.10	2.36	5.57	2.96	8.76	3.56	12.67
0.57	0.32	1.17	1.37	1.77	3.13	2.37	5.62	2.97	8.82	3.57	12.74
0.58	0.34	1.18	1.39	1.78	3.17	2.38	5.66	2.98	8.88	3.58	12.82
0.59	0.35	1.19	1.42	1.79	3.20	2.39	5.71	2.99	8.94	3.59	12.89
0.60	0.36	1.20	1.44	1.80	3.24	2.40	5.76	3.00	9.00	3.60	12.96

Squares of Numbers to two decimal places.

No.	Square	No.	Square	No.	Square	No.	Square	No.	Square
3.60	12.96	4.20	17.64	4.80	23.04	5.40	29.16	6.00	36.00
3.61	13.03	4.21	17.72	4.81	23.14	5.41	29.27	6.01	36.12
3.62	13.10	4.22	17.81	4.82	23.23	5.42	29.38	6.02	36.24
3.63	13.18	4.23	17.89	4.83	23.33	5.43	23.48	6.03	36.36
3.64	13.25	4.24	17.98	4.84	23.43	5.44	29.59	6.04	36.48
3.65	13.32	4.25	18.06	4.85	23.52	5.45	29.70	6.05	36.60
3.66	13.40	4.26	18.15	4.86	23.62	5.46	29.81	6.06	36.72
3.67	13.47	4.27	18.23	4.87	23.72	5.47	29.92	6.07	36.84
3.68	13.54	3.28	18.32	4.88	23.81	5.48	30.03	6.08	36.97
3.69	13.62	4.29	18.40	4.89	23.91	5.49	30.14	6.09	37.09
3.70	13.69	4.30	18.49	4.90	24.01	5.50	30.25	6.10	37.21
3.71	13.76	4.31	18.58	4.91	24.11	5.51	30.36	6.11	37.33
3.72	13.84	4.32	18.66	4.92	24.21	5.52	30.47	6.12	37.45
3.73	13.91	4.33	18.75	4.93	24.30	5.53	30.58	6.13	37.58
3.74	13.99	4.34	18.84	4.94	24.40	5.54	30.69	6.14	37.70
3.75	14.06	4.35	18.92	4.95	24.50	5.55	30.80	6.15	37.82
3.76	14.14	4.36	19.01	4.96	24.60	5.56	30.91	6.16	37.95
3.77	14.21	4.37	19.10	4.97	24.70	5.57	31.02	6.17	38.07
3.78	14.29	4.38	19.18	4.98	24.80	5.58	31.14	6.18	38.19
3.79	14.36	4.39	19.27	4.99	24.90	5.59	31.25	6.19	38.32
3.80	14.44	4.40	19.36	5.00	25.00	5.60	31.36	6.20	38.44
3.81	14.52	4.41	19.45	5.01	25.10	5.61	31.47	6.21	38.56
3.82	14.59	4.42	19.54	5.02	25.20	5.62	31.58	6.22	38.69
3.83	14.67	4.43	19.62	5.03	25.30	5.63	31.70	6.23	38.81
3.84	14.75	4.44	19.71	5.04	25.40	5.64	31.81	6.24	38.94
3.85	14.82	4.45	19.80	5.05	25.50	5.65	31.92	6.25	39.06
3.86	14.90	4.46	19.89	5.06	25.60	5.66	32.04	6.26	39.19
3.87	14.98	4.47	19.98	5.07	25.70	5.67	32.15	6.27	39.31
3.88	15.05	4.48	20.07	5.08	25.81	5.68	32.26	6.28	39.44
3.89	15.13	4.49	20.16	5.09	25.91	5.69	32.38	6.29	39.56
3.90	15.21	4.50	20.25	5.10	26.01	5.70	32.49	6.30	39.69
3.91	15.29	4.51	20.34	5.11	26.11	5.71	32.60	6.31	39.82
3.92	15.37	4.52	20.43	5.12	26.21	5.72	32.72	6.32	39.94
3.93	15.44	4.53	20.52	5.13	26.32	5.73	32.83	6.33	40.07
3.94	15.52	4.54	20.61	5.14	26.42	5.74	32.95	6.34	40.20
3.95	15.60	4.55	20.70	5.15	26.52	5.75	33.06	6.35	40.32
3.96	15.68	4.56	20.79	5.16	26.63	5.76	33.18	6.36	40.45
3.97	15.76	4.57	20.88	5.17	26.73	5.77	33.29	6.37	40.58
3.98	15.84	4.58	20.98	5.18	26.83	5.78	33.41	6.38	40.70
3.99	15.92	4.59	21.07	5.19	26.94	5.79	33.52	6.39	40.83
4.00	16.00	4.60	21.16	5.20	27.04	5.80	33.64	6.40	40.96
4.01	16.08	4.61	21.25	5.21	27.14	5.81	33.76	6.41	41.09
4.02	16.16	4.62	21.34	5.22	27.25	5.82	33.87	6.42	41.22
4.03	16.24	4.63	21.44	5.23	27.35	5.83	33.99	6.43	41.34
4.04	16.32	4.64	21.53	5.24	27.46	5.84	34.11	6.44	41.47
4.05	16.40	4,65	21.62	5.25	27.56	5.85	34.22	6.45	41.60
4.06	16.48	4.66	21.72	5.26	27.67	5.86	34.34	6.46	41.73
4.07	16.56	4.67	21.81	5.27	27.77	5.87	34.46	6.47	41.86
4.08	16.65	4.68	21.90	5.28	27.88	5.88	34.57	6.48	41.99
4.09	16.73	4.69	22.00	5.29	27.98	5.89	34.69	6.49	42.12
4.10	16.81	4.70	22.09	5.30	28.09	5.90	34.81	6.50	42.25
4.11	16.89	4.71	22.18	5.31	28.20	5.91	34.93	6.51	42.38
4.12	16.97	4.72	22.28	5.32	28.30	5.92	35.05	6.52	42.51
4.13	17.06	4.73	22.37	5.33	28.41	5.93	35.16	6.53	42.64
4.14	17.14	4.74	22.47	5.34	28.52	5.94	35.28	6.54	42.77
4.15	17.22	4.75	22.56	5.35	28.62	5.95	35.40	6.55	42.90
4.16	17.31	4.76	22.66	5.36	28.73	5.96	35.52	6.56	43.03
4.17	17.39	4.77	22.75	5.37	28.84	5.97	35.64	6.57	43.16
4.18	17.47	4.78	22.85	5.38	28.94	5.98	35.76	6.58	43.30
4.19	17.56	4.79	22.94	5.39	29.05	5.99	35.88	6.59	43.43
4.20	17.64	4.80	23.04	5.40	29.16	6.00	36.00	6.60	43.56

Squares of Numbers to two decimal places.

No.	Square	No.	Square	No.	Square	No.	Square
6.60	43.56	7.20	51.84	7.80	60.84	8.40	70.56
6.61	43.69	7.21	51.98	7.81	61.00	8.41	70.73
6.62	43.82	7.22	52.13	7.82	61.15	8.42	70.90
6.63	43.96	7.23	52.27	7.83	61.31	8.43	71.06
6.64	44.09	7.24	52.42	7.84	61.47	8.44	71.23
6.65	44.22	7.25	52.56	7.85	61.62	8.45	71.40
6.66	44.36	7.26	52.71	7.86	61.78	8.46	71.57
6.67	44.49	7.27	52.85	7.87	61.94	8.47	71.74
6.68	44.62	7.28	53.00	7.88	62.09	8.48	71.91
6.69	44.76	7.29	53.14	7.89	62.25	8.49	72.08
6.70	44.89	7.30	53.29	7.90	62.41	8.50	72.25
6.71	45.02	7.31	53.44	7.91	62.57	8.51	72.42
6.72	45.16	7.32	53.58	7.92	62.73	8.52	72.59
6.73	45.29	7.33	53.73	7.93	62.88	8.53	72.76
6.74	45.43	7.34	59.88	7.94	63.04	8.54	72.93
6.75	45.56	7.35	54.02	7.95	63.20	8.55	73.10
6.76	45.70	7.36	54.17	7.96	63.36	8.56	73.27
6.77	45.83	7.37	54.32	7.97	63.52	8.57	73.44
6.78	45.97	7.38	54.46	7.98	63.68	8.58	73.62
6.79	46.10	7.39	54.61	7.99	63.84	8.59	78.79
6.80	46.24	7.40	54.76	8.00	64.00	8.60	73.96
6.81	46.38	7.41	54.91	8.01	64.16	8.61	74.13
6.82	46.51	7.42	55.06	8.02	64.32	8.62	74.30
6.83	46.65	7.43	55.20	8.03	64.48	8.63	74.48
6.84	46.79	7.44	55.35	8.04	64.64	8.64	74.65
6.85	46.92	7.45	55.50	8.05	64.80	8.65	74.82
6.86	47.06	7.46	55.65	8.06	64.96	8.66	75.00
6.87	47.20	7.47	55.80	8.07	65.12	8.67	75.17
6.88	47.33	7.48	55.95	8.08	65.29	8.68	75.34
6.89	47.47	7.49	56.10	8.09	65.45	8.69	75.52
6.90	47.61	7.50	56.25	8.10	65.61	8.70	75.69
6.91	47.75	7.51	56.40	8.11	65.77	8.71	75.86
6.92	47.89	7.52	56.55	8.12	65.93	8.72	76.04
6.93	48.02	7.53	56.70	8.13	66.10	8.73	76.21
6.94	48.16	7.54	56.85	8.14	66.26	8.74	76.39
6.95	48.30	7.55	57.00	8.15	66.42	8.75	76.56
6.96	48.44	7.56	57.15	8.16	66.59	8.76	76.74
6.97	48.58	7.57	57.30	8.17	66.75	8.77	76.91
6.98	48.72	7.58	57.46	8.18	66.91	8.78	77.09
6.99	48.86	7.59	57.61	8.19	67.08	8.79	77.26
7.00	49.00	7.60	57.76	8.20	67.24	8.80	77.44
7.01	49.14	7.61	57.91	8.21	67.40	8.81	77.62
7.02	49.28	7.62	58.06	8.22	67.57	8.82	77.79
7.03	49.42	7.63	58.22	8.23	67.73	8.83	77.97
7.04	49.56	7.64	58.37	8.24	67.90	8.84	78.15
7.05	49.70	7.65	58.52	8.25	68.06	8.85	78.32
7.06	49.84	7.66	58.68	8.26	68.23	8.86	78.50
7.07	49.98	7.67	58.83	8.27	68.39	8.87	78.68
7.08	50.13	7.68	58.98	8.28	68.56	8.88	78.85
7.09	50.27	7.69	59.14	8.29	68.72	8.89	79.03
7.10	50.41	7.70	59.29	8.30	68.89	8.90	79.21
7.11	50.55	7.71	59.44	8.31	69.06	8.91	79.39
7.12	50.69	7.72	59.60	8.32	69.22	8.92	79.57
7.13	50.84	7.73	59.75	8.33	69.39	8.93	79.74
7.14	50.98	7.74	59.91	8.34	69.56	8.94	79.92
7.15	51.12	7.75	60.06	8.35	69.72	8.95	80.10
7.16	51.27	7.76	60.22	8.36	69.89	8.96	80.28
7.17	51.41	7.77	60.37	8.37	70.06	8.97	80.46
7.18	51.55	7.78	60.53	8.38	70.22	8.98	80.64
7.19	51.70	7.79	60.68	8.39	70.39	8.99	80.82
7.20	51.84	7.80	60.84	8.40	70.56	9.00	81.00

Squares of Numbers from 0.01 *to* 6.60.

No.	Square.	No.	Square.	No.	Square.	No.	Square.	No.	Square.
0.00	0.0000	0.50	0.2500	1.00	1.0000	1.50	2.2500	2.00	4.0000
0.01	0.0001	0.51	0.2601	1.01	1.0201	1.51	2.2801	2.01	4.0401
0.02	0.0004	0.52	0.2704	1.02	1.0404	1.52	2.3104	2.02	4.0804
0.03	0.0009	0.53	0.2809	1.03	1.0609	1.53	2.3409	2.03	4.1209
0.04	0.0016	0.54	0.2916	1.04	1.0816	1.54	2.3716	2.04	4.1616
0.05	0.0025	0.55	0.3025	1.05	1.1025	1.55	2.4025	2.05	4.2025
0.06	0.0036	0.56	0.3136	1.06	1.1236	1.56	2.4336	2.06	4.2436
0.07	0.0049	0.57	0.3249	1.07	1.1449	1.57	2.4649	2.07	4.2849
0.08	0.0064	0.58	0.3364	1.08	1.1664	1.58	2.4964	2.08	4.3264
0.09	0.0081	0.59	0.3481	1.09	1.1881	1.59	2.5281	2.09	4.3681
0.10	0.0100	0.60	0.3600	1.10	1.2100	1.60	2.5600	2.10	4.4100
0.11	0.0121	0.61	0.3721	1.11	1.2321	1.61	2.5921	2.11	4.4521
0.12	0.0144	0.62	0.3844	1.12	1.2544	1.62	2.6244	2.12	4.4944
0.13	0.0169	0.63	0.3969	1.13	1.2769	1.63	2.6569	2.13	4.5369
0.14	0.0196	0.64	0.4096	1.14	1.2996	1.64	2.6896	2.14	4.5796
0.15	0.0225	0 65	0.4225	1.15	1.3225	1.65	2.7225	2.15	4.6225
0.16	0.0256	0.66	0.4356	1.16	1.3456	1.66	2.7556	2.16	4.6656
0.17	0.0289	0.67	0.4489	1.17	1.3689	1.67	2.7889	2.17	4.7089
0.18	0.0324	0.68	0.4624	1.18	1.3924	1.68	2.8224	2.18	4.7524
0.19	0.0361	0.69	0.4761	1.19	1.4161	1.69	2.8561	2.19	4.7961
0.20	0.0400	0.70	0.4900	1.20	1.4400	1.70	2.8900	2.20	4.8400
0.21	0.0441	0.71	0.5041	1.21	1.4641	1.71	2.9241	2.21	4.8841
0.22	0.0484	0.72	0.5184	1.22	1.4884	1.72	2.9584	2.22	4.9284
0.23	0.0529	0.73	0.5329	1.23	1.5129	1.73	2.9929	2.23	4.9729
0.24	0.0576	0.74	0.5476	1.24	1.5376	1.74	3.0276	2.24	5.0176
0.25	0.0625	0.75	0.5625	1.25	1.5625	1.75	3.0625	2.25	5.0625
0.26	0.0676	0.76	0.5776	1.26	1.5876	1.76	3.0976	2.26	5.1076
0.27	0.0729	0.77	0.5929	1.27	1.6129	1.77	3.1329	2.27	5.1529
0.28	0.0784	0.78	0.6084	1.28	1.6384	1.78	3.1684	2.28	5.1984
0.29	0.0841	0.79	0.6241	1.29	1.6641	1.79	3.2041	2.29	5.2441
0.30	0.0900	0.80	0.6400	1.30	1.6900	1.80	3.2400	2.30	5.2900
0.31	0.0961	0.81	0.6561	1.31	1.7161	1.81	3.2761	2.31	5.3361
0.32	0.1024	0.82	0.6724	1.32	1.7424	1.82	3.3124	2.32	5.3824
0.33	0.1089	0.83	0.6889	1.33	1.7689	1.83	3.3489	2.33	5.4289
0.34	0.1156	0.84	0.7056	1.34	1.7956	1.84	3.3856	2.34	5.4756
0.35	0.1225	0.85	0.7225	1.35	1.8225	1.85	3.4225	2.35	5.5225
0.36	0.1296	0.86	0.7396	1.36	1.8496	1.86	3.4596	2.36	5.5696
0.37	0.1369	0.87	0.7569	1.37	1.8769	1.87	3.4969	2.37	5.6169
0.38	0.1444	0.88	0.7744	1.38	1.9044	1.88	3.5344	2.38	5.6644
0.39	0.1521	0.89	0.7921	1.39	1.9321	1.89	3.5721	2.39	5.7121
0.40	0.1600	0.90	0.8100	1.40	1.9600	1.90	3.6100	2.40	5.7600
0.41	0.1681	0.91	0.8281	1.41	1.9881	1.91	3.6481	2.41	5.8081
0.42	0.1764	0.92	0.8464	1.42	2.0164	1.92	3.6864	2.42	5.8564
0.43	0.1849	0.93	0.8649	1.43	2.0449	1.93	3.7249	2.43	5.9049
0.44	0.1936	0.94	0.8836	1.44	2.0736	1.94	3.7636	2.44	5.9536
0.45	0.2025	0.95	0.9025	1.45	2.1025	1.95	3.8025	2.45	6.0025
0.46	0.2116	0.96	0.9216	1.46	2.1316	1.96	3.8416	2.46	6.0516
0.47	0.2209	0.97	0.9409	1.47	2.1609	1.97	3.8809	2.47	6.1009
0.48	0.2304	0.98	0.9604	1.48	2.1904	1.98	3.9204	2.48	6.1504
0.49	0.2401	0.99	0.9801	1.49	2.2201	1.99	3.9601	2.49	6.2001
0.50	0.2500	1.00	1.0000	1.50	2.2500	2.00	4.0000	2.50	6.2500

Squares of Numbers from 0.01 *to* 6.60.

No.	Square.	No.	Square.	No.	Square.	No.	Square.
2.50	6.2500	3.00	9.0000	3.50	12.2500	4.00	16.0000
2.51	6.3001	3.01	9.0601	3.51	12.3201	4.01	16.0801
2.52	6.3504	3.02	9.1204	3.52	12.3904	4.02	16.1604
2.53	6.4009	3.03	9.1809	3.53	12.4609	4.03	16.2409
2.54	6.4516	3.04	9.2416	3.54	12.5316	4.04	16.3216
2.55	6.5025	3.05	9.3025	3.55	12.6025	4.05	16.4025
2.56	6.5536	3.06	9.3636	3.56	12.6736	4.06	16.4836
2.57	6.6049	3.07	9.4249	3.57	12.7449	4.07	16.5649
2.58	6.6564	3.08	9.4864	3.58	12.8164	4.08	16.6464
2.59	6.7081	3.09	9.5481	3.59	12.8881	4.09	16.7281
2.60	6.7600	3.10	9.6100	3.60	12.9600	4.10	16.8100
2.61	6.8121	3.11	9.6721	3.61	13.0321	4.11	16.8921
2.62	6.8644	3.12	9.7344	3.62	13.1044	4.12	16.9744
2.63	6.9169	3.13	9.7969	3.63	13.1769	4.13	17.0569
2.64	6.9696	3.14	9.8596	3.64	13.2496	4.14	17.1396
2.65	7.0225	3.15	9.9225	3.65	13.3225	4.15	17.2225
2.66	7.0756	3.16	9.9856	3.66	13.3956	4.16	17.3056
2.67	7.1289	3.17	10.0489	3.67	13.4689	4.17	17.3889
2.68	7.1824	3.18	10.1124	3.68	13.5424	4.18	17.4724
2.69	7.2361	3.19	10.1761	3.69	13.6161	4.19	17.5561
2.70	7.2900	3.20	10.2400	3.70	13.6900	4.20	17.6400
2.71	7.3441	3.21	10.3041	3.71	13.7641	4.21	17.7241
2.72	7.3984	3.22	10.3684	3.72	13.8384	4.22	17.8084
2.73	7.4529	3.23	10.4329	3.73	13.9129	4.23	17.8929
2.74	7.5076	3.24	10.4976	3.74	13.9876	4.24	17.9776
2.75	7.5625	3.25	10.5625	3.75	14.0625	4.25	18.0625
2.76	7.6176	3.26	10.6276	3.76	14.1376	4.26	18.1476
2.77	7.6729	3.27	10.6929	3.77	14.2129	4.27	18.2329
2.78	7.7284	3.28	10.7584	3.78	14.2884	4.28	18.3184
2.79	7.7841	3.29	10.8241	3.79	14.3641	4.29	18.4041
2.80	7.8400	3.30	10.8900	3.80	14.4400	4.30	18.4900
2.81	7.8961	3.31	10.9561	3.81	14.5161	4.31	18.5761
2.82	7.9524	3.32	11.0224	3.82	14.5924	4.32	18.6624
2.83	8.0089	3.33	11.0889	3.83	14.6689	4.33	18.7489
2.84	8.0656	3.34	11.1556	3.84	14.7456	4.34	18.8356
2.85	8.1225	3.35	11.2225	3.85	14.8225	4.35	18.9225
2.86	8.1796	3.36	11.2896	3.86	14.8996	4.36	19.0096
2.87	8.2369	3.37	11.3569	3.87	14.9769	4.37	19.0969
2.88	8.2944	3.38	11.4244	3.88	15.0544	4.38	19.1844
2.89	8.3521	3.39	11.4921	3.89	15.1321	4.39	19.2721
2.90	8.4100	3.40	11.5600	3.90	15.2100	4.40	19.3600
2.91	8.4681	3.41	11.6281	3.91	15.2881	4.41	19.4481
2.92	8.5264	3.42	11.6964	3.92	15.3664	4.42	19.5364
2.93	8.5849	3.43	11.7649	3.93	15.4449	4.43	19.6249
2.94	8.6436	3.44	11.8336	3.94	15.5236	4.44	19.7136
2.95	8.7025	3.45	11.9025	3.95	15.6025	4.45	19.8025
2.96	8.7616	3.46	11.9716	3.96	15.6816	4.46	19.8916
2.97	8.8209	3.47	12.0409	3.97	15.7609	4.47	19.9809
2.98	8.8804	3.48	12.1104	3.98	15.8404	4.48	20.0704
2.99	8.9401	3.49	12.1801	3.99	15.9201	4.49	20.1601
3.00	9.0000	3.50	12.2500	4.00	16.0000	4.50	20.2500

Squares of Numbers from 0.01 *to* 6.60

No.	Square	No.	Square	No.	Square	No.	Square
4.50	20.2500	5.00	25.0000	5.50	30.2500	6.00	36.0000
4.51	20.3401	5.01	25.1001	5.51	30.3601	6.01	36.1201
4.52	20.4304	5.02	25.2004	5.52	30.4704	6.02	36.2404
4.53	20.5209	5.03	25.3009	5.53	30.5809	6.03	36.3609
4.54	20.6116	5.04	25.4016	5.54	30.6916	6.04	36.4816
4.55	20.7025	5.05	25.5025	5.55	30.8025	6.05	36.6025
4.56	20.7936	5.06	25.6036	5.56	30.9136	6.06	36.7236
4.57	20.8849	5.07	25.7049	5.57	31.0249	6.07	36.8449
4.58	20.9764	5.08	25.8064	5.58	31.1364	6.08	36.9664
4.59	21.0681	5.09	25.9081	5.59	31.2481	6.09	37.0881
4.60	21.1600	5.10	26.0100	5.60	31.3600	6.10	37.2100
4.61	21.2521	5.11	26.1121	5.61	31.4721	6.11	37.3321
4.62	21.3444	5.12	26.2144	5.62	31.5844	6.12	37.4544
4.63	21.4369	5.13	26.3169	5.63	31.6969	6.13	37.5769
4.64	21.5296	5.14	26.4196	5.64	31.8096	6.14	37.6996
4.65	21.6225	5.15	26.5225	5.65	31.9225	6.15	37.8225
4.66	21.7156	5.16	26.6256	5.66	32.0356	6.16	37.9456
4.67	21.8089	5.17	26.7289	5.67	32.1489	6.17	38.0689
4.68	21.9024	5.18	26.8324	5.68	32.2624	6.18	38.1924
4.69	21.9961	5.19	26.9361	5.69	32.3761	6.19	38.3161
4.70	22.0900	5.20	27.0400	5.70	32.4900	6.20	38.4400
4.71	22.1841	5.21	27.1441	5.71	32.6041	6.21	38.5641
4.72	22.2784	5.22	27.2484	5.72	32.7184	6.22	38.6884
4.73	22.3729	5.23	27.3529	5.73	32.8329	6.23	38.8129
4.74	22.4676	5.24	27.4576	5.74	32.9476	6.24	38.9376
4.75	22.5625	5.25	27.5625	5.75	33.0625	6.25	39.0625
4.76	22.6576	5.26	27.6676	5.76	33.1776	6.26	39.1876
4.77	22.7529	5.27	27.7729	5.77	33.2929	6.27	39.3129
4.78	22.8484	5.28	27.8784	5.78	33.4084	6.28	39.4384
4.79	22.9441	5.29	27.9841	5.79	33.5241	6.29	39.5641
4.80	23.0400	5.30	28.0900	5.80	33.6400	6.30	39.6900
4.81	23.1361	5.31	28.1961	5.81	33.7561	6.31	39.8161
4.82	23.2324	5.32	28.3024	5.82	33.8724	6.32	39.9424
4.83	23.3289	5.33	28.4089	5.83	33.9889	6.33	40.0689
4.84	23.4256	5.34	28.5156	5.84	34.1056	6.34	40.1956
4.85	23.5225	5.35	28.6225	5.85	34.2225	6.35	40.3225
4.86	23.6196	5.36	28.7296	5.86	34.3396	6.36	40.4496
4.87	23.7169	5.37	28.8369	5.87	34.4569	6.37	40.5769
4.88	23.8144	5.38	28.9444	5.88	34.5744	6.38	40.7044
4.89	23.9121	5.39	29.0521	5.89	34.6921	6.39	40.8321
4.90	24.0100	5.40	29.1600	5.90	34.8100	6.40	40.9600
4.91	24.1081	5.41	29.2681	5.91	34.9281	6.41	41.0881
4.92	24.2064	5.42	29.3764	5.92	35.0464	6.42	41.2164
4.93	24.3049	5.43	29.4849	5.93	35.1649	6.43	41.3449
4.94	24.4036	5.44	29.5936	5.94	35.2836	6.44	41.4736
4.95	24.5025	5.45	29.7025	5.95	35.4025	6.45	41.6025
4.96	24.6016	5.46	29.8116	5.96	35.5216	6.46	41.7316
4.97	24.7009	5.47	29.9209	5.97	35.6409	6.47	41.8609
4.98	24.8004	5.48	30.0304	5.98	35.7604	6.48	41.9904
4.99	24.9001	5.49	30.1401	5.99	35.8801	6.49	42.1201
5.00	25.0000	5.50	30.2500	6.00	36.0000	6.50	42.2500

For the Aberration of a Star in Right Ascension and Declination.

ARGUMENT. Sun's true longitude.

	0^s	VI^s	I^s	VII^s	II^s	$VIII^s$	
	Log. a −	x +	Log. a −	x +	Log. a −	x +	
0°	1.2690	0° 0′	1.2790	2° 11′	1.2977	2° 6′	30′
1	1.2690	0 5	1.2796	2 14	1.2983	2 3	29
2	1.2691	0 11	1.2802	2 16	1.2988	2 0	28
3	1.2692	0 16	1.2808	2 18	1.2993	1 57	27
4	1.2692	0 22	1.2815	2 20	1.2998	1 54	26
5	1.2693	0 27	1.2821	2 21	1.3003	1 51	25
6	1.2695	0 32	1.2827	2 23	1.3008	1 47	24
7	1.2696	0 37	1.2834	2 24	1.3012	1 44	23
8	1.2698	0 43	1.2840	2 25	1.3017	1 40	22
9	1.2700	0 48	1.2847	2 26	1.3021	1 36	21
10	1.2703	0 53	1.2853	2 27	1.3025	1 32	20
11	1.2705	0 58	1.2860	2 28	1.3028	1 28	19
12	1.2708	1 3	1.2866	2 28	1.3032	1 24	18
13	1.2711	1 8	1.2873	2 28	1.3036	1 20	17
14	1.2714	1 12	1.2879	2 28	1.3039	1 16	16
15	1.2718	1 17	1.2886	2 28	1.3042	1 11	15
16	1.2721	1 22	1.2892	2 28	1.3045	1 7	14
17	1.2725	1 26	1.2899	2 27	1.3048	1 3	13
18	1.2729	1 30	1.2905	2 27	1.3050	0 58	12
19	1.2733	1 34	1.2912	2 26	1.3053	0 53	11
20	1.2738	1 39	1.2918	2 25	1.3055	0 49	10
21	1.2742	1 42	1.2924	2 24	1.3057	0 44	9
22	1.2747	1 46	1.2931	2 22	1.3059	0 39	8
23	1.2752	1 50	1.2938	2 21	1.3060	0 34	7
24	1.2757	1 53	1.2944	2 19	1.3061	0 30	6
25	1.2762	1 57	1.2949	2 17	1.3063	0 25	5
26	1.2768	2 0	1.2956	2 15	1.3064	0 20	4
27	1.2773	2 3	1.2961	2 13	1.3064	0 15	3
28	1.2779	2 6	1.2966	2 11	1.3065	0 10	2
29	1.2785	2 9	1.2972	2 8	1.3065	0 5	1
30	1.2790	2 11	1.2977	2 6	1.3065	0 0	0
	− Log. a	− x	− Log. a	− x	− Log. a	− x	
	V^s	XI^s	IV^s	X^s	III^s	IX^s	

For the Aberration of a Star in Right Ascension and Declination.

Argument. Sun's Longitude, more or less the Star's Declination.

	0s —	Is —	IIs —	IIIs +	IVs +	Vs +	
°	″	″	″	″	″	″	°
0	4.03	3.49	2.02	0.00	2.02	3.49	30
1	4.03	3.46	1.95	0.07	2.08	3.53	29
2	4.03	3.42	1.89	0.14	2.14	3.56	28
3	4.03	3.38	1.83	0.21	2.20	3.59	27
4	4.02	3.34	1.77	0.28	2.26	3.63	26
5	4.02	3.30	1.70	0.35	2.31	3.66	25
6	4.01	3.26	1.64	0.42	2.37	3.68	24
7	4.00	3.22	1.58	0.49	2.43	3.71	23
8	3.99	3.18	1.51	0.56	2.48	3.74	22
9	3.98	3.13	1.45	0.63	2.54	3.77	21
10	3.97	3.09	1.38	0.70	2.59	3.79	20
11	3.96	3.04	1.31	0.77	2.65	3.81	19
12	3.95	3.00	1.25	0.84	2.70	3.84	18
13	3.93	2.95	1.18	0.91	2.75	3.86	17
14	3.91	2.90	1.11	0.98	2.80	3.88	16
15	3.90	2.85	1.04	1.04	2.85	3.90	15
16	3.88	2.80	0.98	1.11	2.90	3.91	14
17	3.86	2.75	0.91	1.18	2.95	3.93	13
18	3.84	2.70	0.84	1.25	3.00	3.95	12
19	3.81	2.65	0.77	1.31	3.04	3.96	11
20	3.79	2.59	0.70	1.38	3.09	3.97	10
21	3.77	2.54	0.63	1.45	3.13	3.98	9
22	3.74	2.48	0.56	1.51	3.18	3.99	8
23	3.71	2.43	0.49	1.58	3.22	4.00	7
24	3.68	2.37	0.42	1.64	3.26	4.01	6
25	3.66	2.31	0.35	1.70	3.30	4.02	5
26	3.63	2.26	0.28	1.77	3.34	4.02	4
27	3.59	2.20	0.21	1.83	3.38	4.03	3
28	3.56	2.14	0.14	1.89	3.42	4.03	2
29	3.53	2.08	0.07	1.95	3.46	4.03	1
30	3.49	2.02	0.00	2.02	3.49	4.03	0
	— XIs	— Xs	— IXs	+ VIIIs	+ VIIs	+ VIs	

For the Nutation in Right Ascension and Declination.

Argument. Mean Longitude of Moon's Ascending Node.

	0^{s}	VI^{s}	I^{s}	VII^{s}	II^{s}	$VIII^{s}$	
	Log. *b* —	B —	Log. *b* —	B —	Log. *b* —	B —	
0°	0.9844	0° 0′	0.9588	6°45′	0.8960	7°48′	30°
1	0.9844	0 15	0.9571	6 54	0.8939	7 40	29
2	0.9843	0 31	0.9554	7 3	0.8917	7 32	28
3	0.9842	0 46	9.9536	7 12	0.8896	7 23	27
4	0.9840	1 1	0.9518	7 20	0.8875	7 14	26
5	0.9837	1 16	0.9500	7 28	0.8854	7 4	25
6	0.9834	1 32	0.9481	7 36	0.8834	6 53	24
7	0.9830	1 47	0.9462	7 43	0.8814	6 42	23
8	0.9825	2 2	0.9442	7 49	0.8795	6 29	22
9	0.9821	2 17	0.9422	7 55	0.8776	6 17	21
10	0.9815	2 31	0.9402	8 1	0.8758	6 3	20
11	0.9809	2 46	0.9382	8 6	0.8740	5 49	19
12	0.9802	3 1	0.9361	8 10	0.8723	5 35	18
13	0.9795	3 15	0.9340	8 14	0.8707	5 20	17
14	0.9787	3 29	0.9318	8 17	0.8692	5 4	16
15	0.9779	3 45	0.9297	8 20	0.8677	4 48	15
16	0.9770	3 57	0.9275	8 23	0.8663	4 31	14
17	0.9760	4 11	0.9253	8 24	0.8650	4 14	13
18	0.9750	4 24	0.9231	8 25	0.8637	3 56	12
19	0.9739	4 37	0.9208	8 25	0.8625	3 38	11
20	0.9728	4 50	0.9186	8 25	0.8615	3 20	10
21	0.9716	5 3	0.9163	8 24	0.8605	3 1	9
22	0.9704	5 16	0.9140	8 23	0.8596	2 41	8
23	0.9691	5 28	0.9118	8 21	0.8588	2 22	7
24	0.9678	5 40	0.9095	8 18	0.8582	2 2	6
25	0.9664	5 51	0.9072	8 15	0.8576	1 42	5
26	0.9650	6 3	0.9050	8 11	0.8571	1 22	4
27	0.9635	6 14	0.9027	8 6	0.8568	1 2	3
28	0.9620	6 24	0.9005	8 1	0.8565	0 41	2
29	0.9604	6 35	0.8983	7 55	0.8563	0 21	1
30	0.9588	6 45	0.8960	7 48	0.8563	0 0	0
	— Log. *b*	+ B	— Log. *b*	+ B	— Log. *b*	+ B	
	V^{s}	XI^{s}	IV^{s}	X^{s}	III^{s}	IX^{s}	

For the Nutation in Right Ascension and Declination.

Argument. **Mean Longitude of Moon's Ascending Node.**

	0^s —	I^s —	II^s —	III^s —	IV^s —	V^s —	
°	″	″	″	″	″	″	°
0	0.00	8.27	14.33	16.54	14.33	8.27	30
1	0.29	8.52	14.47	16.54	14.18	8.02	29
2	0.58	8.77	14.61	16.53	14.03	7.77	28
3	0.87	9.01	14.74	16.52	13.83	7.51	27
4	1.15	9.25	14.87	16.50	13.72	7.25	26
5	1.44	9.49	14.99	16.48	13.55	6.99	25
6	1.75	9.72	15.11	16.45	13.38	6.73	24
7	2.02	9.96	15.23	16.42	13.21	6.46	23
8	2.30	10.19	15.34	16.38	13.04	6.20	22
9	2.59	10.41	15.45	16.34	12.86	5.93	21
10	2.87	10.63	15.55	16.29	12.67	5.66	20
11	3.16	10.85	15.64	16.24	12.49	5.39	19
12	3.44	11.07	15.73	16.18	12.30	5.11	18
13	3.72	11.28	15.82	16.12	12.10	4.84	17
14	4.00	11.49	15.90	16.05	11.90	4.56	16
15	4.28	11.70	15.98	15.98	11.70	4.28	15
16	4.56	11.90	16.05	15.90	11.49	4.00	14
17	4.84	12.10	16.12	15.82	11.28	3.72	13
18	5.11	12.30	16.18	15.73	11.07	3.44	12
19	5.39	12.49	16.24	15.64	10.85	3.16	11
20	5.66	12.67	16.29	15.55	10.63	2.87	10
21	5.93	12.86	16.34	15.45	10.41	2.59	9
22	6.20	13.04	16.38	15.34	10.19	2.30	8
23	6.46	13.21	16.42	15.23	9.96	2.02	7
24	6.73	13.38	16.45	15.11	9.72	1.75	6
25	6.99	13.55	16.48	14.99	9.49	1.44	5
26	7.25	13.72	16.50	14.87	9.25	1.15	4
27	7.51	13.83	16.52	14.74	9.01	0.87	3
28	7.77	14.03	16.53	14.61	8.77	0.58	2
29	8.02	14.18	16.54	14.47	8.52	0.29	1
30	8.27	14.33	16.54	14.33	8.27	0.00	0
	+ XI^s	+ X^s	+ IX^s	+ $VIII^s$	+ VII^s	+ VI^s	

Plate 1.

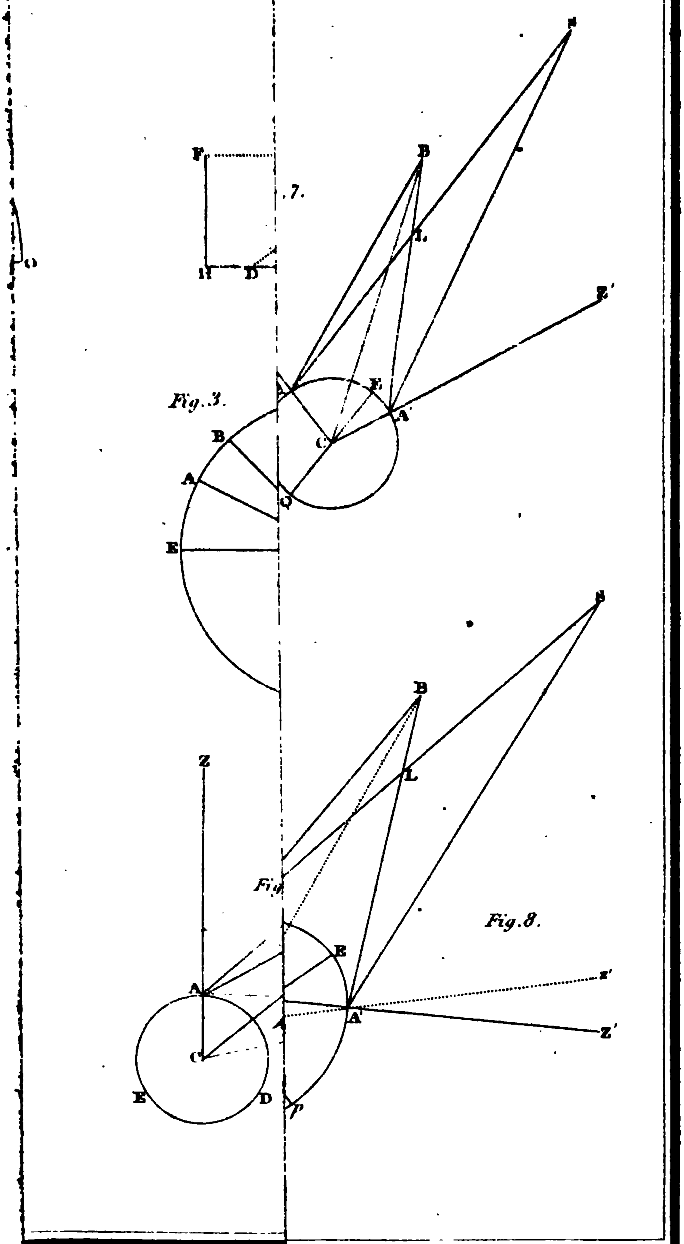

Fig. 3.

Fig. 8.

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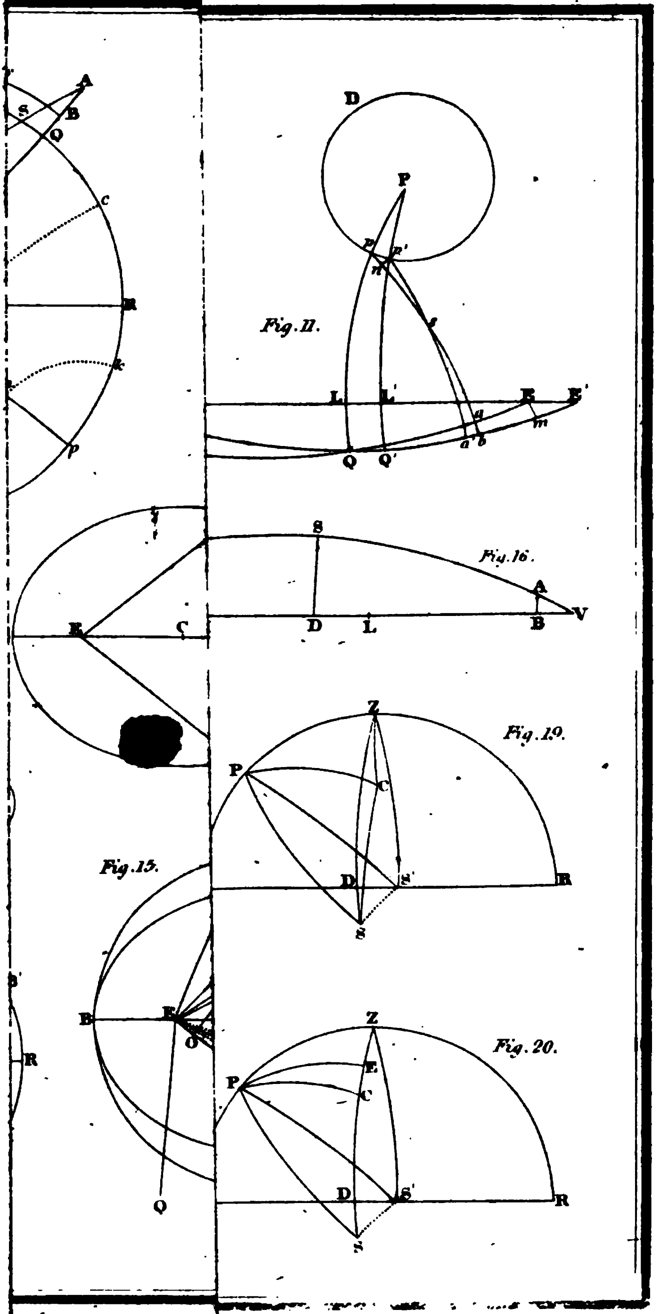
Plate 2.



D
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Fig. 11.
L
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S
B
Q
c
R
k
p
S
Fig. 16.
A
V
D
L
B
E
C
Z
Fig. 19.
P
C
D
S'
R
S
Fig. 15.
B'
B
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O
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Q
Z
Fig. 20.
P
E
C
D
S'
R
S

Plate 3.

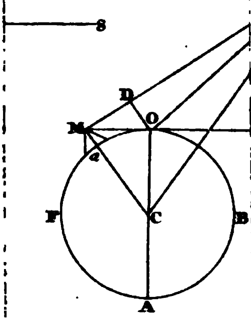


Fig. 28.

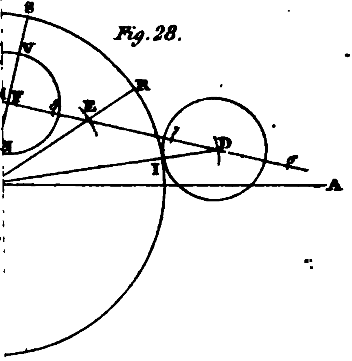


Fig. 25.

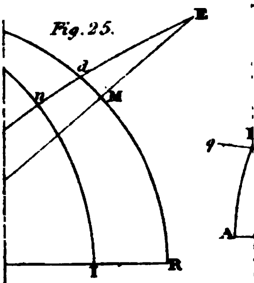


Fig. 30.

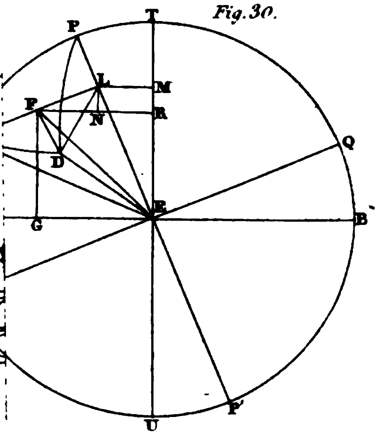
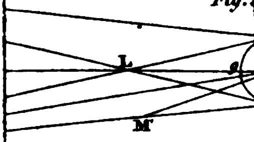
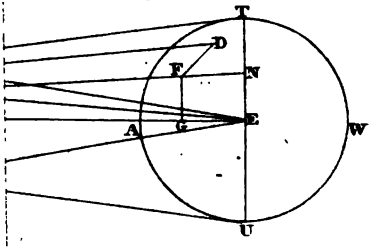
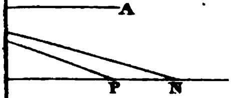


Fig.







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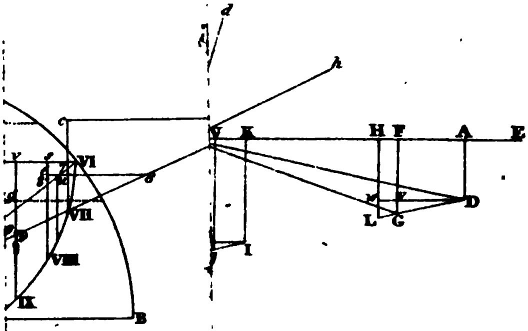
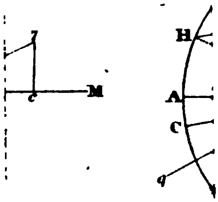




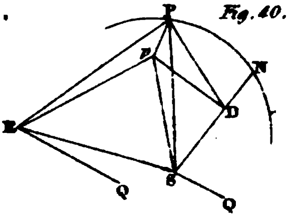
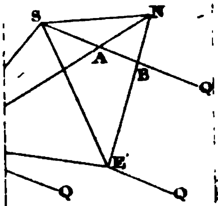


Fig. 40.



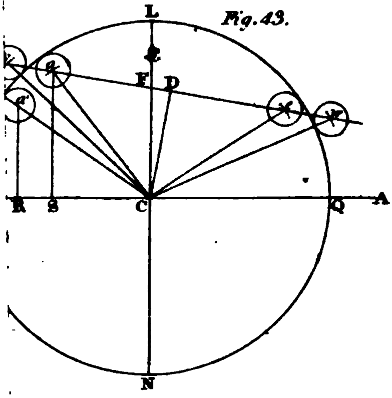


Fig. 43.

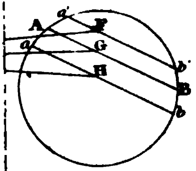


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Plate 5.

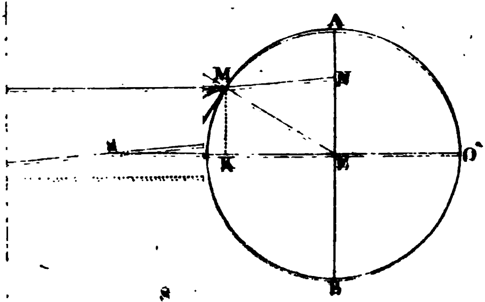


Fig.

Fig. 54.

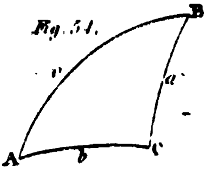


Fig. 47.

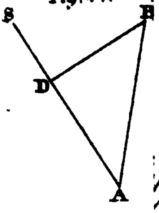
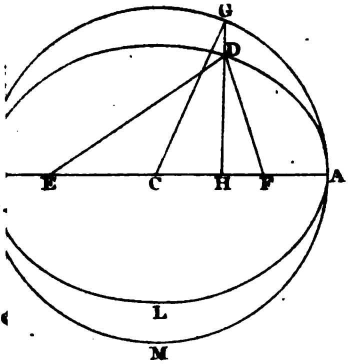


Fig. 55.



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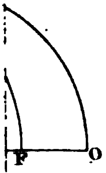

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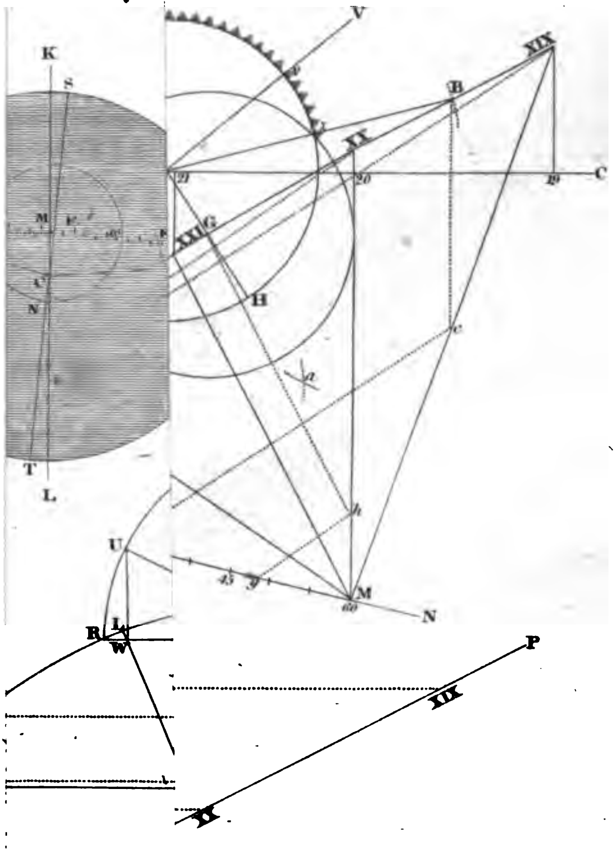

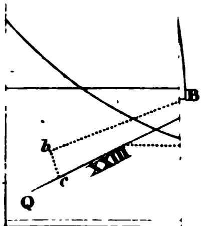

Fig. 59.

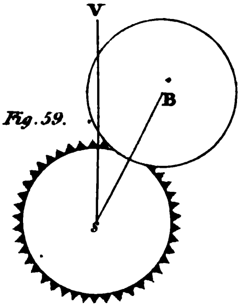

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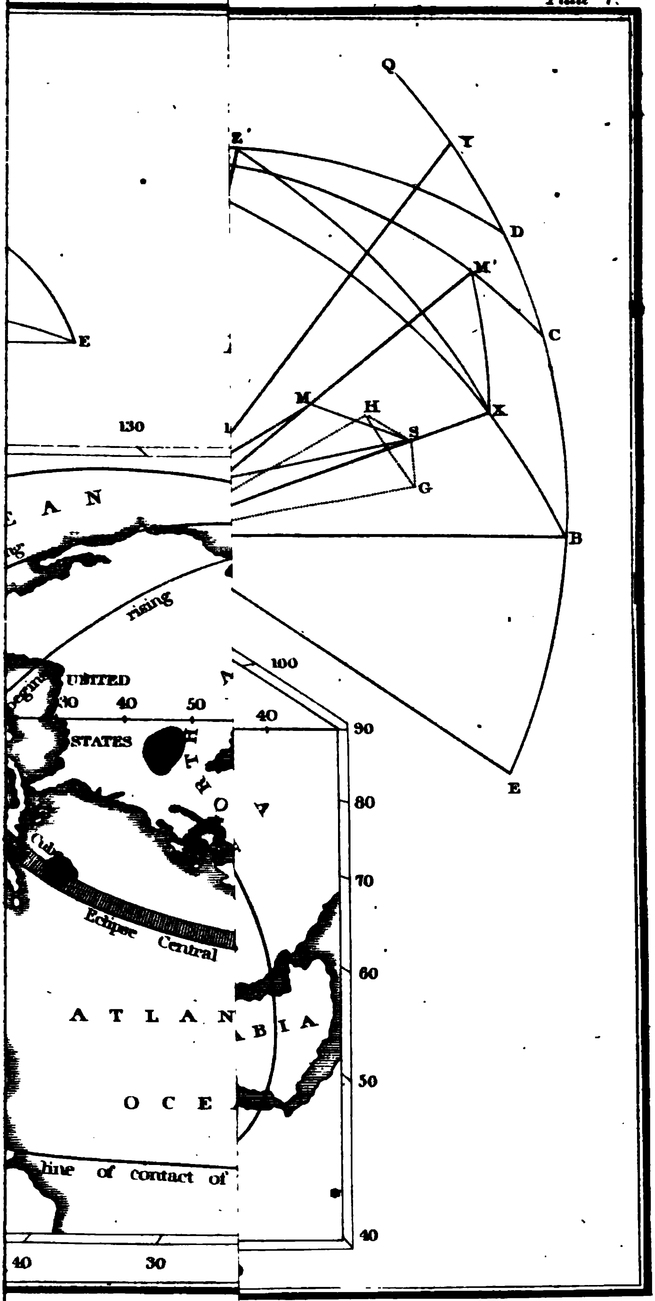
Plate 7.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